Distribution analysis of stresses across the stretching edge of die body and bending radius of deforming roll during profiling and drawing of cylindrical workpiece

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Abstract
There considered questions concerning determination of stress field during form alteration of wheel crowns from steel cylindrical works at the first pass of radial- rotary profiling. There also fulfilled process analysis of drawing process of cylindrical pieces made of sheet metal. Determination of stresses at bending radius of profiling roll while shaping and on stretching edge of die body while drawing is of great interest, as coupling areas of various profile elements of crown are the most loaded and on the value of meridian stress depends the possibility of getting of high-quality piece per stretching pass. Results of theoretical research allow to analyze stresses, which arise while deforming, and determine group of technological and constructional parameters of profiling and stretching process, which affect greatly the value and distribution of normal and shear stresses.
Key words: CROWN, PROFILING, WORKPIECE, BENDING RADIUS, DEFORMATION, DRAWING.

Introduction

Extent of drawing process of cylindrical pieces is conditioned by enough level of knowledge of phenomena, occurring in deformation zone, great amount of experimental data and also information recommendations, which allow to consider all the peculiarities of means and designing of passes, basing on them.

One of the determining process parameters of profiling of wheel crowns is minimum allowable thickness of metal in the places of radius blends of profile, that is why the thickness of workpiece should be chosen such as this condition was fulfilled and not to assume weighing of crowns, having overestimated it initially.

Analysis of latest researches and literature

Several works are devoted to theoretical analysis of stretching process of cylindrical pieces [1, 2, 3, 4], where for determination of strain-stress state there solved simplified equilibrium equations together with the plasticity criterion according to the theory of maximum shearing stresses, which gives an opportunity for authors to determine analytically stress distribution at any workpiece area, reveal technological factors of this method, which influence on the getting of qualitative half-finished product. As the researches concerning theory and experiment of radial-rotary profiling [5-7] show, and also calculations for strength of wheel crown while operation [8,9], the most interesting are the publications directed to setting of stress and deformation field, affecting bending radius of the wheel in the process of radial-rotary profiling and in the ready workpiece during its loading by external forces.

Aim of research, problem statement

The aim of work is to settle stress field at stretching edge of die body during drawing of cylindrical pieces made of sheet metal without pressing of piece flange and also stresses on the bending radius of half-finished product during radial-rotary profiling.

Materials of research

To solve the task concerning determination of stress components at bending radii of pieces, they judged from the hypotheses and assumptions of momentless technical theory of shells of rotation [10]. Half-finished product profile during shaping and drawing was cut by cross sections perpendicular and parallel to the piece axis to obtain simple geometrical bodies.

Let us use for solution of this task general equilibrium equations of rotation shells. We denote through a the radius of cross section and R – as the distance from the axis to the center of section. Let us introduce curvilinear coordinates θ and φ; θ is azimuth angle in the plane of meridian, φ – angle in the plane of parallel circle. One of the main radii of curve, corresponding to meridian, coincides with cross section radius and is equal to $R_2 = a$; the second, of parallel circle one, according to Meusnier theorem is equal to $R_1 = \frac{r}{\sin\theta}$, where r – is the distance from symmetry axis to some point of median surface [12], $r = R + a\sin\theta = a\frac{(1 + k\sin\theta)}{k}$, where $k = \frac{a}{R}$, (fig. 1).

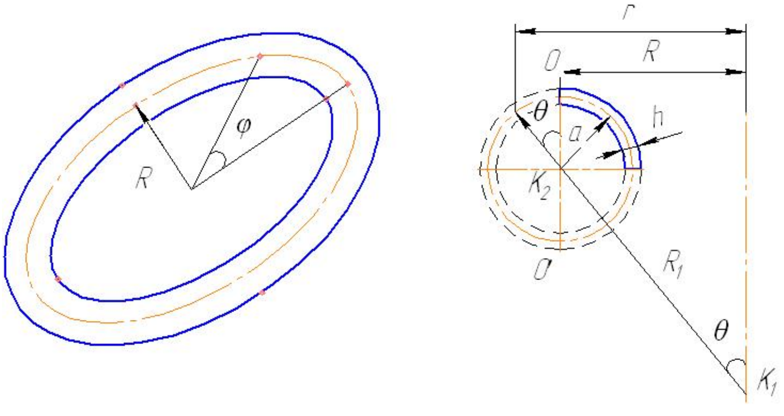

Figure 1. Toroidal shell with agreed notations

In general equilibrium equations it is not considered bending and rotational moments, shear strains and also surface loading, than they in projections for tangent lines θ and φ will look as follows [12]

$$a\frac{\partial N_1}{\partial\varphi}+2a\cos\theta\cdot S+\frac{a}{k}(1+k\sin\varphi)\frac{\partial S}{\partial\theta}=0,$$

$$a\cos\theta\cdot N_2+\frac{a}{k}(1+k\sin\varphi)\frac{\partial N_2}{\partial\theta}+a\frac{\partial S}{\partial\varphi}-a\cos\theta\cdot N_1=0,$$

$$\frac{k\sin\theta\cdot N_1}{a(1+k\sin\theta)}+\frac{N_2}{a}=0.$$

(1)

where N_1, N_2, S – inner meridonial, tangential and shearing forces.

Solution for the process of radial-rotary profiling. Deforming of work piece during this process is fulfilled by the pair of rotating rolls having necessary profile, where the part of half-ready product is gradually introduced into deformation zone due to friction between workpiece and tool, where deformation zone makes 2-3% from side surface of cylindrical piece, i.e. there is local deformation. To solve the equations (1) in this case is impossible with the usage of assumption about axial symmetry of deformation process. There also accepted the following assumptions: work material is plastic-rigid, isotrope in deformation zone and elastic beyond its limits, we may neglect friction forces in tangential and meridian directions, sizes of deformation zone correspond to the tool size, which contacts with work material.

Let us express from the third simultaneous equations N_2 and substitute it into the second equilibrium equation, carrying out simple changing, take the solution of equations as function of one θ. After their substitution into simultaneous equation (1), we will obtain ordinary differential equation to determine these functions. Adding and subtracting these equations, we will obtain two ordinary differential equations, each of which has one unknown. Integration of equations we conduct with the help of variables separation and substitution of $u=tg\frac{\theta}{2}$, herein the integrals are being rationalized and suppressing unnecessary computation, we will obtain

$$N_1=\left[\left(tg^2\frac{\theta}{2}+4ktg\frac{\theta}{2}+1\right)^{4k}\left(\frac{e^{-A}C_1}{2}-\frac{e^{-B}C_2}{2}\right)\right]\cos\varphi,$$

$$S=\left[\left(tg^2\frac{\theta}{2}+4ktg\frac{\theta}{2}+1\right)^{4k}\left(\frac{e^{-A}C_1}{2}+\frac{e^{-B}C_2}{2}\right)\right]\sin\varphi.$$

(2)

where $A=\frac{4k}{\sqrt{1-4k^2}}arctg\frac{tg\frac{\theta}{2}+2k}{\sqrt{1-4k^2}}(4k+1)$;

$$B=\frac{4k}{\sqrt{1-4k^2}}arctg\frac{tg\frac{\theta}{2}+2k}{\sqrt{1-4k^2}}(4k+3).$$

Left end of toroidal shell is interfited with cylinder, this means that the following equations should be true: $N_1=N_x$, $S_c=S_T$, where N_x, S_c - solutions for meridonial and tangential forces obtained for cylindrical workpiece [13].

To define the value of tangential stresses in deformation zone, we will use transition conditions into elastic condition according to hypothesis of maximum shearing stresses

$$\sigma_y=\sigma_x-\sqrt{\sigma_S^2-4\tau^2}. \qquad (3)$$

Beyond the deformation zone, we may find tangential stresses from the third equation of the system (1).

Distribution of stresses at bending radius of profiling roll according to the results of expression (2) depends on the first and second multiplier and also on the boundary condition in the zones of coupling. As meridional stresses are the most dangerous from the point of view of getting of qualitative pieces, which lead to excessive thinning of profile, buckling failure and even appearance of cracks and damage of workpiece, than let us analyze the first equation of the system (2).

The first and second multipliers of the equation consider the change of stresses during deformation at bending radius of the profile, integration constants – the value of meridional stresses at cylinder area. With the increase of θ value the first and second multipliers grow and meridional stresses increase. The first multiplier of the expression (2), when increasing θ from 90 to 175º (at $\theta=180^o$ – tangent does not exist), changes from $\left(tg^2\frac{\theta}{2}+4ktg\frac{\theta}{2}+1\right)^{4k}\approx 1$ to $\left(tg^2\frac{\theta}{2}+4ktg\frac{\theta}{2}+1\right)^{4k}\approx 3{,}5$, it affects significantly the distribution of stresses at rounded area. The result obtained corresponds to the location sheet-metal stamping theory, where meridional stresses increase while increasing angle of contact of matrix bending radius during drawing. The second multiplier - $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)$ with the increase of θ value, reduces, as for $\theta=0^o$, $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=1$;

$\theta=5^o$, $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=0{,}92$, for $\theta=20^o$, $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=0{,}88$; $\theta=90,^o$ $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=0{,}73$. Besides, the value $k=\frac{a}{R}$, characterizing the ratio of roll bending radius to the radius of workpiece, is not a constant value, but changes dynamically.

For upper rounded area *R* increases, as there occur distribution of some pieces, *a* – remains constant, *k* – reduces. For wheel crown *W*12x24, for which the dimensions of sidewall are equal to: $l=380\ mm$; $D=568\ mm$; $h=4{,}5\ mm$ according to factory engineering process after the first pass of profiling

$D_c=610\ mm$; $D_p=556\ mm$; $a_c=15mm$; $a_p=30mm$

where D_c, D_p – diameters of cap and pass of the crown respectively; a_c, a_p – bending radii of crown profile from the cap to the wall of adjusting pass and from the wall of adjusting pass to adjusting pass respectively. For two border-line cases of distribution area we will obtain: at $\theta=90^o$, $k=\frac{15}{305}=0{,}049$, $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=1$, as at the input into deformation area the values *A*, *B* and A_1, B_1 are equal; at $\theta=175^o$, $k=\frac{15}{305-15}=0{,}051$, $\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=0{,}76$. There observed also great reduction of the level of meridional stresses at the output from rounded area. But in the second border-line case, when $\theta=175^o$ the first multiplier is equal to *3.5* and we will finally obtain

$$\left(tg^2\frac{\theta}{2}+4ktg\frac{\theta}{2}+1\right)^{4k}\times\left(\frac{e^{-A}}{2e^{-A_1}}+\frac{e^{-B}}{2e^{-B_1}}\right)=3{,}5\times0{,}76\approx2{,}7$$

These calculations show that with the increase of range of bending radius of deforming roll by workpiece metal in the zone of distribution, the value of meridional stresses 2.7 times increases till the end of deformation process, when workpiece material will fully cover bending radius of lower roll.

For the area of workpiece pressing we will have similar dependencies, both for the distribution area, where meridional stresses increase and at the end of pass of radial- rotary profiling will be 5.2 times greater than at cylindrical area of half-finished product profile for the wheel crown *W*12x24.

Solution for the drawing of cylindrical workpiece.

Let us consider that deformation is axisymmetric, and this means that the components of equation (1) disappear, containing $\partial\varphi$, and also shear load, than we will have:

$$N_2 a\cos\theta+\frac{\partial N_2}{\partial\theta}\frac{a}{k}\left(1+k\sin\theta\right)-N_1 a\cos\theta=0 \qquad (4)$$

For solution of obtained equation, we will use plasticity equation according to the hypothesis of maximum shearing stresses:

$$\sigma_t-\sigma_\theta=\sigma_s \qquad (5)$$

where σ_t – maximum tensile meridional stress;

σ_θ – minimum main normal tangential stresses;

σ_s – material yield point.

Joint solution of equations taking into account that $\sigma=\frac{N}{h}$ gives differential equation:

$$d\sigma_t=\sigma_s\frac{k\cos\theta}{1+k\sin\phi}d\theta, \qquad (6)$$

where $N_1=h\sigma_t$, $N_2=h\sigma_\theta$.

Integrated equation (6) with the usage of boundary condition $\theta=0$ $\sigma_s=\sigma_s\ln\frac{R_з}{R_Д+s+r_m}$ (at the boundary of flange zones and beginning of formation of toroidal area) allows to obtain the formula for formation of value σ_s in the function of angle θ by metal wrap of matrix crown

$$\sigma_t=\sigma_s\left(\ln\frac{R_w}{R_s+s+r_m}+\frac{\frac{r_m}{R_s+s+r_m}\sin\theta}{1+\frac{r_m}{R_s+s+r_m}\sin\theta}\right), \qquad (7)$$

where R_w – radius of workpiece;

R_s – radius of stretchable workpiece;

s – workpiece thickness;

r_m – radius of matrix lead-in shoulder (fig.2).

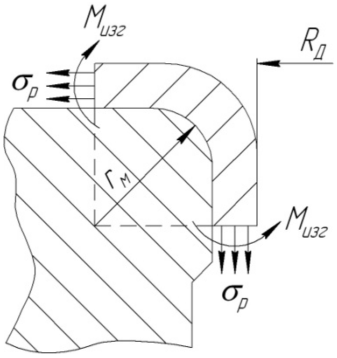

Figure 2. Workpiece area at matrix bending radius

As the workpiece is deformed axisymmetric, than the summand, containing $sin\varphi$ may be omitted, than the formula (7) is simplified:

$$\sigma_t = \sigma_s \left(\ln \frac{R_w}{R_s + s + r_m} + \frac{r_m}{R_s + s + r_m} \sin\theta \right) \quad (8)$$

From plasticity equation we may find the expression for determination of the value of tangential stresses σ_θ in the function of angle of contact θ

$$\sigma_\theta = -\sigma_s \left(1 - \ln \frac{R_w}{R_s + s + r_m} - \frac{r_m}{R_s + s + r_m} \sin\theta \right) \quad (9)$$

In the case when contact angle of workpiece of matrix round edge reaches the value $\theta = \frac{\pi}{2}$, $sin\theta = 1$, meridional stresses will have maximum values

$$\sigma_t = \sigma_s \left(\ln \frac{R_w}{R_s + s + r_m} + \frac{r_m}{R_s + s + r_m} \right). \quad (10)$$

Let us compare the results of obtained solution with research results of Popov E.A. [4].

During passing of flange area of workpiece on the rounded edge of matrix, meridional stresses skippingly increase for the value $\Delta\sigma_t$, which takes into account stress increment depending on the bending and rectification at the entrance into die hole [4] and their maximum value will be expressed by dependence

$$\Delta\sigma_t = \frac{s}{2r_m + s} \sigma_s, \quad (11)$$

For the piece with R = 50 mm, s = 2 mm, R_s = 25 mm and r_m = 10 mm acoording to the dependence (10) $\Delta\sigma_t = 0{,}09\sigma_s$, and according to formula (9) $\Delta\sigma_t = 0{,}27\sigma_s$, i.e. three times greater.

Conclusions

As fulfilled theoretical researches of the process of drawing of cylindrical pieces showed that meridonial tensile stresses during coming to the die cavity lips do not increase skippingly, but smoothly from the value $\sigma_s \ln \frac{R_w}{R_s + s + r_m}$ till the value $\sigma_s \left(\ln \frac{R_w}{R_s + s + r_m} + \frac{r_m}{R_s + s + r_m} \right)$ and change at the drawing edge according to sine law. Moreover their maximum value reduces with the increase of diameter of drawing small glass, which corresponds to the increase of drawing coefficient $m = R_s / R_w$ – reduction of reduction rate. With the increase of bending radius of lead-in shoulder, maximum tensile stress also increases, but with smaller intensity, than during reduction of workpiece diameter. Obtained dependency reveals more accurate the matter of influence of drawing parameters on the value of critical stresses and may be used in engineering simulation.

Generalizing the results obtained concerning radial-rotary profiling, one may say that the contact angle of bending radius of roller by workpiece material θ and $k = \frac{a}{R}$ value, which is equal to the ration of profile bending to workpiece radius, affect the distribution and value of meridional stresses. That is why during radial-rotary profiling for reduction of the value of meridional stresses at rounded areas, it is necessary to increase k value and reduction of θ angle for the area of distribution and pressing of workpiece.

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