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Application of numerical methods for the calculation of core structural elements with the discrete nature of rigidities, Winkler coefficients and detachment of the elastic supports foundation

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Abstract
The paper presents methodology for proof of the calculation results of the stress-strain state of engineering structural elements typical method of finite and boundary elements. In the role of investigating construction there chosen design scheme combined arch systems with different rigidities Rigil and racks, as well as four times statically indeterminate beam, which changes discretely lateral stiffness ratio of the Winkler coefficient elastic base has a resilient support and a portion of the beam is free from the base. The techniques of taking into account additional factors structures in the boundary element method, and in the finite element method are suggested.
Key words: FINITE ELEMENT METHOD, BOUNDARY ELEMENT METHOD, ARCH SYSTEM, CONTINUOUS BEAM, THE STRESS-STRAIN STATE

In various engineering structures there used elements such elements as arched combined system continuous beams with different combinations of restraints, and the ones taking into account the elastic foundation. Moves of such structures is described by complex functions. So urgent is the question of the accuracy of calculation of stress-strain state, caused by an external load.

In this paper we proposed to use for this purpose numerical-analytical boundary element method [1] and the program complex ANSYS, which implements the finite element method [2]. Each of these methods has its advantages and disadvantages, but if the results of two different methods will be similar, with a high probability it will be possible to assert that the accuracy of determining the stress-strain state of the construction will be proved. It is especially important to identify correctly the parameters of the stress-strain state at the design stage, when it is necessary to assign reasonable dimensions similar design.

As the object of study, let us consider a combined system with curved rod bolts, outlined a circular arc, with different stiffness elements and loaded uniformly distributed load.

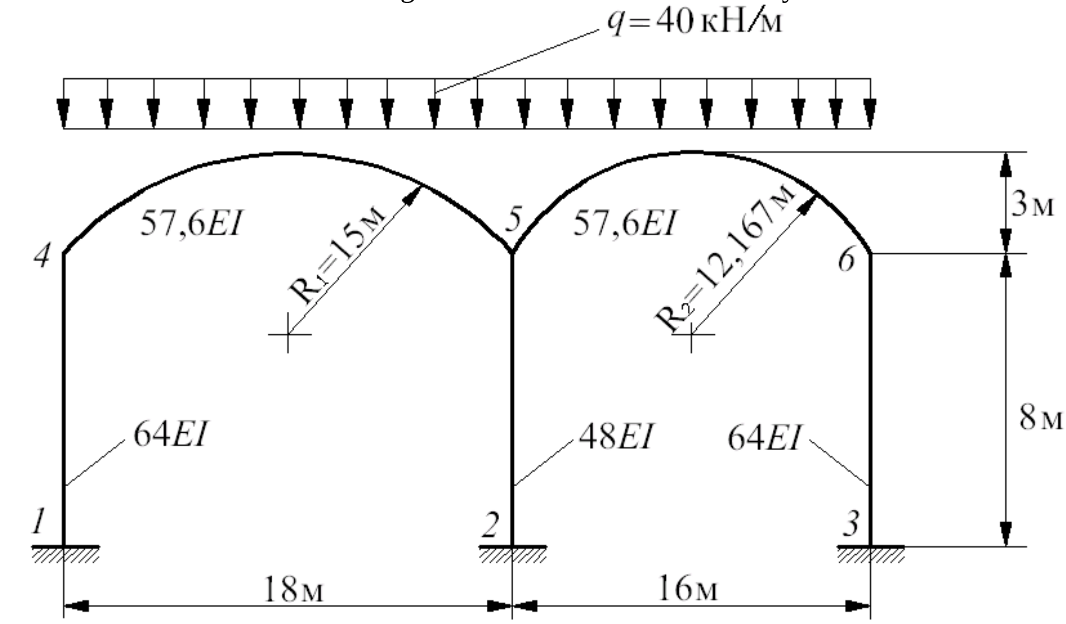

Figure 1. Combined arched structure

Calculation method of boundary elements

Form the equation of BEM scheme

$$\mathrm{Y}(l) = \mathrm{A}\,(l)\ \mathrm{X}\,(0) + \mathrm{B}\,(l) \rightarrow \mathrm{A}\,(l)\ \mathrm{X}\,(0) - \mathrm{Y}\,(l) = \mathrm{B}\,(l) \rightarrow \mathrm{A}_*\,(l)\ \mathrm{X}_*\,(0,l) = -\mathrm{B}\,(l)\,. \qquad (1)$$

For curved rods there used equation of the form

$$\begin{pmatrix} EIv(\alpha) \\ EI\varphi(\alpha) \\ M(\alpha) \\ Q(\alpha) \\ EAu(\alpha) \\ N(\alpha) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & -A_{13} & -A_{14} & A_{15} & A_{16} \\ & A_{22} & -A_{23} & -A_{13} & & A_{26} \\ & & A_{12} & & & -A_{56} \\ & & A_{11} & & & -A_{46} \\ A_{51} & A_{52} & -A_{53} & -A_{54} & A_{11} & A_{56} \\ & & & -A_{64} & & A_{11} \end{pmatrix} \begin{pmatrix} EIV(0) \\ EI\varphi(0) \\ M(0) \\ Q(0) \\ EAU(0) \\ N(0) \end{pmatrix} + \int_0^{\alpha} \begin{pmatrix} B_{11} \\ B_{21} \\ -B_{31} \\ -B_{41} \\ -B_{51} \\ -B_{61} \end{pmatrix} d\xi\,, \qquad (2)$$

where the fundamental functions are given in [1,3]

For the equation of straight bars bending with normal forces

$$EIv(x) = \begin{pmatrix} 1 & x & x^2/2 & x^3/6 & \end{pmatrix} \ EIv(0) \ \left| (x-\xi)^3 q_y(\xi)/6 \right| d\xi \qquad (3)$$

$$\begin{array}{|c|}\hline EI\varphi(x)\\ \hline M(x)\\ \hline Q(x)\\ \hline N(x)\\ \hline\end{array}\quad \begin{array}{|c|c|c|c|c|}\hline & 1 & x & x^2/2 & \\ \hline & & 1 & x & \\ \hline & & & 1 & \\ \hline & & & & 1\\ \hline\end{array}\quad \begin{array}{|c|}\hline EI\varphi(0)\\ \hline M(0)\\ \hline Q(0)\\ \hline N(0)\\ \hline\end{array} + \int_0^x \begin{array}{|c|}\hline (x-\xi)^2 q_y(\xi)/2\\ \hline (x-\xi)^2 q_y(\xi)\\ \hline q_y(\xi)\\ \hline -q_y(\xi)\\ \hline\end{array}.$$

From the system of linear algebraic equations (1) there defined boundary parameters, and the state structure in the interior of - according to the equations (2), (3). The calculation results of the IGE are summarized in Table 1.

Calculation of the finite element method

To implement the algorithm FEM, there chosen a program ANSYS. Approximation of calculation model is made from the library element BEAM3 of finite element program ANSYS. This is a two-node element used mainly to calculate beam structures.

The calculation of the FEM, for each variant partitioning CE, a deformed shape of the system diagrams of normal forces and shear forces, bending moments diagram (Figure 2). The calculation results for n> 1000 are not specified.

Table 1. Summary table of the results of the calculation of the combined arch

IFF	BEM	Method Rogitskogo [1]	FEM (10fe)	FEM (20fe)	FEM (40fe)	FEM (100fe)	FEM (1000fe)
N_{z_1}	383,8		322,11	340,1	349,09	354,49	357,73
Q_{y_1}	169,6		160,68	161,31	161,47	161,51	161,52
M_{x_1}	642,32	618,1	610,52	612,8	613,37	613,53	613,56
N_{z_2}	741,1		688,95	650,99	668,0	678,20	684,32
Q_{y_2}	30,6		30,109	30,263	30,301	30,312	30,314
M_{x_2}	110,1858	98,6	108,31	108,85	108,99	109,02	108,79
N_{z_3}	345,6		284,93	300,91	308,91	313,71	316,59
Q_{y_3}	138,9		130,57	131,05	131,17	131,20	131,21
M_{x_3}	547,7885	524,7	517,3	519,09	519,22	519,68	519,71
N_{z_4}	365,98		296,19	318,24	325,62	330,06	332,73
Q_{y_4}	205,2838		193,45	201,03	204,73	206,91	208,21
M_{x_4}	714,6036	665,9	674,92	677,69	678,38	678,58	678,62
N_{z_5}	741,1		616,96	650,99	668,0	678,20	684,32
Q_{y_5}	30,6		30,109	30,263	30,301	30,312	30,314
M_{x_5}	135,2156	136,9	132,56	133,25	133,42	133,47	133,48
N_{z_6}	336,62		284,93	300,91	308,91	313,71	316,59
Q_{y_6}	138,94		130,57	131,05	131,17	131,2	131,21
M_{x_6}	563,7337	524,7	527,3	529,29	529,8	529,94	529,97

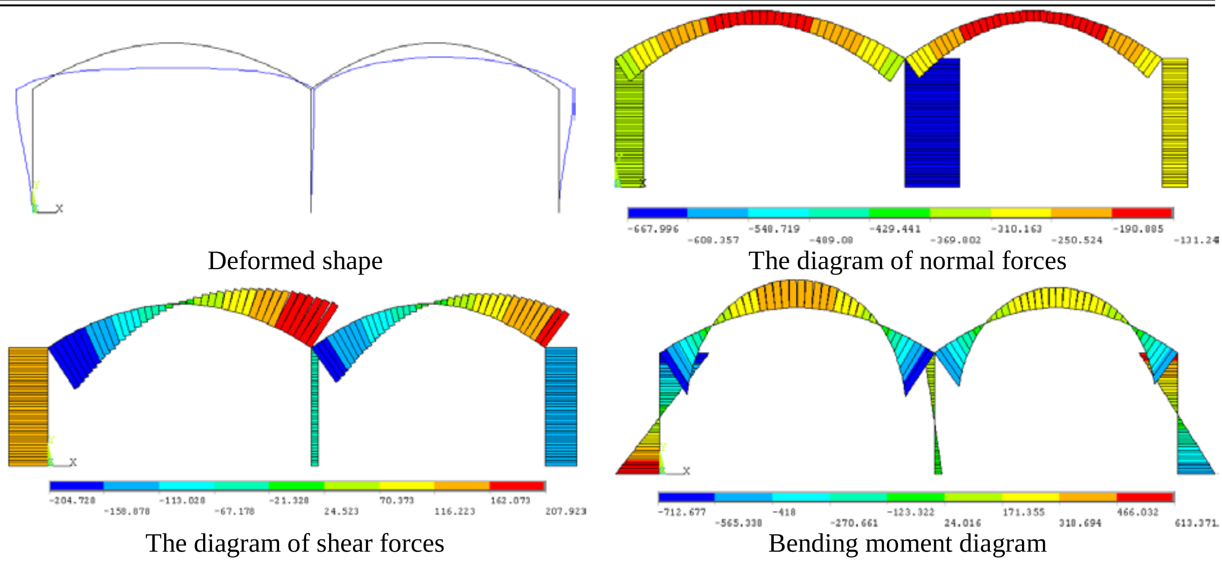

Figure 2. Diagrams of internal force factors arch structure by FEM

FEM results in crushing a finite element mesh shown in Table 1. Comparison of these calculations shows that the results of FEM can be considered the most reliable. They are in good agreement with the method Rogitskogo. At the same time, the results of the IGE somewhat overstated due to appear to neglect of tensile deformation of straight bars bending in the initial equations (3).

As another structure, which is of great practical importance in the engineering practice, consider the design scheme statically indeterminate continuous beam with elastic support and a different configuration of the Winkler foundation (Figure 3).

Using the boundary element method (BEM) [1], implemented in MATLAB package and the finite element method (FEM), implemented in the package ANSYS [2].

Scientific mission 1. Continuous beam (Figure 3) in the algorithm BEM we split into five rods, enumerate nodes and indicate the beginning and end of each element with arrows.

The elastic support is taken into account with correlations:

$$\overset{2-3}{v}(l)=\overset{3-4}{v}(0)=-\frac{R_3}{15EI};\; R_3=-\overset{2-3}{Q}(l)+\overset{3-4}{Q}(0);\; \overset{2-3}{v}(l)=\overset{3-4}{v}(0)=\frac{\overset{2-3}{Q}(l)-\overset{3-4}{Q}(0)}{15EI}L^3.$$

Matrices X_*; Y, which take into account the boundary conditions, the equations of equilibrium and compatibility displacement units 1, 2, 3, 4, and elastic support will take the form (4).

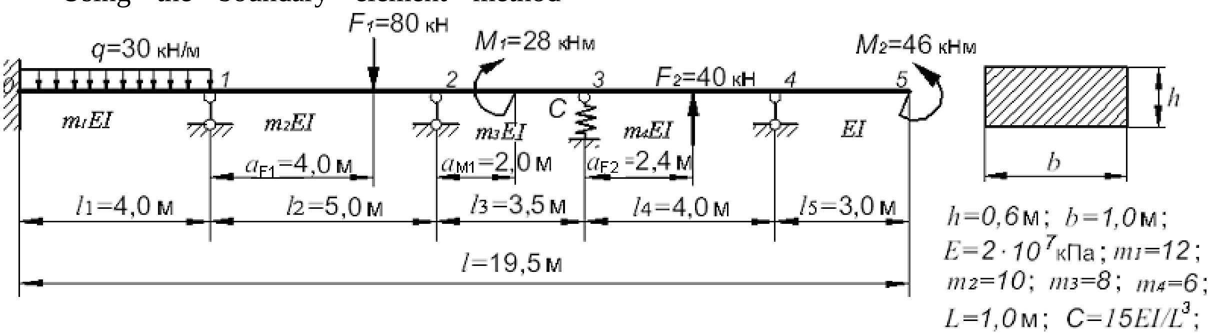

Figure 3. Continuous beam with elastic support

$$X_* = \begin{array}{c|c|} 1 & m_1EIv_{(0)}^{0-1}=0;\; Q_{(l)}^{0-1} \\ 2 & m_1EI\varphi_{(0)}^{0-1}=0;\; Q_{(l)}^{1-2} \\ 3 & M_{(0)}^{0-1}; \\ 4 & Q_{(0)}^{0-1} \end{array};\; Y = \begin{array}{|c|} m_1EIv_{(0)}^{0-1}=0 \\ m_1EI\varphi_{(l)}^{0-1}=m_1EI\varphi_{(0)}^{1-2} \\ M_{(0)}^{0-1}=M_{(0)}^{1-2} \\ Q_{(l)}^{0-1} \end{array} \qquad (4)$$

5	$m_2 EI v_{(0)}^{0-2} = 0;\ Q_{(l)}^{2-3}$	$m_2 EI\varphi_{(l)}^{1-2} = 0$
6	$m_2 EI\varphi_{(0)}^{1-2}$	$m_2 EI\varphi_{(l)}^{1-2} = m_2 I\varphi_{(0)}^{2-3}$
7	$M_{(0)}^{1-2};$	$M_{(l)}^{1-2} = M_{(0)}^{2-3}$
8	$Q_{(0)}^{1-2}$	$Q_{(l)}^{1-2}$
9	$m_3 EI v_{(0)}^{2-3} = 0;\ Q_{(l)}^{3-4}$	$m_3 EI v_{(l)}^{2-3} = \frac{m_3 L^3}{15}\left[Q_{(l)}^{2-3} - Q_{(0)}^{3-4}\right]$
10	$m_3 EI\varphi_{(0)}^{2-3}$	$m_3 EI\varphi_{(l)}^{2-3} = m_3 I\varphi_{(0)}^{3-4}$
11	$M_{(0)}^{2-3};$	$M_{(l)}^{2-3} = M_{(0)}^{3-4}$
12	$Q_{(0)}^{2-3}$	$Q_{(l)}^{2-3}$
13	$m_4 EI v_{(0)}^{3-4} = \frac{m_4 L^3}{15}\left[Q_{(l)}^{2-3} - Q_{(0)}^{3-4}\right];\ EI v_{(l)}^{4-5}$	$m_4 EI v_{(l)}^{3-4} = 0$
14	$m_4 EI\varphi_{(0)}^{3-4}$	$m_4 EI\varphi_{(l)}^{3-4} = m_4 EI\varphi_{(0)}^{4-5}$
15	$M_{(0)}^{3-4};$	$M_{(l)}^{3-4} = M_{(0)}^{4-5}$
16	$Q_{(0)}^{3-4}$	$Q_{(l)}^{3-4}$
17	$EI v_{(0)}^{4-5} = 0;\ Q_{(l)}^{4-5}$	$EI v_{(l)}^{4-5}$
18	$EI\varphi_{(0)}^{4-5}$	$EI\varphi_{(l)}^{4-5}$
19	$M_{(0)}^{4-5};$	$M_{(l)}^{4-5} = 0$
20	$Q_{(0)}^{4-5}$	$Q_{(l)}^{4-5} = 0$

Resolving equations for a beam (Figure 3) is formed by the rules of the BEM [1].

To implement the chosen FEM calculation software package ANSYS. The program allows to perform the calculation of almost any structures for durability, stability and dynamic loads [2]. For the calculation of continuous beam (Figure 3) from the library of standard finite element program is selected a two-node beam element BEAM54, designed to solve two-dimensional problems. Element properties BEAM54 are defined by describing the cross-sectional characteristics, material properties (modulus of elasticity and Poisson's ratio), and the resilient base. The construction was divided into 90 finite elements.

The results of solving of boundary value problem and calculate the parameters of the state of the beam is shown in Table 2. There is a comparison of the results of the two methods FEM and BEM.

Table 2. Stress-strain state of a beam with elastic support

Global coordinate x, m	Stress strain state parameters of the beam					
	Bending moment M, kNm			Transverse force Q, kN		
	FEM	BEM	Difference Δ, %	FEM	BEM	Difference Δ, %
0,00	43,752	44,09	0,77	62,814	63,06	0,39
4,00	32,495	31,8	2,186	57,186 -18,636	56,9 -18,1	0,5 2,88
8,00	42,049	40,5	3,82	-18,636 61,364	-18,1 61,9	2,88 0,867
9,00	19,495	21,3	8,47	61,364	61,9	0,867

				-0,07526	-0,12	4,099
11,00	20,82 6,841	21,1 6,89	1,32 0,79	-0,07526	-0,12	37,3
12,5	6,9366	7,07	1,89	6,013	6,27	4,27
16,5	46,0	46,0	0,00	-33,987	-33,73	0,76
19,5	46,0	46,0	0,00	0,00	0,00	0,00
x, m	Sag υ, m			Angle of rotation φ, radian		
2	$0{,}5498\cdot10^{-5}$	$0{,}557\cdot10^{-5}$	1,31	$0{,}4343\cdot10^{-6}$	$0{,}473\cdot10^{-6}$	8,18
6	$0{,}7676\cdot10^{-5}$	$0{,}7188\cdot10^{-5}$	6,78	$0{,}5962\cdot10^{-5}$	$0{,}5706\cdot10^{-5}$	4,48
7	$0{,}1158\cdot10^{-4}$	$0{,}1147\cdot10^{-4}$	0,98	$0{,}324\cdot10^{-5}$	$0{,}2\cdot10^{-5}$	62
8	$0{,}902\cdot10^{-5}$	$0{,}952\cdot10^{-5}$	1,89	$0{,}7045\cdot10^{-5}$	$0{,}675\cdot10^{-5}$	29,5
11	$0{,}6\cdot10^{-5}$	$0{,}4075\cdot10^{-5}$	0,79	$0{,}47537\cdot10^{-5}$	$0{,}53\cdot10^{-5}$	10,3
12,5	$0{,}162\cdot10^{-5}$	$0{,}1832\cdot10^{-5}$	11,57	$0{,}1746\cdot10^{-6}$	$0{,}1678\cdot10^{-5}$	4,05
19,5	$0{,}608\cdot10^{-3}$	$0{,}6107\cdot10^{-3}$	0,44	$0{,}39423\cdot10^{-3}$	$0{,}395\cdot10^{-3}$	0,195

Table 2 shows sufficient correspondence of results of two different methods.

Scientific mission 2. Let us add in the calculation scheme of the beam (Figure 3) the elastic base with discrete variable gain bed (Figure 4): a rod 0-1 – $k_1 = 1\cdot10^4\ kN/m^3$; 1-2 – $k_2 = 2\cdot10^4\ kN/m^3$; 2-3 – $k_3 = 3\cdot10^4\ kN/m^3$; 3-4 – $k_4 = 4\cdot10^4\ kN/m^3$; 4-5 – $k_5 = 5\cdot10^4\ kN/m^3$. To consider elastic base, it is sufficient to replace fundamental functions of the bending task in resolving equations 1 to the fundamental functions of the beam on elastic foundation with a single Winkler coefficient [1]. The calculation results of the stress-strain state of the beam are shown in Table 3.

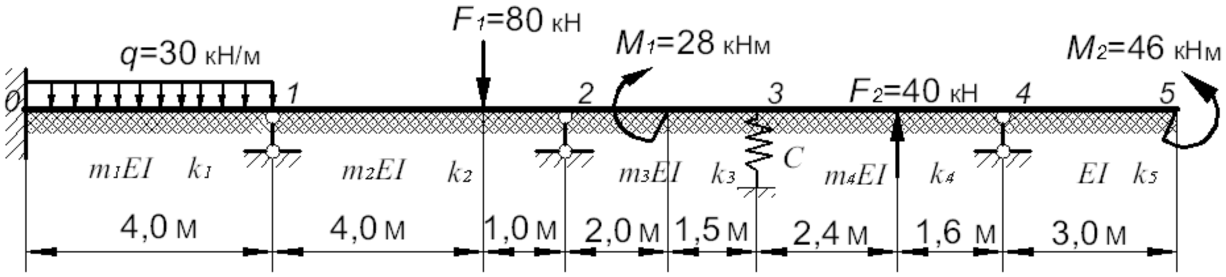

Figure 4. continuous beam on elastic foundation

Table 3. Stress-strain state of a beam with elastic support and the elastic base

Global coordinate x, m	Stress strain state parameters of the beam					
	Bending moment M, kNm			Transverse force Q, kN		
	FEM	BEM	Difference Δ, %	FEM	BEM	Difference Δ, %
0,00	44,201	44,8	1,34	-63,13	-63,6	0,739
4,00	31,441	30,02	4,73	56,745 -17,743	56,3 -16,7	0,79 6,245
8,00	40,137	37,4	7,3	-18,157 61,742	-17,2 62,8	5,56 1,685
9,00	21,558	25,3	14,79	61,664 -2,394	62,7 -4,76	1,65 49,7
11,00	17,412 11,336	15,9 12,1	9,5 6,3	-2,394	-4,76	49,7
12,5	14,108	19,1	26,1	-2,13 15,5	-4,75 17,3	55,16 10,4
14,9	19,778	20,1	1,6	16,192 -23,74	17,3 -22,7	6,4 4,58

16,5	14,038	13,4	4,76	-23,428 13,648	-22,3 14,0	0,76 5,06
19,5	46,0	46,0	0,00	0,00	0,00	0,00
x, m	Sag υ, m			Angle of rotation φ, radian		
2	$0{,}56071\cdot10^{-5}$	$0{,}57477\cdot10^{-5}$	2,45	$0{,}4921\cdot10^{-6}$	$0{,}5624\cdot10^{-6}$	12,5
6	$0{,}69571\cdot10^{-5}$	$0{,}60752\cdot10^{-5}$	14,5	$0{,}5637\cdot10^{-5}$	$0{,}5216\cdot10^{-5}$	8,07
7	$0{,}1068\cdot10^{-4}$	$0{,}10042\cdot10^{-4}$	6,35	$0{,}21406\cdot10^{-5}$	$0{,}1934\cdot10^{-5}$	10,68
8	$0{,}93246\cdot10^{-5}$	$0{,}83676\cdot10^{-5}$	11,4	$0{,}6607\cdot10^{-5}$	$0{,}6076\cdot10^{-5}$	8,74
11	$0{,}1912\cdot10^{-5}$	$0{,}12258\cdot10^{-5}$	55,9	$0{,}41625\cdot10^{-5}$	$0{,}6548\cdot10^{-5}$	36,4
12,5	$0{,}2961\cdot10^{-5}$	$0{,}40834\cdot10^{-5}$	27,487	$0{,}23885\cdot10^{-6}$	$0{,}1547\cdot10^{-5}$	54,9
19,5	$0{,}3212\cdot10^{-3}$	$0{,}325\cdot10^{-3}$	1,17	$0{,}2679\cdot10^{-3}$	$0{,}2687\cdot10^{-3}$	0,3

Table 3 also shows that the results of FEM and BEM are quite consistent.

Scientific mission 3. Computational model (Figure 4) let us complicate by detachment phenomenon, i.e. in some parts of the beam (0-1 and 2-3) elastic base is absent (Figure 5). In this case, the equation of the boundary value problem, according to BEM [1], the fundamental functions return for bend sections 0-1 to 2-3. The calculation results of the beam condition is shown in Table 4.

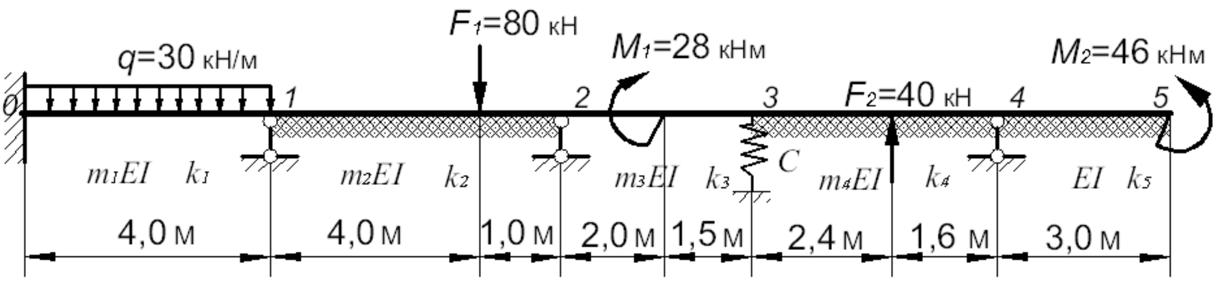

Figure 5. Continuous beam on elastic foundation with the effect of detachment

Table 4. Stress-strain state of the beam on elastic foundation with the detachment

Global coordinate x, m	Stress strain state parameters of the beam					
	Bending moment M, kNm			Transverse force Q, kN		
	FEM	BEM	Difference Δ, %	FEM	BEM	Difference Δ, %
0,00	44,252	44,87	1,38	-63,189	-63,66	0,74
4,00	31,495	30,25	4,116	56,811 -17,775	56,34 -16,8	0,836 5,8
8,00	40,214	37,4	7,52	-18,19 61,709	-17,3 62,7	5,145 1,58
9,00	21,447	25,3	15,23	61,629 -2,1697	62,68 -4,66	1,68 53,4
11,00	17,542 11,218	15,9 12,1	10,3 7,29	-2,1697	-4,66	53,4
12,5	14,147	19,0	25,5	-2,1697 15,469	-4,66 17,3	53,4 10,58
14,9	19,835	20,2	1,8	16,202 -23,729	17,5 -22,5	7,417 5,462
16,5	14,037	13,4	4,75	-23,418 13,649	-22,3 14,0	5,46 2,507
19,5	46,0	46,0	0,00	0,00	0,00	0,00

x, m	Sag υ, m			Angle of rotation φ, radian		
2	$0,5614\cdot10^{-5}$	$0,57576\cdot10^{-5}$	2,49	$0,49218\cdot10^{-6}$	$0,564\cdot10^{-6}$	12,73
6	$0,69752\cdot10^{-5}$	$0,6076\cdot10^{-5}$	14,8	$0,5649\cdot10^{-5}$	$0,522\cdot10^{-5}$	8,22
7	$0,10706\cdot10^{-4}$	$0,10047\cdot10^{-4}$	6,56	$0,2056\cdot10^{-5}$	$0,1937\cdot10^{-5}$	6,14
8	$0,9349\cdot10^{-5}$	$0,83731\cdot10^{-5}$	11,655	$0,66196\cdot10^{-5}$	$0,6078\cdot10^{-5}$	8,91
11	$0,1976\cdot10^{-5}$	$0,1377\cdot10^{-5}$	43,5	$0,41635\cdot10^{-5}$	$0,6548\cdot10^{-5}$	36,42
12,5	$0,2991\cdot10^{-5}$	$0,40666\cdot10^{-5}$	26,45	$0,2357\cdot10^{-6}$	$0,155\cdot10^{-5}$	52,06
19,5	$0,3212\cdot10^{-3}$	$0,32491\cdot10^{-3}$	1,142	$0,26794\cdot10^{-3}$	$0,2687\cdot10^{-3}$	0,283

Analysis of the results according to the tables 2,3,4 indicates that the data of FEM and BEM is quite consistent with each other, taking into account the various structural and power factors. This suggests that FEM and BEM allow to obtain very precise and reliable results about the internal state structure. In this case, the base has little effect on the stress strain state of beams, due to the presence of rigid supports and small deflections in the spans. In the console area, where there are large deflections, the influence of the elastic base is significant. Reaction R_4 is 4 times reduced, and the maximum deflection decreased almost twice. In such circumstances the absence of a base in the areas 0-1 and 2-3 is almost invisible.

Suggested approach ensures the accuracy of determining of stress-strain state. From the material presented it follows that the attraction for research on the nature of the different construction methods simplifies significantly the design stage when choosing a structural dimensions.

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