Application of ENO-Biorthogonal Wavelet Transform Algorithm in Image Compression Processing

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Abstract
Because of the bad image phase distortion caused by high pixel image edge when use EZW algorithm or SPIHT algorithm or ENO- orthogonal wavelet transform for image compression processing, the overall effect of the reconstructed image in image compression process is not ideal. This paper put forward a new image compression processing algorithm based on the ENO-biorthogonal wavelet transformation, use the biorthogonal wavelet transformation to carry on image compression processing with the characteristics of completely restructures and the biorthogonal wavelet filter linear phase, perfectly optimized the edge of image, improved the compression ratio, And the experimental result has confirmed that the ENO-biorthogonal wavelet transformation algorithm is very quality.
Key words: ENO THOUGHTS, ORTHOGONAL WAVELET TRANSFORMATION, ENO-BIORTHOGONAL WAVELET TRANSFORMATION, IMAGE COMPRESSION PROCESSING

1. Introduction

Wavelet analysis technology began to develop since the late 80's, is a focus of current research, widely used in signal analysis, image processing, computer recognition, edge detection, speech synthesis, quantum mechanics, and electron rivalry.

In 1992, by Lewis and Knowles [1] first proposed the concept of zero tree image in wavelet transform domain, the zero tree coding idea. The famous EZW[2] algorithm and SPIHT algorithm[3], is the use of the zero tree structure search important coefficients according to the progressive image coding method of transfer, save these important coefficients, so as to achieve higher compression ratio. However, when the image contains many jumps (i.e. two adjacent pixel value is large) information, the wavelet coefficients of high frequency space will have a lot of modulus larger coefficient. In order to ensure the quality of reconstructed image, the EZW algorithm and SPIHT algorithm and based on these two methods, many improved algorithms more space to store the high frequency wavelet coefficients, which will reduce their compression ratio.

Hao-Min Zhou [4] describes the ENO idea systematically, and puts forward the algorithm of ENO- orthogonal wavelet transform. Using the ENO-Haar wavelet transform and ENO-DB (Daubechis wavelet DB wavelet) for reconstruction of wavelet transform, found edge information of the image is better than using Haar wavelet and DB wavelet processing results improved, but the overall effect of the reconstructed image, the image boundary and edge information is not ideal. Because orthogonal wavelets have no linear phase characteristics, so it will produce phase distortion when used for image processing [5]. The following is a brief introduction about ENO- orthogonal wavelet transform algorithm and process.

In this paper, According the character of linear phase and perfect reconstruction of biorthogonal wavelet filter, We used ENO- biorthogonal wavelet transform algorithm for image compression processing, have overcome the limited that ENO- orthogonal wavelet transform algorithm is only suitable for the filter length is only even, optimized image boundary, and improved the overall effect of the reconstructed image, the experimental results verify the Excellent performance and catholicity for image processing of ENO- biorthogonal wavelet transform algorithm and ENO- biorthogonal wavelet .

2. ENO- orthogonal wavelet transform algorithm transform principle and process

Essentially Non-Oscillatory [6] is applied to the shock of computational fluid dynamics to capture. The method is based on the smoothness adaptive given signal function definition of piecewise polynomial approximation to the signal function. You can use the ENO method to modify the wavelet coefficient of wavelet transform in signal jumps in to remove the Gibbs' effect, can make the image ENO method is applied to the wavelet compression, the principle and the process is as follows:

For each i

⑴ According to the formula (1) to calculate the $\beta_{j,i}$:

$$\beta_{j,\mathrm{i}} = \sum_{k=0}^{l} g_k \alpha_{j+1,2i+k} \quad (1)$$

⑵ If $|\beta_{j,i}| > \beta_{j,i-1}|$ and $|\beta_{j,i}| \geq \lambda$, then :

i) According to the formula (1)to calculate the $\beta_{j,i+k}$ and $\beta_{j,i+k+1}$:

ii) Calculation of its jump, jump in determining, for i≤m≤i+k or i≤m≤i+k+1

a)From the jump on the left, extension can be calculated by extrapolation $\hat{\alpha}_{j,m}$, According to the formula (2)to calculate the $\hat{\beta}_{j,m}$, make $\beta_{j,m} = \hat{\beta}_{j,m}$,si=1;

$$\begin{pmatrix} \hat{\alpha}_{j,i} \\ \hat{\beta}_{j,i} \end{pmatrix} = \begin{pmatrix} \sum_{k=0}^{l-2} h_k \alpha_{j+1,2i+k} + h_{l-1} \hat{\alpha}_{j+1,2i+l-1} + h_l \hat{\alpha}_{j+1,2i+l} \\ \sum_{k=0}^{l-2} g_k \alpha_{j+1,2i+k} + g_{l-1} \hat{g}_{j+1,2i+l-1} + g_l \hat{\alpha}_{j+1,2i+l} \end{pmatrix} \quad (2)$$

b) From the jump on the right, make $\bar{\beta}_{j,m} = 0$, And according to the formula (2)to calculate the $\bar{\alpha}_{j,m}$, make $\alpha_{j,m} = \bar{\alpha}_{j,m}$;

iii) else, According to the formula(3)to calculate the $\alpha_{j,i}$, make si= 0;

$$\alpha_{j,\mathrm{i}} = \sum_{k=0}^{l} h_k \alpha_{j+1,2i+k} \quad (3)$$

Note: λ is a predefined small normal weight, when $|\beta_{j,i}|$ and $|\beta_{j,i-1}|$ are less than λ, believes that the current module is smooth. si is only a bit marker variables, used to the labeled module is a jump.

Thus, gives the lemma:

Lemma (1): if the filter length is l+1 (l is odd), l=2K-1. The processed data to meet the DSP (Discontinuity Separation Property) conditions, for a given i value index, assumed to satisfy:$|\beta_{j,i-1}| \leq \alpha|\beta_{j,i-2}|$ and $|\beta_{j,i}| > \alpha|\beta_{j,i-1}|$ (α is a constant parameter that greater than 1),then:

i) if $|\beta_{j,i+k-1}| > \alpha|\beta_{j,i+k}|$, there are k continuous standard module contains the jump point, and the jump point can be located in the{x(2i+l-1),x(2i+l)}.

ii) if $|\beta_{j,i+k-1}| \leq \alpha|\beta_{j,i+k}|$, there are k-1 continuous standard module contains the jump point, and the jump point can be located in the {x(2i+l-2),x(2i+l-1)}.

ENO-Orthogonal wavelet inverse transform process :

For each i

⑴if si=0 and Sj=0,j=i-l,i-l+1,…,i-1, then standard inverse wavelet formula is available :

$$\alpha_{j+1,2i} = \sum_{k=0}^{l} (g_{2k+1}\alpha_{j,i-k} + h_{2k+1}\beta_{j,i-k}) \quad (4)$$

$$\alpha_{j+1,2i+1} = \sum_{k=0}^{l} (g_{2k}\alpha_{j,i-k} + h_{2k}\beta_{j,i-k}) \quad (5)$$

$$\varphi(x) = \sqrt{2}\sum_{k=0}^{l} h_k \varphi(2x\text{-}k) \quad (6)$$

⑵ if S_i=1, i-l≤j≤i,

i) From Si=1,we can calculate the number of consecutive blocks, and according to lemma(1) positioned jumps;

ii) The $\hat{\alpha}_{j,i}$ extrapolation from the jump at the left

iii) Jump at the right of the $\overline{\beta}_{j,i} = 0$;

iv) According to the formula (5), by using $\hat{\alpha}_{j,k}$ and $\beta_{j,k}$ reconstructed jump at the left;

v) According to the formula (6) by using $\alpha_{j,k}$ and $\overline{\beta}_{j,k}$ reconstructed jump at the right.

In ENO- orthogonal wavelets transform, in order to maintain the reversible of wavelet transform is good, in addition to saving the wavelet coefficients, the corresponding jump positioning location information is required to be preserved. In actual operation, for each module to open 1 bit space to identify whether it contains jump. For each module that contained jump, will to be increased the complexity of O (L) overhead (floating point) when to be ENO transformation. Suppose that each of length N in the data have d jumps, then, the calculation complexity of ENO- orthogonal wavelet transform is O (DL); and the calculation complexity of orthogonal wavelet transform itself is O (NL). Therefore, the ratio of the complexity of ENO- orthogonal wavelet transform and orthogonal wavelet transform is O(d/n), and calculation complexity of ENO- orthogonal wavelet transform is independent of the length of the module, and the computation of ENO- orthogonal wavelet transform can be ignored when it very large, this is the advantages of ENO method. However, because the orthogonal wavelet have no linear phase characteristics, which will produce phase distortion when ENO- orthogonal wavelet transform be used in image processing. For its shortage, combining the idea of ENO, this paper proposes a ENO- biorthogonal wavelet image compression algorithm.

3. ENO- biorthogonal wavelet transform algorithm transform principle and process

The method will not be correctly positioning when the signal that does not satisfy the DSP condition to be using ENO- orthogonal wavelet transform algorithm to locate the jump, and the ENO- orthogonal wavelet transform algorithm is only suitable for the filter length is even, there are obvious limitations. Now, Leave the DSP condition, put forward a new kind of method to lactation jump, which overcomes the limitation that the ENO- orthogonal wavelet transform algorithm is only suitable for the filter length is only even.

The first, may consider the filter number of EON- biorthogonal wavelet algorithm is odd, when it is even followed by analogy.

Biorthogonal wavelet filter is:$\{h, \tilde{h}, g, \tilde{g}\}$, here{h, g} is the low-pass and high-pass filter of split terminal,$\{\tilde{h}, \tilde{g}\}$is the filter of reconstruction terminal, and the length is odd, about 0 symmetry. The relationship between high pass filter and low pass filter:

$$g_n = (-1)^{n-1}\tilde{h}_{1-n},\ \tilde{g}_n = (-1)^{n-1}h_{1-n} \quad (7)$$

Thus we can deduce: $g_n = g_{2-n}$, $\tilde{g}_n = \tilde{g}_{2-n}$

Assume the length of biorthogonal wavelet split terminal's filter is s, l (both are odd, and not less than 3).The Mallat algorithm for biorthogonal wavelet:

Formula for split:

$$\alpha_{j,i} = \sum_{k=(1-s)/2}^{(s-1)/2} h_k \alpha_{j+1,2k+1} \quad (8)$$

$$\beta_{j,i} = \sum_{k=(3-l)/2}^{(l+1)/2} g_k \alpha_{j+1,2k+1} \quad (9)$$

Formula for restructure:

$$\alpha_{j+1,2i} = \sum_{k=(1-l)/2}^{(l-1)/2} \alpha_{j,i-k}\tilde{h}_{2k} + \sum_{k=(3-s)/2}^{(s-1)/2} \beta_{j,i-k}\tilde{g}_{2k} \quad (10)$$

$$\alpha_{j+1,2i+1} = \sum_{k=(1-l)/2}^{(l-1)/2} \alpha_{j,i-k}\tilde{h}_{2k+1} + \sum_{k=(3-s)/2}^{(s-1)/2} \beta_{j,i-k}\tilde{g}_{2k+1} \quad (11)$$

We can defined the original signal whose length is l as current module, the original signal is matched with the high frequency wavelet coefficients ($\beta_{j,i}$) that is generated by biorthogonal wavelet transform. the intermediate data of the current module is x(2i+1).Assume k=(l-1)/2,then the module's data of high frequency wavelet coefficients (βj,i)is begin as x(2i+1-k),and end as x(2i+1+k). Because the wavelet coefficients of the high pass filter in wavelet transform for smooth areas generated value is small, but is larger when for area that include jumps. So, given an appropriate positive threshold θ, if $|\beta_{j,i}| \geq \theta$, then current module must include jump points, we select the maximum difference value between the current module adjacent two signal values of the absolute value for the looking for jump point, and record Its location information. For the preservation of position information, open up bits, its length equal to the original length of signal, and set st=1, t is the location information of left signal value of jump points, and 2i+1-k≤t≤2i+1+k.the other location information values of no jump are 0.here,no more than one jump be defined for one module. The next jump is beginning with judgment of mode value of ($\beta_{j,i+1}$),if the modulus value is less than threshold, then judged $|\beta_{j,i+k+1}|$.From the above analysis, we obtain the following location method:

Positioning method: if the filter length is l, and l is odd, set k=(l−1)/2, a predetermined thresholdθ>0:

1. For a given index value i, if $|\beta_{j,i}| > \theta$, the

data of current module corresponding to($\beta_{j,i}$) is from x(2i+1-k) to x(2i+1+k). Select the maximum difference value between the current module adjacent two signal values of the absolute value for the looking for jump point, and record Its location information as the left of jump. Set the next index value i=i+k.

2. Else, set the next index value i=i+1, and then executing step 1.

In the jump at the ENO- biorthogonal wavelet transformation, still using the outward interpolation extrapolation method, just jump point may be different. The formula of jump at the ENO- biorthogonal wavelet transform is as follows:

$$\hat{\alpha}_{j,i} = \sum_{k=(1-s)/2}^{(s-1)/2-2} h_k \alpha_{j+1,2i+2} + h_{(s-1)/2-1} \hat{\alpha}_{j+1,2i+(s-1)/2-1} + h_{(s-1)/2} \hat{\alpha}_{j+1,2i+(s-1)/2} \quad (12)$$

$$\bar{\alpha}_{j,i} = \sum_{k=(1-s)/2}^{(s-1)/2-2} h_k \bar{\alpha}_{j+1,2i+2} + h_{(s-1)/2-1} \hat{\alpha}_{j+1,2i+(s-1)/2-1} + h_{(s-1)/2} \hat{\alpha}_{j+1,2i+(s-1)/2} \quad (13)$$

Given the ENO- biorthogonal wavelet transform algorithm:

Assume the signal length is M , Open up a string space(st) its length is M bits ,make 0≤t≤M , for each i:

(1) According to the formula (8) to calculate the value of $\alpha_{j,i}$, According to the formula (9) to calculate the value of $\beta_{j,I}$;

(2) if $|\beta_{j,i}|>\theta$,

then ascertain the jump point(t) by using the positioning method above, set st=1, because *mod(2i+1-k,M)≤t≤mod(2i+1+k,M).* the numbers of coefficients of low frequency wavelet that need be modified at jump point is r=(s−1)/2.at the left of jump point, by using formula (12) we can calculate $\hat{\alpha}_{j,m}$; at the right of jump point, by using formula (13) we can calculate $\bar{a}_{j,m}$. to determine the subscript of coefficients of low frequent that include extensive module of jump point as following:

1. if r is even, then there are r/2 low frequency wavelet coefficient need to be modified at the both sides of jump point:

i) the modify method of low frequency wavelet coefficient at the left of jump point:

$\alpha_{j,m} = \hat{\alpha}_{j,m}$ m = [t/2], [t/2]-1,…, [t/2]-r/2+1;

ii) the modify method of low frequency wavelet coefficient at the right of jump point:

$\alpha_{j,m} = \bar{\alpha}_{j,m}$ m = [t/2]+1, [t/2]+2 ,…, [t/2]+r/2;

2. if r is odd, then need to judge the distribution of low frequency wavelet coefficients that need to be modified at the both sides of jump point according to the parity of t:

i) if t is even, then there are (r+1)/2 low frequency wavelet coefficient need to be modified at the left of jump point, and there are (r-1)/2 at the right:

a) the modify method of low frequency wavelet coefficient at the left of jump point:

$\alpha_{j,m} = \hat{\alpha}_{j,m}$ m=t/2, t/2-1,…, t/2-(r+1)/2+1;

b) the modify method of low frequency wavelet coefficient at the right of jump point:

$\alpha_{j,m} = \bar{\alpha}_{j,m}$ m=t/2+1,t/2+2,…, t/2+(r-1)/2;

ii) if t is odd, then there are (r-1)/2 low frequency wavelet coefficient need to be modified at the left of jump point, and there are (r+1)/2 at the right:

a) the modify method of low frequency wavelet coefficient at the left of jump point:

$\alpha_{j,m} = \hat{\alpha}_{j,m}$ m=t/2, t/2-1,…, t/2-(r-1)/2+1;

b) the modify method of low frequency wavelet coefficient at the right of jump point:

$\alpha_{j,m} = \bar{\alpha}_{j,m}$ m=t/2+1,t/2+2,…, t/2+(r+1)/2

Below is the ENO- biorthogonal wavelet inverse transform process:

(1) the first, for each i:

$$\alpha_{j+1,2i} = \sum_{k=(1-l)/2}^{(l-1)/2} \tilde{h}_{2k} \alpha_{j,i-k} \quad (14)$$

$$\alpha_{j+1,2i+1} = \sum_{k=(1-l)/2}^{(l-1)/2} \tilde{h}_{2k+1} \alpha_{j,i-k} \quad (15)$$

(2) if si=1, then the jump point ahead is {$\alpha_{j+1,i}$, $\alpha_{j+1,i+1}$ }, the reconstruction data numbers that need to be modified at jump point is (l-2).

1. if si is odd, there are (l-3)/2 reconstruction data need to be modified at the left of jump point and (l-1)/2 at the right.

i) the modify method of reconstruction data at the left of jump point:

for m=i,i-1,…,i-(l-3)/2+1, use formula (16) when m is odd, else use formula (17):

$$\alpha_{j+1,m} = \sum_{k=(1-l)/2}^{(l-1)/2} \tilde{h}_{2k} \alpha_{j,m/2-k} \quad (16)$$

$$\alpha_{j+1,m} = \sum_{k=(1-l)/2}^{(l-1)/2} \tilde{h}_{2k+1} \alpha_{j,(m-1)/2-k} \quad (17)$$

ii) the modify method of reconstruction data at the right of jump point:

With the known interpolation formula to calculate:

$\bar{\alpha}_{j,i/2}, \ldots, \bar{\alpha}_{j,(i-l+3)/2}$

And set:

$\alpha_{j,i/2} = \bar{\alpha}_{j,i/2}, \ldots,\ \alpha_{j,(i-l+3)/2} = \bar{\alpha}_{j,(i-l+3)/2}$

For m=i+1,i+2,…,i-(l-1)/2,using formula (16) to modify when m is odd, if m is even, then use formula (17).

2. if *i* is even, then there are (l-1)/2 reconstruction data will be modified at the left of jump point, and there are (l-3)/2 at the right. the modify method like 1.

4. The achieve of image compression by using the ENO- biorthogonal wavelet transform

In the process of using ENO-biorthogonal wavelet transform to compression image, save only the wavelet coefficients of coarse scale space LL after ENO transformation, as well as the jump location marker information. In fact, the mark information structure is very special, it is composed of a large number of continuous 0 and a few of 1. The mark information can be lossless compression by using run length encoding [7]. For the special structure data of marking information, the compression rate of run length encoding is very high. Due to the occupation of the compressed marker information space is very small, so we leave aside the storage amount of marker information. The ratio between the storage of the original image with the LL produced by ENO - biortogonal wavelet transform is about 4:1.

The compression procedure of one layer ENO-biorthogonal wavelet transform image is as follows:

1. the first, decompose the row of image data with ENO-biorthogonal wavelet, get the low frequency wavelet coefficients L, the mark information at jump point stored in S1.

2. decompose the Column of L with ENO-biorthogonal wavelet, get the low frequency wavelet coefficients LL, the mark information at jump point stored in S2.

3. The reconstruction order is opposite: reconstruct the column of LL with S2 back to L by using ENO-biorthogonal wavelet contrary transform in the first, then reconstruct the row of L with S1 back to image by using ENO-biorthogonal wavelet contrary transform. (reconstruction values are of approximate the original data).

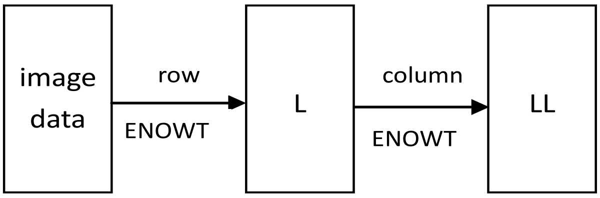

Figure 1. Image compression model of one layer of ENO-biorthogonal wavelet transform

The above steps are only one layer ENO-biorthogonal wavelet transform for image. If transformation is multiple layers, repeat step 1,2 , and reconstruction according to the steps of 3 layer by layer.

In the experiment, the Test image is decomposed into one layer and two layer with CDF53, CDF97 and ENO-CDF53, ENO-CDF97 algorithm of wavelet transform, and then reconstruction by using decomposition or the LL , table 1 is experimental results, numerical value in the table is the mean square error of MSE, numerical value in parentheses is the threshold.

Table 1 and table 2 shows the superiority of ENO- biorthogonal wavelet transform, especially, the result of processing image that include jump points is very well.

Table 1. Mean square error of Test image after the four transformation

Decompose layer	Compression ratio	Test(512×512pixels)			
		CDF53	ENO-CDF53 (50)	CDF97	ENO-CDF97 (40/30)
One layer	4:1	□□□□□□	□□□□□□□□□	□□□□□□□	□□□□□□□□
Two layer	16:1	□□□□□□□□	□□□□□□□□□	□□□□□□□□	□□□□□□□□□

Table 2. The result of comparison with test image

Decompose layer	Threshold θ	NEZW-CDF53		ENO-CDF53	
		MSE	The number of high frequency coefficient that	MSE	The number of location information

			greater than θ		
One layer	10	11.861984	2794	3.286114	2031
	20	170.731285	2308	3.286114	2031
	30	188.097870	2268	3.286114	2031
	40	265.928154	2085	3.214008	2029
	50	377.947819	1950	3.202801	2024
Two layer	10	79.530941	4762	6.049480	3040
	20	521.394810	3745	5.948719	3029
	30	717.269836	3425	5.871258	3024
	40	845.762287	3093	5.762703	3017
	50	924.711792	2877	5.675087	3008

The following three figures are the difference of experimental effect by using different method.

Figure 2. The original 512 × 512 pixel Lena image

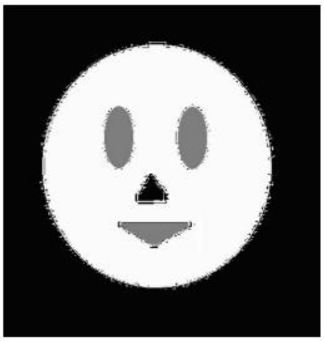

Figure 3. The construction image by two wavelet transform on θ=20 of the NEZW-CDF53, MSE =521.394810

Figure 4. The LL reconstruction image by a wavelet transform on should θ = 11, ENO-CDF53, MSE=5.948719

5. Conclusion

In fact, this paper has constructed a kind of special linear phase filter by using ENO-biorthogonal wavelet transform algorithm, after filter, it through move time scale to achieve zero phase filtering. As the phase is proportional to frequency, so the linear phase does not change the waveform. In normal band, frequency versus phase shift is linear, and the cycle must be an integer multiple of 2π. This filter produces no phase distortion. If the cycle is odd times π, the filter will put the wavelet phase. Linear phase filter is sometimes used as a part of the polarization filter, in order to remove the horizontal or vertical component vibration.

According the character of linear phase and perfect reconstruction of biorthogonal wavelet filter, using ENO- biorthogonal wavelet transform algorithm for image compression processing, has overcome the limited that ENO- orthogonal wavelet transform algorithm is only suitable for the filter length is only even, the experimental results verify the Excellent performance and catholicity for image processing of ENO- biorthogonal wavelet transform algorithm and ENO-biorthogonal wavelet.

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