

## The evaluation of measurement uncertainty of total stations permanence

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### Abstract

The algorithm of evaluation of measurement uncertainty of total stations permanence during calibration procedure is presented. The procedure of evaluation of uncertainty using two methods of calibration of total stations is considered.

Key words: TOTAL STATION, UNCERTAINTY, CALIBRATION

Evaluation of measurement uncertainty of total stations permanence is conducted during the calibration of distance measuring part of total stations. In various countries of the European Union and world we can often find two procedures metrological control of total stations – verification or calibration. During of the verification procedure is controlled mainly mean square error of measuring distances and angles, and during of the calibration - mostly estimated uncertainty in the measurement of various constants and corrections, which in turn affect the accuracy.

For a comprehensive evaluation of the quality of the measurements of distances by means of total stations is suggested to consider the

principles of evaluation of measurement uncertainty of total stations permanence.

*Evaluation of measurement uncertainty of total stations during calibration on geodesic polygons.*

Evaluation of measurement uncertainty of total stations permanence is carried out using a linear basis of geodesic polygon in the following order:

according to the requirements of operational documentation total station set on the zero point basis and steer crosshair in the center of a special reflector located sequentially at distances of 24 m, 96 m, 150 m, 480 m, 984 m and 1528 m (the largest controlled distance limited measuring

range total station). Measuring distances conducted by tenfold each controlled distance measurement basis from the zero point.

Evaluation of uncertainty according to DSTU-N RMG 43: 2006 "Metrology. The use of "Guide to the evaluation of measurement uncertainty".

Systematic error for each distance measure is defined as the difference between the measured value and reference value. But for EDM total station important his constant value (constant), or in other words - the corrections, which in turn is opposite in sign to the error. Corrections added to measurement result range.

Systematic correction for each distance measure defined by the formula 1.

$$\Delta_{Si} = S_{Ei} - \bar{S}_i, \quad (1)$$

where

$S_{Ei}$  – the length of the reference controlled distance;  $\bar{S}_i$  – the mean value of 10 measurements a distance.

An EDM constant is determined using all obtained systematic corrections of all controlled distances.

$$R = \frac{\sum_{i=1}^n \Delta_{Si}}{n}, \quad (2)$$

where  $n$  – number of controlled distances.

Model equation  $y$  measurements for each controlled distance:

$$y = u_{\bar{\Delta}_S} + u_B + u_t + u_h + \Delta_d \quad (3)$$

where

$u_B$  – input value, standard error (uncertainty of reference value controlled distance);  $u_t$  – input value, uncertainty caused by the influence of temperature and atmospheric pressure that differs from the "standard";  $u_h$  – input value, the uncertainty caused by different height total station and reflector;  $\Delta_d$  – input value, uncertainty associated with discreteness of the total station.

Standard deviation of correction for each distance is defined by the formula 4.

$$m_{\Delta_{Si}} = \sqrt{\frac{\sum_{i=1}^r (\Delta_{S_{ir}} - \bar{\Delta}_{Si})^2}{r-1}}, \quad (4)$$

where

$$\Delta_{S_{ir}} = S_{Ei} - S_i, \quad (5)$$

$$\bar{\Delta}_{Si} = \frac{1}{r} \sum_{i=1}^r \Delta_{S_{ir}}, \quad (6)$$

where  $r$  – number of measurements controlled distance.

Standard uncertainty corrections (type A) for each distance can be expressed as:

$$u_{\bar{\Delta}_{Si}} = \frac{m_{\Delta_{Si}}}{\sqrt{r}}, \quad (7)$$

Total uncertainty corrections for each distance is defined by the formula:

$$u_{Si} = \sqrt{u_{\bar{\Delta}_{Si}}^2 + u_B^2 + u_t^2 + u_h^2 + u_d^2}, \quad (8)$$

The standard uncertainty of the type B ( $u_t$ ), caused by the influence of temperature and atmospheric pressure defined by the formula:

$$u_t = \sqrt{\left(\frac{D}{273,15+t} \bar{S} \cdot 10^{-6}\right)^2 \Delta_p^2 + \left(\frac{D \cdot P \cdot \bar{S} \cdot 10^{-6}}{(273,15+t)^2}\right)^2 \Delta_t^2}, \quad (9)$$

where

$D$  – parameter of EDM total station with technical documentation of the manufacturer (for example, for some models total station Sokkia company  $D = 79,4$ );  $P$  – pressure, hPa;  $t$  – temperature, °C;  $\Delta_t$  – error temperature observations, °C (for example,  $\pm 0,2$ °C);  $\Delta_p$  – error correction of atmospheric pressure, hPa (for example,  $\pm 2$  hPa).

The standard uncertainty of the type B ( $u_h$ ), caused by different height total station and reflector defined by the formula 10:

$$u_h = \sqrt{\left(-\frac{\Delta H^2}{2S} - \frac{\Delta H^4}{8S^3} - \frac{\Delta H^6}{16S^5}\right)^2}, \quad (10)$$

where  $H$  – the difference in height of total station and reflector.

The standard uncertainty of the type B, input value – standard error ( $u_B$ ) and error associated with discreteness of the total station ( $u_d$ ) is defined by the formula:

$$u_B = \frac{b-a}{2\sqrt{3}}, \quad (11)$$

For direct measurement, sensitivity coefficients ( $c_j$ ) components of the model equations equal to 1.

The contribution of the uncertainty of each input value in uncertainty values measured is defined by the formula 12:

$$u_j(y) = c_j \cdot u(x_j), \quad (12)$$

None of the input quantities is not correlated with other in any great extent.

To calculate the coverage factor for repeated measurements can be taken for Student's distribution 0,95 confidence level and the effective degrees of freedom  $v_{eff}$ .

Effective degrees of freedom  $v_{eff}$  for direct multiple measurements is defined by the formula Welch-Satterthwaite:

$$v_{eff} = (n-1) \cdot \left(\frac{u_c(y)}{u_A(y)}\right)^4, \quad (13)$$

Often, in practice, there is an assumption of normality of the distribution of possible measured values and believe that the coverage factor is 2 to 95% level of confidence.

# Standardization

Expanded uncertainty corrections for each distance with a 95% confidence level is defined by the formula:

$$U_i = k \cdot u_{Si}, \quad (14)$$

The uncertainty budget is presented in Table 1:

**Table 1.** The uncertainty budget

| Input value   | Evaluation of the input value  | Standard uncertainty $u(x_j)$           |      | The number of degrees of freedom, $\nu_j$ | Law probability distribution of the input value | Sensitivity coefficients $c_j$ | Uncertainties contribution $u_j(y)$ | $u_j^2(y)$ |
|---|--------------------------------|---|------|---|---|--------------------------------|-------------------------------------|------------|
|   |                                | Value                                   | Type |   |   |                                |                                     |            |
| Measured value X  | $\bar{x}_e$                    | $u_A(\bar{x}_e)$                        | A    | $n-1$                                     | Normal distribution                             | 1                              | $c_j \cdot u(x_j)$                  | $u_j^2(y)$ |
| Standard error (uncertainty of reference value controlled distance ( $u_B$ ))         | $u_B$                          | $u_e = \frac{u_B}{\sqrt{3}}$            | B    | $\infty$                                  | Uniform distribution                            | 1                              | $u_2(y) = u_e$                      | $u_e^2(y)$ |
| Uncertainty caused by the influence of temperature and atmospheric pressure ( $u_t$ ) | $u_t$                          | according to the formula 9              | B    | $\infty$                                  | Uniform distribution                            | 1                              | $u_3(y) = u_t$                      | $u_t^2(y)$ |
| Uncertainty caused by different height total station and reflector ( $u_h$ )          | $u_h$                          | according to the formula 10             | B    | $\infty$                                  | Uniform distribution                            | 1                              | $u_4(y) = u_h$                      | $u_h^2(y)$ |
| Error associated with discreteness of the total station, ( $\Delta_d$ )               | $\Delta_d$                     | $u_d = \frac{\Delta_d}{2\sqrt{3}}$      | B    | $\infty$                                  | Uniform distribution                            | 1                              | $u_5(y) = u_d$                      | $u_d^2(y)$ |
| Output value  | Evaluation of the output value | Standart total uncertainty              |      | Effective degrees of freedom              | Level of confidence                             | Coverage factor                | Expanded uncertainty                |            |
| Y   | $\bar{x}_e = y$                | $u_c(y) = \sqrt{\sum_{i=1}^n u_i^2(y)}$ |      | $\nu_{eff}$                               | $p=0,95$  | $k$                            | $U=k \cdot u_c$                     |            |

Standard of uncertainty of mean constant EDM total station is defined by the formula:

$$u_{\bar{R}} = \sqrt{\frac{\sum_{i=1}^n (\Delta_{Si} - R)^2}{n(n-1)}}, \quad (15)$$

where  $n$  – number of controlled distances.

Thus the resulting uncertainty constant EDM is defined by the formula:

$$u_R = \sqrt{u_{\bar{R}}^2 + \frac{1}{n} \sum_{i=1}^n u_{Si}^2}, \quad (16)$$

Expanded uncertainty constant EDM with coverage probability of approximately 95% is

defined by the formula:

$$U_R = k \cdot u_R, \quad (17)$$

*Evaluation of measurement uncertainty of total stations in laboratory.*

Evaluation of measurement uncertainty constant EDM total station is carried out by means of controlling zigzag basis distances in the laboratory in the following order:

- bring the temperature of the environment in laboratory under "standard" according

to the information of the manufacturer of the instrument,

- according to the requirements of operational documentation set total station zero point at the same height with a special reflector,
- steer crosshair in the center reflection special reflector which is located at the end of zigzag reproducible distances,
- measurement of the distance conducted by tenfold each controlled zigzag distance from the zero point.

Systematic correction for each distance measure defined by the formula 1.

An EDM constant is determined using all obtained systematic corrections of all controlled distances by the formula 2.

Model equation  $y$  measurements for each controlled distance:

$$y = u_{\Delta S} + u_B + \Delta_d \quad (18)$$

where

$u_B$  – input value, standard error (uncertainty of reference value controlled distance);  $\Delta_d$  – input value, uncertainty associated with discreteness of the total station.

Standard deviation of correction for each

distance is defined by the formula 4.

Standard uncertainty corrections (type A) for each distance can be expressed as:

$$u_{\Delta S_i} = \frac{m_{\Delta S_i}}{\sqrt{r}}, \quad (19)$$

Total uncertainty corrections for each distance is defined by the formula:

$$u_{si} = \sqrt{u_{\Delta S_i}^2 + u_B^2 + u_d^2}, \quad (20)$$

The standard uncertainty of the type B, input value – standard error ( $u_B$ ) and error associated with discreteness of the total station ( $u_d$ ) is defined by the formula 11.

For direct measurement, sensitivity coefficients ( $c_j$ ) components of the model equations equal to 1. The contribution of the uncertainty of each input value in uncertainty values measured is defined by the formula 12.

None of the input quantities is not correlated with other in any great extent. In the normal distribution, for a 95% level of confidence, assume that coverage factor is 2.

Expanded uncertainty corrections for each distance with a 95% confidence level is defined by the formula 14.

The uncertainty budget is presented in Table 2:

Table 2

| Input value   | Evaluation of the input value  | Standard uncertainty $u(x_j)$           |      | The number of degrees of freedom, $\nu_j$ | Law probability distribution of the input value | Sensitivity coefficients $c_j$ | Uncertainties contribution $u_j(y)$ | $u_j^2(y)$ |
|---|--------------------------------|---|------|---|---|--------------------------------|-------------------------------------|------------|
|   |                                | Value                                   | Type |   |   |                                |                                     |            |
| Measured value X  | $\bar{x}_e$                    | $u_A(\bar{x}_e)$                        | A    | $n-1$                                     | Normal distribution                             | 1                              | $c_j \cdot u(x_j)$                  | $u_j^2(y)$ |
| Standard error (uncertainty of reference value controlled distance ( $u_B$ )) | $u_B$                          | $u_e = \frac{u_B}{\sqrt{3}}$            | B    | $\infty$                                  | Uniform distribution                            | 1                              | $u_2(y) = u_e$                      | $u_e^2(y)$ |
| Error associated with discreteness of the total station, ( $\Delta_d$ )       | $\Delta_d$                     | $u_d = \frac{\Delta_d}{2\sqrt{3}}$      | B    | $\infty$                                  | Uniform distribution                            | 1                              | $u_3(y) = u_d$                      | $u_d^2(y)$ |
| Output value  | Evaluation of the output value | Standart total uncertainty              |      | Effective degrees of freedom              | Level of confidence                             | Coverage factor                | Expanded uncertainty                |            |
| Y   | $\bar{x}_e = y$                | $u_c(y) = \sqrt{\sum_{i=1}^n u_i^2(y)}$ |      | -   | p=0,95  | 2                              | $U = k \cdot u_c$                   |            |

Standard of uncertainty of mean constant

EDM total station is defined by the formula 15.

## Standardization

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Resulting uncertainty constant EDM is defined by the formula 16.

Expanded uncertainty constant EDM with coverage probability of approximately 95% is defined by the formula 17.

### *Registration of calibration results.*

- The measurement results during calibration are registered in the record of primary observation by hand and are the part of the calibration protocol.

- General data and results of calculations during the calibration, results of determining compliance to technical conditions (regulations on use and operational documents) are registered in the calibration protocol.

- The calibration certificate is provided and it is based on the calibration protocol.

### **Conclusions**

Having analyzed the input values of uncertainties and errors during measurements on

linear basis of geodesic polygon we can conclude the existence of a very large impact of external factors on the measurement results. During calibration in laboratory it is possible to reduce the impact of external conditions to a minimum and exclude the impact of certain errors and uncertainties, and thus increase the accuracy of measurements.

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