An Approach of Associated Continued Fractions Blending Rational Interpolation

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Abstract
Interpolation has widely application in signal processing, numerical integration, Computer Aided Geometric Design (CAGD), engineering technology and electrochemistry. As we know, Newton's polynomial interpolation may be the favorite linear interpolation, Thiele continued fractions interpolation is the favorite nonlinear rational interpolation. An approach to associated continued fractions rational interpolation–three associated continued fractions rational interpolation is constructed. Recursive algorithm, interpolation theorem and unique theorem are discussed. Error estimation is worked out and numerical example shows the method can avoid the high times of polynomial and deal with the interpolation problems where inverse differences are nonexistent or unattainable points occur. Multivariate case and some new interpolation scheme as its generalization are given in remarks.
Key words: ASSOCIATED CONTINUED FRACTIONS; DIVIDED DIFFERENCES; INVERSE DIFFERENCES

1. Introduction

The problem of constructing a continuously defined function from given discrete data is unavoidable whenever one wishes to manipulate the data in a way that requires information not included explicitly in the data. In this age of ever-increasing digitization in the storage, processing, analysis, and communication of information, it is not difficult to find examples of applications where this problem occurs. So interpolation method plays an important pole in numerical analysis. The classical method is polynomial interpolation. Rational interpolation and approximation have better property than polynomial interpolation and approximation.

Many people studied rational interpolation and approximation via Thiele type continued fractions, associated continued fractions and barycentric rational interpolation[1-8].The problem of constructing a continuously defined function from given discrete data is unavoidable whenever one wishes to manipulate the data in a way that requires information not included explicitly in the data. In this age of ever-increasing digitization in the storage, processing, analysis, and communication of information, it is not difficult to find examples of applications where this problem occurs. Some scholars have been studying their application in image processing, numerical integration, Computer Aided Geometric Design (CAGD), engineering technology, curves/surface[1,8-10]. Our contribution in this paper is to construct a new type of associated continued fractions interpolation.

The organization of the paper is as follows. We obtain the interpolation algorithm and interpolant theorem in section 2. Unique theorem is discussed in section 3. Error estimation is provided in section 4. Section 5 presents some remarks. Numerical example is given in section 6 to show the method can avoid the high times of polynomial and deal with the interpolation problems where inverse differences are nonexistent or unattainable points occur.

2. Three Associated Continued Fractions Interpolation

Given a set of real points $X_n = \{x_0, x_1, x_2, x_3, x_4, x_5, \cdots, x_{3[n/3]+1}, x_{3[n/3]+2}\} \subset [a,b] \subset R$ and a function $f(x_i)(i = 0,1,\cdots,3[n/3]+2)$. We can establish an associated continued fractions

$$r_{3[\frac{n}{3}]+2}(x) = \omega_0[x_0] + \omega_1[x_1](x-x_0) + \omega_2[x_2](x-x_0)(x-x_1) + \frac{(x-x_0)(x-x_1)(x-x_2) \mid}{\mid \omega_3[x_3] + \omega_4[x_4](x-x_3) + \omega_5[x_5](x-x_3)(x-x_4)} + \cdots + \frac{(x-x_{3[\frac{n}{3}]-3})(x-x_{3[\frac{n}{3}]-2})(x-x_{3[\frac{n}{3}]-1}) \mid}{\mid \omega_{3[\frac{n}{3}]}[x_{3[\frac{n}{3}]}] + \omega_{3[\frac{n}{3}]+1}[x_{3[\frac{n}{3}]+1}](x-x_{3[\frac{n}{3}]}) + \omega_{3[\frac{n}{3}]+2}[x_{3[\frac{n}{3}]+2}](x-x_{3[\frac{n}{3}]})(x-x_{3[\frac{n}{3}]+1})}, \quad (1)$$

where

$$\omega_0[x_i] = f(x_i) \qquad \forall x \in X_n, \quad (2)$$

$$\omega_{3k}[x_{3k}] = \omega_{3k}[x_0, \cdots, x_{3k-1}, x_{3k}] = \frac{x_{3k} - x_{3k-1}}{\omega_{3k+1}[x_0, \cdots, x_{3k-2}, x_{3k}] - \omega_{3k-1}[x_0, \cdots, x_{3k-2}, x_{3k-1}]}, \quad (3)$$

$$\omega_{3k+1}[x_{3k+1}] = \omega_{3k+1}[x_0, \cdots, x_{3k}, x_{3k+1}] = \frac{\omega_{3k}[x_0, \cdots, x_{3k-1}, x_{3k+1}] - \omega_{3k}[x_0, \cdots, x_{3k-1}, x_{3k}]}{x_{3k+1} - x_{3k}}, \quad (4)$$

$$\omega_{3k+2}[x_{3k+2}] = \omega_{3k+2}[x_0, \cdots, x_{3k+1}, x_{3k+2}] = \frac{\omega_{3k+1}[x_0, \cdots, x_{3k}, x_{3k+2}] - \omega_{3k+1}[x_0, \cdots, x_{3k}, x_{3k+1}]}{x_{3k+2} - x_{3k+1}}, \quad (5)$$

where $k = 0,1,\cdots,[n/3]$, $[\mu]$ is entire function. We call the interpolation $r_{3[n/3]+2}(x)$ is three associated continued fractions interpolation.

From the algorithm as showed above, we can get the following theorem.

Theorem 1: Given the sets $X_n = \{x_0, x_1, x_2, x_3, x_4, x_5, \cdots, x_{3[n/3]+1}, x_{3[n/3]+2}\} \subset [a,b] \subset R$ and a function $f(x)$ defined in $[a,b]$, $\omega_i[x_i]$ are defined by (2)-(5), then we have

$$r_l(x_i) = f(x_i), \quad i = 0,1,\cdots,l, \quad l = 0,1,\cdots,3[n/3]+2. \quad (6)$$

Proof: when $l = 0$, it is obvious we have $r_0(x_0) = f(x_0)$, when $l = 1$, from (3)(4) we can get

$$r_1(x_0) = f(x_0),$$

$$r_1(x_1) = \omega_0(x_0) + \omega_1[x_0, x_1](x_1 - x_0) = \omega_0(x_0) + \frac{\omega_0[x_1] - \omega_0[x_0]}{x_1 - x_0}(x_1 - x_0) = \omega_0[x_1],$$

When $l = 2$, from (3)-(5) we can get

$$r_2(x_0) = f(x_0),$$

$$r_2(x_1) = \omega_0(x_0) + \omega_1[x_0, x_1](x_1 - x_0) = \omega_0(x_0) + \frac{\omega_0[x_1] - \omega_0[x_0]}{x_1 - x_0}(x_1 - x_0) = \omega_0[x_1],$$

$$r_2(x_2)=\omega_0(x_0)+\omega_1[x_0,x_1](x_2-x_0)+(x_2-x_0)(x_2-x_1)\omega_2[x_0,x_1,x_2]$$
$$=\omega_0(x_0)+\omega_1[x_0,x_1](x_2-x_0)+(x_2-x_0)(x_2-x_1)\frac{\omega_1[x_0,x_2]-\omega_1[x_0,x_1]}{x_2-x_1}=\omega_0[x_2],$$

Supposed $l=3t+2, t=s<[n/3]$, interpolation formula (1) interpolate $f(x)$ at $\left\{x_0,x_1,x_2,x_3,x_4,x_5,\cdots,x_{3[n/3]+1},x_{3[n/3]+2}\right\}$,

For $t=s+1$,

$$r_{3s+5}(x)=\omega_0[x_0]+\omega_1[x_1](x-x_0)+\omega_2[x_2](x-x_0)(x-x_1)+$$
$$\sum_{k=0}^{s}\frac{(x-x_{3k-3})(x-x_{3k-2})(x-x_{3k-1})\big|}{\big|\ \omega_{3k}[x_{3k}]+\omega_{3k+1}[x_{3k+1}](x-x_{3k})+\omega_{3k+2}[x_{3k+2}](x-x_{3k})(x-x_{3k+1})}$$
$$+\frac{(x-x_{3s})(x-x_{3s+1})(x-x_{3s+2})}{\omega_{3s+3}[x_{3s+3}]+\omega_{3s+4}[x_{3s+4}](x-x_{3s+3})+\omega_{3s+5}[x_{3s+5}](x-x_{3s+3})(x-x_{3s+4})},$$

from hypothesis we can get $r_{3s+5}(x_i)=f(x_i), i=0,1,\cdots,3s+2$. Now we will prove $r_{3s+5}(x_i)=f(x_i), i=3s+3,3s+4,3s+5$,

for $i=3s+3$,

$$r_{3s+5}(x_{3s+3})=\omega_0[x_0]+\omega_1[x_1](x_{3s+3}-x_0)+\omega_2[x_2](x_{3s+3}-x_0)(x_{3s+3}-x_1)+$$
$$\sum_{k=0}^{s}\frac{(x_{3s+3}-x_{3k-3})(x_{3s+3}-x_{3k-2})(x_{3s+3}-x_{3k-1})\big|}{\big|\ \omega_{3k}[x_{3k}]+\omega_{3k+1}[x_{3k+1}](x_{3s+3}-x_{3k})+\omega_{3k+2}[x_{3k+2}](x_{3s+3}-x_{3k})(x_{3s+3}-x_{3k+1})}$$
$$+\frac{(x_{3s+3}-x_{3s})(x_{3s+3}-x_{3s+1})(x_{3s+3}-x_{3s+2})}{\omega_{3s+3}[x_{3s+3}]}$$
$$=\omega_0[x_0]+\omega_1[x_1](x_{3s+3}-x_0)+\omega_2[x_2](x_{3s+3}-x_0)(x_{3s+3}-x_1)+$$
$$\sum_{k=0}^{s}\frac{(x_{3s+3}-x_{3k-3})(x_{3s+3}-x_{3k-2})(x_{3s+3}-x_{3k-1})\big|}{\big|\ \omega_{3k}[x_{3k}]+\omega_{3k+1}[x_{3k+1}](x_{3s+3}-x_{3k})+\omega_{3k+2}[x_{3k+2}](x_{3s+3}-x_{3k})(x_{3s+3}-x_{3k+1})}$$
$$+\frac{(x_{3s+3}-x_{3s})(x_{3s+3}-x_{3s+1})(x_{3s+3}-x_{3s+2})}{\frac{x_{3s+3}-x_{3s+2}}{\omega_{3s+2}[x_0,\cdots,x_{3s+1},x_{3s+3}]-\omega_{3s+2}[x_0,\cdots,x_{3s+1},x_{3s+2}]}}$$
$$=\omega_0[x_0]+\omega_1[x_1](x_{3s+3}-x_0)+\omega_2[x_2](x_{3s+3}-x_0)(x_{3s+3}-x_1)+$$
$$\sum_{k=0}^{s-1}\frac{(x_{3s+3}-x_{3k-3})(x_{3s+3}-x_{3k-2})(x_{3s+3}-x_{3k-1})\big|}{\big|\ \omega_{3k}[x_{3k}]+\omega_{3k+1}[x_{3k+1}](x_{3s+3}-x_{3k})+\omega_{3k+2}[x_{3k+2}](x_{3s+3}-x_{3k})(x_{3s+3}-x_{3k+1})}$$
$$+\frac{(x_{3s+3}-x_{3s-3})(x_{3s+3}-x_{3s-2})(x_{3s+3}-x_{3s-1})\big|}{\big|\ \omega_{3s}[x_{3s}]+\omega_{3s+1}[x_{3s+1}](x_{3s+3}-x_{3s})+\omega_{3s+2}[x_{3s+2}](x_{3s+3}-x_{3s})(x_{3s+3}-x_{3s+1})}$$
$$+\frac{(x_{3s+3}-x_{3s})(x_{3s+3}-x_{3s+1})(x_{3s+3}-x_{3s+2})}{\frac{x_{3s+3}-x_{3s+2}}{\omega_{3s+2}[x_0,\cdots,x_{3s+1},x_{3s+3}]-\omega_{3s+2}[x_0,\cdots,x_{3s+1},x_{3s+2}]}}$$

$$= \omega_0[x_0] + \omega_1[x_1](x_{3s+3} - x_0) + \omega_2[x_2](x_{3s+3} - x_0)(x_{3s+3} - x_1) +$$
$$\sum_{k=0}^{s-1} \frac{(x_{3s+3} - x_{3k-3})(x_{3s+3} - x_{3k-2})(x_{3s+3} - x_{3k-1})\Big|}{\Big| \ \omega_{3k}[x_{3k}] + \omega_{3k+1}[x_{3k+1}](x_{3s+3} - x_{3k}) + \omega_{3k+2}[x_{3k+2}](x_{3s+3} - x_{3k})(x_{3s+3} - x_{3k+1})} +$$
$$\frac{(x_{3s+3} - x_{3s-3})(x_{3s+3} - x_{3s-2})(x_{3s+3} - x_{3s-1})}{\omega_{3s}[x_{3s}] + \omega_{3s+1}[x_{3s+1}](x_{3s+3} - x_{3s}) + \omega_{3s+2}[x_0, \cdots, x_{3s+1}, x_{3s+3}](x_{3s+3} - x_{3s})(x_{3s+3} - x_{3s+1})}$$
$$= \cdots = \omega_0(x_0) + \omega_1[x_0, x_1](x_{3s+3} - x_0) + \omega_2[x_0, x_1, x_{3s+3}](x_{3s+3} - x_0)(x_{3s+3} - x_1)$$
$$= \omega_0[x_{3s+3}] = f(x_{3s+3})$$

for $i = 3s+4$,

$$r_{3s+5}(x_{3s+4}) = \omega_0[x_0] + \omega_1[x_1](x_{3s+4} - x_0) + \omega_2[x_2](x_{3s+4} - x_0)(x_{3s+4} - x_1) +$$
$$\sum_{k=0}^{s} \frac{(x_{3s+4} - x_{3k-3})(x_{3s+4} - x_{3k-2})(x_{3s+4} - x_{3k-1})\Big|}{\Big| \ \omega_{3k}[x_{3k}] + \omega_{3k+1}[x_{3k+1}](x_{3s+4} - x_{3k}) + \omega_{3k+2}[x_{3k+2}](x_{3s+4} - x_{3k})(x_{3s+4} - x_{3k+1})}$$
$$+ \frac{(x_{3s+4} - x_{3s})(x_{3s+4} - x_{3s+1})(x_{3s+4} - x_{3s+2})}{\omega_{3s+3}[x_{3s+3}] + \omega_{3s+4}[x_{3s+4}](x_{3s+4} - x_{3s+3})}$$
$$= \omega_0[x_0] + \omega_1[x_1](x_{3s+4} - x_0) + \omega_2[x_2](x_{3s+4} - x_0)(x_{3s+4} - x_1) +$$
$$\sum_{k=0}^{s} \frac{(x_{3s+4} - x_{3k-3})(x_{3s+4} - x_{3k-2})(x_{3s+4} - x_{3k-1})\Big|}{\Big| \ \omega_{3k}[x_{3k}] + \omega_{3k+1}[x_{3k+1}](x_{3s+4} - x_{3k}) + \omega_{3k+2}[x_{3k+2}](x_{3s+4} - x_{3k})(x_{3s+4} - x_{3k+1})}$$
$$+ \frac{(x_{3s+4} - x_{3s})(x_{3s+4} - x_{3s+1})(x_{3s+4} - x_{3s+2})}{\omega_{3s+3}[x_{3s+3}] + (x_{3s+4} - x_{3s+3})\frac{\omega_{3s+3}[x_0, \cdots, x_{3s+2}, x_{3s+4}] - \omega_{3s+3}[x_0, \cdots, x_{3s+2}, x_{3s+3}]}{x_{3s+4} - x_{3s+3}}}$$
$$= \omega_0[x_0] + \omega_1[x_1](x_{3s+3} - x_0) + \omega_2[x_2](x_{3s+3} - x_0)(x_{3s+3} - x_1) +$$
$$\sum_{k=0}^{s} \frac{(x_{3s+3} - x_{3k-3})(x_{3s+3} - x_{3k-2})(x_{3s+3} - x_{3k-1})\Big|}{\Big| \ \omega_{3k}[x_{3k}] + \omega_{3k+1}[x_{3k+1}](x_{3s+3} - x_{3k}) + \omega_{3k+2}[x_{3k+2}](x_{3s+3} - x_{3k})(x_{3s+3} - x_{3k+1})}$$
$$+ \frac{(x_{3s+3} - x_{3s})(x_{3s+3} - x_{3s+1})(x_{3s+3} - x_{3s+2})}{\omega_{3s+3}[x_0, \cdots, x_{3s+2}, x_{3s+4}]}$$
$$= \cdots = \omega_0(x_0) + \omega_1[x_0, x_1](x_{3s+4} - x_0) + \omega_2[x_0, x_1, x_{3s+4}](x_{3s+3} - x_0)(x_{3s+4} - x_1)$$
$$= \omega_0[x_{3s+4}] = f(x_{3s+4}),$$

For $i = 3s+5$, we can prove the theorem similarly.

So we have proved the theorem.

3. Unique Theorem

We can obtain unique theorem of three associated continued fractions interpolation, if all the $\varphi_i[x_i](i = 0,1,\cdots,3[n/3]+2)$ exist.

Theorem 2: (unique theorem) three associated continued fractions interpolation $r_{3[n/3]+2}(x)$ of function $f(x)$, if it exists then it is unique.

Proof: Suppose

$$r_{3[n/3]+2}(x) = b_0 + b_1[x_1](x - x_0) + b_2(x - x_0)(x - x_1) + \frac{(x - x_0)(x - x_1)(x - x_2)\Big|}{\Big| \ b_3 + b_4(x - x_3) + b_5(x - x_3)(x - x_4)} + \cdots +$$
$$\frac{(x - x_{3[n/3]-3})(x - x_{3[n/3]-2})(x - x_{3[n/3]-1})\Big|}{\Big| \ b_{3[n/3]} + b_{3[n/3]+1}(x - x_{3[n/3]}) + b_{3[n/3]+2}(x - x_{3[n/3]})(x - x_{3[n/3]+1})}$$
$$\tilde{r}_{3[n/3]+2}(x) = \tilde{b}_0 + \tilde{b}_1[x_1](x - x_0) + \tilde{b}_2(x - x_0)(x - x_1) + \frac{(x - x_0)(x - x_1)(x - x_2)\Big|}{\Big| \ \tilde{b}_3 + \tilde{b}_4(x - x_3) + b_5(x - x_3)(x - x_4)} + \cdots$$
$$+ \frac{(x - x_{3[n/3]-3})(x - x_{3[n/3]-2})(x - x_{3[n/3]-1})\Big|}{\Big| \ \tilde{b}_{3[n/3]} + \tilde{b}_{3[n/3]+1}(x - x_{3[n/3]}) + \tilde{b}_{3[n/3]+2}(x - x_{3[n/3]})(x - x_{3[n/3]+1})},$$

are two different associated continued fractions interpolation of function $f(x)$, we have

$$r_{3[n/3]+2}(x_i) = \tilde{r}_{3[n/3]+2}(x_i) = f(x_i), \quad i = 0,1,\cdots,3[n/3]+2$$

From $r_{3[n/3]+2}(x_0) = \tilde{r}_{3[n/3]+2}(x_0) \Rightarrow \tilde{b}_0 = b_0$.

From

$r_{3[n/3]+2}(x_1) = \tilde{r}_{3[n/3]+2}(x_1) \Rightarrow \tilde{b}_0 + \tilde{b}_1(x_1 - x_0) = b_0 + b_1(x_1 - x_0) \Rightarrow \tilde{b}_1 = b_1$

.

We suppose $b_i^* = b_i, i = 0,1,\cdots,k(k \le 3\left[\frac{n}{3}\right]-1)$

Then when $k = 3[n/3]$, we can get

$$
\begin{aligned}
& r_{3[n/3]+2}(x_{3[n/3]}) = \tilde{r}_{3[n/3]+2}(x_{3[n/3]}) \\
& \Rightarrow b_0 + b_1[x_1](x_{3[n/3]} - x_0) + b_2(x_{3[n/3]} - x_0)(x_{3[n/3]} - x_1) + \frac{(x_{3[n/3]} - x_0)(x_{3[n/3]} - x_1)(x_{3[n/3]} - x_2)\big|}{\big| \; b_3 + b_4(x_{3[n/3]} - x_3) + b_5(x_{3[n/3]} - x_3)(x_{3[n/3]} - x_4)} + \cdots \\
& + \frac{(x_{3[n/3]} - x_{3[n/3]-3})(x_{3[n/3]} - x_{3[n/3]-2})(x_{3[n/3]} - x_{3[n/3]-1})\big|}{\big| \; b_{3[n/3]}} \\
& = \tilde{b}_0 + \tilde{b}_1[x_1](x - x_0) + \tilde{b}_2(x - x_0)(x - x_1) + \frac{(x - x_0)(x - x_1)(x - x_2)\big|}{\big| \; \tilde{b}_3 + \tilde{b}_4(x - x_3) + b_5(x - x_3)(x - x_4)} + \cdots + \\
& \frac{(x_{3[n/3]} - x_{3[n/3]-3})(x_{3[n/3]} - x_{3[n/3]-2})(x_{3[n/3]} - x_{3[n/3]-1})\big|}{\big| \; \tilde{b}_{3[n/3]}} \\
& \Rightarrow \tilde{b}_{k+1} = b_{k+1}
\end{aligned}
$$

For $k = 3[n/3]+1, k = 3[n/3]+2$, we can prove similarly $\tilde{b}_{k+1} = b_{k+1}$.
So we have $\tilde{b}_i = b_i$ by inducting for $\forall i \in \{0,1,\cdots,3[n/3]+2\}$.

4. Error Estimation

We now turn to a discussion of the error in the approximation of a function $f(x)$ by its three associated continued fractions interpolation. It is easy to verify the following theorem in terms of Newton interpolation formula[3].

Theorem 3: Suppose $[a,b]$ is the smallest interval containing $X_n = \{x_0, x_1, x_2, x_3, x_4, x_5, \cdots, x_{3[n/3]+2}\}$ and the real function $f(x)$ is differentiable in $[a,b]$ up to $3[n/3]+3$ times. Let

$$
\begin{aligned}
& r_{3[n/3]+2}(x) = \omega_0[x_0] + \omega_1[x_1](x - x_0) + \omega_2[x_2](x - x_0)(x - x_1) + \\
& \frac{(x - x_0)(x - x_1)(x - x_2)\big|}{\big| \; \omega_3[x_3] + \omega_4[x_4](x - x_3) + \omega_5[x_5](x - x_3)(x - x_4)} + \cdots + \\
& \frac{(x - x_{3[n/3]-3})(x - x_{3[n/3]-2})(x - x_{3[n/3]-1})\big|}{\big| \; \omega_{3[n/3]}[x_{3[n/3]}] + \omega_{3[n/3]+1}[x_{3[n/3]+1}](x - x_{3[n/3]}) + \omega_{3[n/3]+2}[x_{3[n/3]+2}](x - x_{3[n/3]})(x - x_{3[n/3]+1})} \\
& = \frac{N(x)}{D(x)}
\end{aligned}
\tag{7}
$$

and satisfies

$$Q(x_i) = f(x_i), i = 0,1,\cdots,3[n/3]+2. \tag{8}$$

Then for every $x \in [a,b]$, there exists a point $\xi \in [a,b]$ such that

$$f(x) - Q(x) = \frac{\omega(x)}{D(x)} \frac{[f(x)D(x) - N(x)]_{x=\xi}^{(3[n/3]+3)}}{(3[n/3]+3)!} \tag{9}$$

where $\omega(x) = \prod_{i=0}^{3[n/3]+2} (x - x_i)$.

5. Some Remarks

The three associated continued fractions rational interpolation we have studied in section 2 has many advantages, such as the method can avoid the high times of polynomial and deal with the interpolation problems where inverse differences are nonexistent or unattainable points occur. In this section, we give some generalization as follows.

Remark 1: We can generalize the result in the paper to multivariate case;

Remark 2: By using the method in paper[11], one can generalize the conclusion to univariate block based interpolation case and multivariate block based blending rational interpolation case;

Remark 3: One can construct many new kinds of symmetry blending rational interpolation by combining three associated continued fractions interpolation with Newton interpolation polynomial, Thiele-type continued fractions rational interpolation, associated continued fractions interpolation and barycentric interpolation;

Remark 4: One can construct Viscovatov-like algorithm[1] for the three-associated continued fractions similar to the method Viscovatov algorithm for a function given the formal power series;

Remark 5: One can generalize the interpolation scheme to vector valued case and matrix valued case[1, 12];

Remark 6: One can construct many kinds of multivariate blending rational interpolation and its dual interpolation and block based case by combining three associated continued fractions interpolation with Newton interpolation polynomial, Thiele-type continued fractions rational interpolation, associated continued fractions interpolation and barycentric interpolation;

Remark 7: We construct three associated continued fractions blending rational interpolation by using divided differences and inverse differences. In this paper, however, one also can construct the interpolation scheme with the general frames of interpolation in[13-15] via choosing parameters appropriately.

6. Numerical Example

In this section, we present a simple example to show the effectiveness of our method and it could be used to solve the interpolation problems where inverse differences are nonexistent or unattainable points occur and avoid the high times of polynomial.

Example Suppose the interpolating points and the prescribed values of $f(x)$ at the support abscissa x_i are given as follows:

Table 1. Interpolation data

$x_0=-2$	$x_1=-1$	$x_2=0$	$x_3=1$	$x_4=2$	$x_5=3$	$x_6=4$
$f_0=-\frac{3}{5}$	$f_1=-\frac{1}{3}$	$f_2=0$	$f_3=3$	$f_4=\frac{7}{3}$	$f_5=\frac{13}{5}$	$f_6=3$

We can find (0,0) is unique unattainable point by methods in [4]. We could construct the interpolation formula by the conclusions in this paper, but we could not construct them with Thiele type continued fraction, and we also could not construct associated continued fraction interpolation by the methods in [14].

Scheme 1, Newton interpolant polynomial,

$$r_1(x)=-\frac{3}{5}+\frac{4}{15}(x+2)+\frac{1}{30}(x+2)(x+1)+\frac{13}{30}(x+2)(x+1)x-\frac{67}{180}(x+2)(x+1)x(x-1)$$
$$+\frac{149}{900}(x+2)(x+1)x(x-1)(x-2)-\frac{181}{3600}(x+2)(x+1)x(x-1)(x-2)(x-3)$$

scheme 2, three associated continued fractions interpolant formula

$$r_2(x)=-\frac{3}{5}+\frac{4}{15}(x+2)+\frac{1}{30}(x+2)(x+1)+\frac{(x+2)(x+1)x\,|}{|\,\frac{30}{13}+\frac{2010}{143}(x-1)+\frac{1400}{143}(x-1)(x-2)}$$
$$+\frac{(x-1)(x-2)(x-3)\,|}{|\,\frac{143}{400}}$$

It is easy to verify $r_i(x),(i=1,2)$ satisfies the conditions of interpolation.

Conclusion

By using divided difference and inverse difference, we construct a new type associated continued fractions rational interpolation–three associated continued fractions rational interpolation. We discussed the interpolation theorem and unique theorem. By applying the method in paper we can generalize the results to multivariate case[1,12]. We can generalize the results to vector valued case and matrix valued case with samlson inverse. We can generalize the results to block based interpolation case, multivariate block based interpolation case, and one can get many new blending rational by combing the results with Newton interpolation, Thiele continued fractions interpolation, associated continued fractions interpolation and symmetry interpolation and barycentric interpolation.

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