A Multi-objective Ant Colony Optimization Algorithm Based on Elitist Selection Strategy

Xiangui Shi, Dekui Kong

School of Information Science and Technology, Anhui Xinhua University, Hefei 230088, Anhui, China

Abstract
Multi-objective optimization problem is a kind of common optimization problem in science and engineering. This paper explores the improvement strategy of multi-objective ant colony algorithm and proposes an Elitist Multi-objective Ant Colony Optimization (EMOACO). This method proposes to improve ant colony fitness based on Pareto non-dominated set, performs local search on every individual generated in the ant colony algorithm and accelerates the parallel search of multiple objectives by adopting elite selection strategy in order to increase its search rate. The experimental result shows that the algorithm of this paper is effective and that it makes some improvements in global optimization capacity and population diversity compared with the basic multi-objective ant colony algorithm, it can quickly converge to Pareto optimal solution and provide a reliable basis for the decision making.
Key words: MULTI-OBJECTIVE, ANT COLONY OPTIMIZATION, ELITIST SELECTION STRATEGY

1. Introduction

At present, multi-objective optimization problem has been applied in an increasing number of fields. In daily life and engineering, not only one index but multiple indexes are required to achieve optimization at the same time[1]. Numerous problems can be summarized into the multi-objective optimization problem with multiple objectives achieving optimization at the same time under certain constraints. In the field of scientific research, scientific experiment in particular, the experimental goal is not merely a single goal but the integration of multiple goals, therefore, the in-depth research of multi-objective optimization problem and its algorithms have significant influence on the economic development and scientific progress[2]. Currently, importance has been attached to the research of the algorithms for multi-objective optimization problems, however, the search on algorithm is to be further improved, therefore, how to realize multi-objective optimization in the combination of algorithm research and the actual engineering problems is a topic worthy of attention[3].

Multiple objectives are a branch of mathematical programming. At the earliest, the optimization problems which are incomparable are investigated. Then some mathematicians have made some in-depth exploration, nevertheless, there isn't a satisfactory definition so far. The methods to seek multi-objective programming are classified as follows.The first is to covert multiple into few, namely to convert multiple objectives into single or dual objectives which are easy to resolve, including main objective method, linear weighting method and ideal point method. The second is called layered sequencing method, which gives a sequence of the objectives according to the importance. It seeks the optimalsolution to the next objective from the optimal solution set of the previous objective every time until it finds the common optimal solution. Revised simplex method can be used in multi-objective linear programming apart from the above methods[4]. The third is analytic hierarchy method, which is a multi-objective decision and analytic method combining qualitative analysis and quantitative analysis and which is very practical in complicated objective structure with lack of necessary data. In recent years, intelligent algorithm has demonstrated increasingly powerful vitality in solving multi-objective optimization problem and those intelligent optimization algorithms not only search Pareto optimal solution from the deduction of pure mathematics but from the derivation of the crossover field formed by development of life science and information science. Intelligent optimization algorithms use iterative computations to resolve multi-objective optimization problems by simulating such life signs as biological evolution and population activities[5].

At first, this paper describes multi-objective optimization problem and the basic ant colony algorithms. Then, on this basis, it improves Multi-objective Ant Colony Optimization (MOACO) through elistist strategy, helping this algorithm to overcome the defects in global search and termination criterion, so as to make the improved algorithm can not only provide more extensive search in the space of the multi-objective optimization problem, but also converge to the optimal solution in a reliable and rapid manner and provide some assistance in the final decision making. At last, it compares Elistist Multi-objective Ant Colony Optimization (EMOACO) with Basic Multi-objective Ant Colony Optimization (BMOACO) under the same test functions.

2. Description of Multi-objective Optimization Problem

In words, multi-objective optimization problem is such an optimization problem constituted by D decision variable parameters, N objective function(s) and $m+n$ constraints where the decision variable has a functional relationship with objective function and constraints. In the non-inferior solution set, the decision-makers can only choose one satisfactory non-inferior solution as the final solution according to the specific problems. The mathematical form of multi-objective optimization problem can be described as follows:

$$\min\ y = f(x) = \left[f_1(x), f_2(x), \ldots, f_n(x) \right],\ n = 1,2,\ldots,N$$
$$\text{st}\ g_i(x) \le 0,\ i = 1,2,\ldots,m$$
$$h_j(x) = 0,\ j = 1,2,\ldots,k x = [x_1, x_2, x_d, \ldots, x_D]$$
$$x_{d\_min} \le x_d \le x_{d\_max},\ d = 1,2,\ldots,D \quad (1)$$

Here, x is a D-dimensional decision vector, y is the objective vector and N is the number of all objectives to be optimized. $g_i(x) \le 0$ is the ith inequality constraint, $h_j(x) = 0$ is the jth equality constraint and $f_n(x)$ is the nth objective function. X is the decision space formed by the decision vectors and Y is the objective space by the objective vectors. $g_i(x) \le 0$ and $h_j(x) = 0$ define the feasible region of solution and x_{d_min} and x_{d_max} are the upper and lower limits of the vector search in every dimension[6].

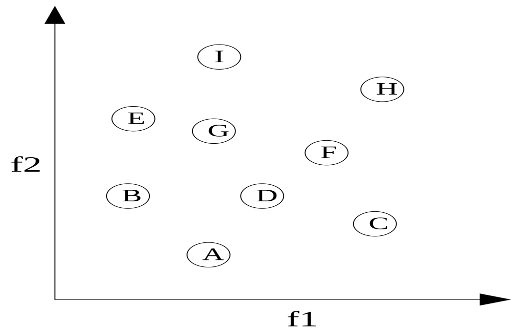


Figure 1. Pareto optimal solution set

There are significant differences between multi-objective optimization problem and single-objective optimization problem. Generally speaking, there is more than one Pareto optimal solution to multi-objective optimization problem but a set, which is named Pareto optimal solution set. The graphical representation of the corresponding objective vectors is called Pareto optimal front-end. When there are many objectives, the optimal solution to all objectives may not exist since the objectives may conflict with each other. Due to the possible conflicts and competition among multiple objectives, Multi-objective Optimization Problem (MOP) has a Pareto optimal solution set instead of a single optimal solution [7]. The Pareto optimal solution set is indicated as Figure 1.

Single-objective decision obtains the optimal solution by comparing different solutions while multi-objective decision needs to use the weighting method to seek the satisfactory solution in addition to determining the inferior and non-inferior solutions.

3. Analysis of Mechanism of Basic Ant Colony Optimization

Artificial ant colony optimization is a simulated evolutionary algorithm based on population which is inspired by the research achievements of the collective behaviors of the real ants in the natural world and it is a random search algorithm.

Ants live together in colonies and they use chemical cues called pheromones to provide a sophisticated communication system, when an ant finds a source of food, it walks back to the colony leaving pheromones that show the path has food. When other ants come across the pheromones, they are likely to follow the path with a certain probability. If they do, they then populate the path with their own pheromones as they bring the food back. As more ants find the path, it gets stronger until there are a couple streams of ants traveling to various and different food sources. The repetition of the above mechanism represents the collective behaviour of a real ant colony which is a form of autocatalytic behaviour where the more the ants follow a trail, the more attractive that trail becomes. Because the ants drop pheromones every time they bring food, shorter paths are more likely to be stronger, hence optimizing the solution that solve the shortest path of multi-objective problem. This algorithm has drawn much attention and it has been incorporated in many optimization problems. Because of the correlation and difference between the artificial ant colony and the real ant colony, to solve the problems with artificial ant colony simulating the real ant colony has made it possible and the practice has proven that this bionic method is effective[8]. Ant colony finding a path from nest to food is shown as following Fig.2.

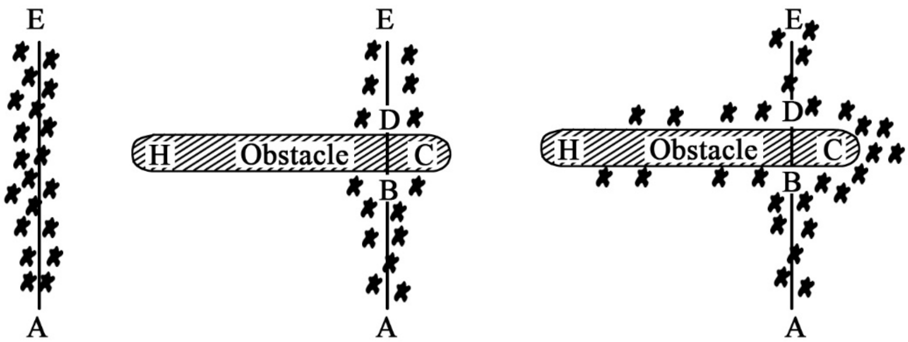


Figure 2. The ant colony can always find a path from its nest to the food

In real life, the capacity and intelligent of a single ant is very simple, however, through collaboration, division of work and cooperation, both the worker ant and the queen ant are fully capable to instruct and complete such complicated behaviors as nesting, foraging, migration and nest cleaning. Additionally, the ant can also get adapted to the environment changes. For example, in the event of sudden obstacles in the motion path of the ant, every ant is uniformly distributed in the first place and then the ants will select different paths at the same probability whether the path is short or not. The information exchange and mutual collaboration among the individuals play an important role in the complicated but orderly behaviors, a similar approach can be used find near-optimal solution to the traveling salesman problem. Assume that there are n cities and m ants and make $d_{ij}(i, j = 1, 2, \cdots, n)$ as the distance between the cities i, j and $\tau_{ij}(t)$ as the pheromone concentration left in the paths between the cities i, j at the time of t. When ant k moves forward, its next step is determined by the pheromone concentration left in every path. Take $p_{ij}^k(t)$ as the probability for the ant k to move from city i to city j at the time of t and then:

$$p_{ij}^k(t) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum_{F \in allowed_k} \tau_{iF}^{\alpha}(t)\eta_{iF}^{\beta}(t)}, j \in allowed_k \\ 0, \qquad otherwise \end{cases} \tag{2}$$

$allowed_k$ is the city set that ant k can move and table $tabu_k$ is the tabu table of ant k. η_{ir} is the heuristic factor, meaning the expectation of ant k to move from city i to city j, which is usually the reciprocal of d_{ij} and α, β are the relatively important procedures of pheromone and heuristic factor in the formula respective. After all the ants finish one traversal, update the pheromone of every path according to formula (3):

$$\tau_{ij}(t+1) = (1-\rho) * \tau_{ij}(t) + \Delta\tau_{ij}$$
$$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^k \tag{3}$$

Here, $\rho(0 \prec \rho \prec 1)$ is the evaporation coefficient, 1-ρ is the persistence coefficient, $\Delta\tau_{ij}$ is the pheromone increment after this iteration and $\Delta\tau_{ij}^{k}$ is the pheromone left by the kth ant in this iteration[9].

4. The Procedures of Multi-objective Ant Colony Optimization Based on Elistist Selection Strategy

The idea of elistist selection originates in the elistist strategy of genetic algorithm.This strategy preserves the excellent potential solutions from the current generation to the next generation in the iterations of the algorithm. One common method is simply copy the corresponding solution from the current generation to the next generation. From the perspective of the entire selection strategy of genetic algorithm, elistist selection is a basic guarantee for the population to converge to the optimal solution of the optimization problem. If the fitness of the optimal individual in the population of the next generation is smaller than that of the current optimal individual, then directly copy the current optimal individual or those individuals with bigger fitness than the next optimal individual to the next generation and randomly replace the worst individual or the same number of individuals in the next generation[10].

Different from single-objective optimization, multi-objective optimization has no absolute optimal solution but relatively superior or inferior solutions, therefore, the amount of pheromone released by the ants in the paper is determined by the solutions and the non-inferior solutions found in every generation are preserved in the elistist set. The individual set generated by every single-objective problem is referred as sub-population and the combination of all sub-populations is called multi-objective population. Every single-objective sub-population is the same. After a new generation of multi-objective population is generated, the individuals in the elistist population will be updated according to the following definition to make sure that the solutions in the elistist population are the Pareto optimal solutions in current meaning. If ant i is a feasible solution and it is non-dominated by Set FL or different from any solution in FL, then ant i can be added to the non-inferior set FL. Besides, when the algorithm fulfills a round of iterations, conduct a neighborhood search to the ants in the elistist set and search the more optimal solution in the Neighborhood δ. If found, replace and update the elistist set, otherwise, keep the set unchanged[11].

Based on the above idea, to improve the ant colony algorithm with elistist selection can accelerate its search speed and effectively find the accurate solution. Its basic steps include the following[12]:

(1) Initialize the population: give the number N of the initial population POP, evaluate the initial population, select the initial position (solution), which is $Rand(start(i), end(i))$. Here, $(start(i), end(i))$ is the scope of the parameters to be optimized. Compute the objective function value $f_i(x)$ and constraint of every ant in POP.

(2) Initialize the elistist set FL, which include all the feasible and non-dominated solutions in POP, namely $X_f = \{x \in POP \mid x \ is \ feasible\}$ and $FL = \left\{x \in X_f \,\middle|\, \nexists x' \in X_f, x' \succ x\right\}$.

(3) Randomly generate a random number s within [0,1] and compare it with parameter, p_0 which is a parameter within the range of [0,1]. If $s \le p_0$, use elistist search method on the current ant i while $s > p_0$, adopt mobile search method within the population on ant i.

(4) After deciding the movement direction, move according to different movement rules. Re-evaluate ant i and compute its objective function value and constraint function value. Search a new position and compute its fitness. If the new position is superior to the previous position, replace the previous position with the new one, otherwise, preserve the previous position.

(5) If ant i meets the update rules of elistist set, add ant i to set FL and delete the solutions dominated by i in FL. Select $loop+1$ (number of $loop$ circulations) positions from the population according to the sequencing fitness and evaluate whether this position is superior to the current selected position with the idea of elistist selection. If so, replace the selected position.

(6) Memorize the optimal position (solution) of the ant search process in the elistist set.

(7) Judge whether it meets the termination conditions. If so, turn to Step (3); otherwise, stop computation.

The following is the basic flowchart of Elistist Multi-objective Ant Colony Optimization (EMOACO)[13,14]:

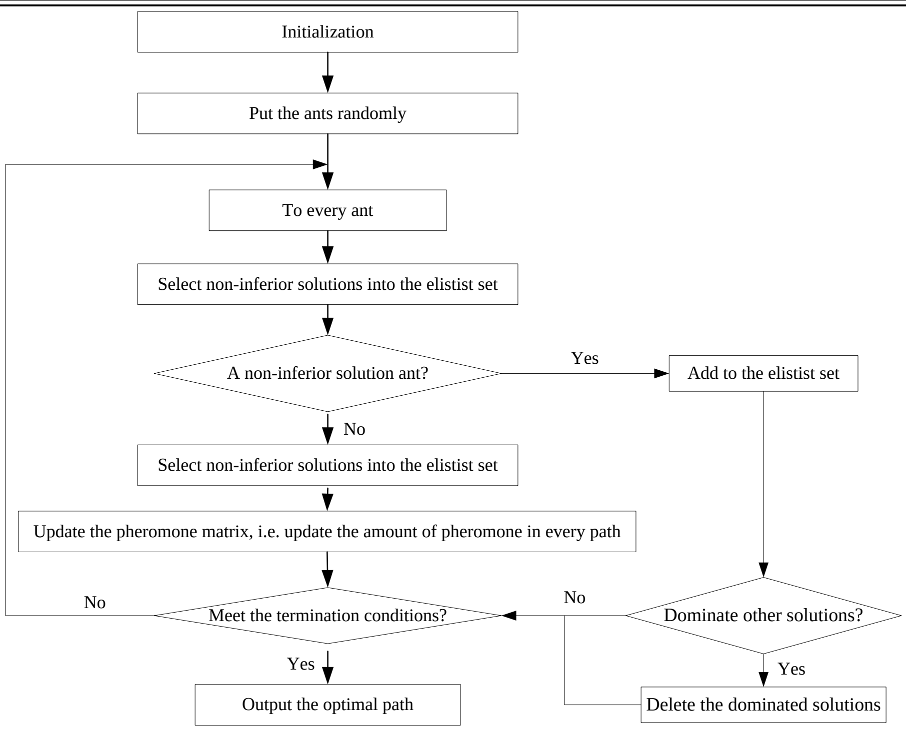


Figure 3. Flowchart of EMOACO

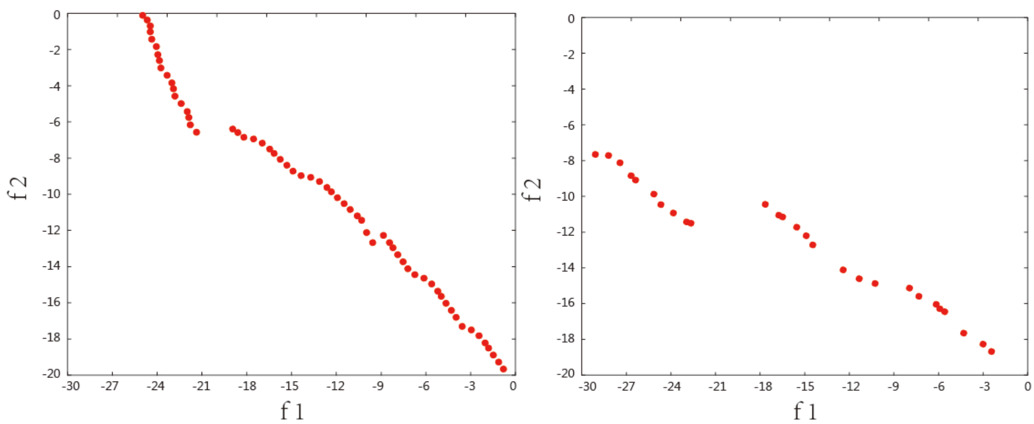
5. Experiment Simulation and Analysis

Basically, the Pareto solution set to multi-objective optimization problem is a continuous curve, therefore, the adoption of EMOACO can select the adjacent two Pareto points as the father to generate new Pareto points in the neighborhoods. In this way, most of the excellent features of the fathers can be passed to the descendants so as to make the entire algorithm converged to Pareto solution set. Termination criterion can greatly reduce the computation amount of the function value. More importantly, it evaluates Pareto set. We have compared EMOACO, the algorithm of this paper with BMOACO (Basic Multi-objective Ant Colony Optimization) under the same test functions, which are as follows:

$$f_1(x) = 1 - \exp\left(-\sum_{i=1}^{3}\left(x_i - \frac{1}{\sqrt{3}}\right)^2\right) \quad (4)$$

$$f_2(x) = 1 - \exp\left(-\sum_{i=1}^{3}\left(x_i + \frac{1}{\sqrt{3}}\right)^2\right) \quad -4 \le x_1, x_2, x_3 \le 4 \quad (5)$$

It can be seen from the Pareto optimal front obtained from the test functions that the algorithm of this paper obtains better and clearer distribution, as shown in Figure 4.



(a) EMOACO (b) BMOACO

Figure 4. The comparison of Pareto optimal front between by EMOACO and BMOACO

It can been seen from the above comparison that the Pareto optimal solution set obtained by the algorithm of this paper has better diversity as well as smooth and uniformly-distributed Pareto optimal solution plane, suggesting that the elistist strategy works and that it can avoid the algorithm from being trapped in local optimum and avoid pre-maturity. Besides, this algorithm can also do a great job in increased complexity. Therefore, when resolving complicated multi-dimensional problem, EMOACO is better than the BMOACO in the global optimization capacity and population diversity, indicating that it is effective in solving multi-objective problems.

6. Conclusion

This paper has proposed an Elitist Multi-objective Ant Colony Optimization (EMOACO), which integrates elitist selection into the iterations and ensures that the solutions of the elitist population are the Pareto optimal solutions. In the final part, this paper verifies this algorithm through basic test functions and the result shows that it is obviously better than the basic multi-objective ant colony optimization in resolving complicated and multi-dimensional optimization problems.

References

1. Juan Rada-Vilela, Manuel Chica, Óscar Cordón, Sergio Damas (2013) A Comparative Study of Multi-Objective Ant Colony Optimization Algorithms for the Time and Space Assembly Line Balancing Problem. *Applied Soft Computing*, 13(11), p.p.4370-4382.
2. Yi-Jun He, Zi-Feng Ma (2014) Optimal Design of Linear Sensor Networks for Process Plants: A Multi-objective Ant Colony Optimization Approach. *Chemometrics and Intelligent Laboratory Systems*, 135(15), p.p.37-47.
3. Yongqiang Gao, Haibing Guan, Zhengwei Qi, Yang Hou, Liang Liu (2013) A Multi-objective Ant Colony System Algorithm for Virtual Machine Placement in Cloud Computing. *Journal of Computer and System Sciences*, 79(8), p.p.1230-1242.
4. Zhaojun Zhang, Na Zhang, Zuren Feng (2014) Multi-satellite Control Resource Scheduling Based on Ant Colony Optimization. *Expert Systems with Applications*, 41(6), p.p.816-2823.
5. Junzhong Ji, Zhijun Liu, Aidong Zhang, Cuicui Yang, Chunnian Liu (2013) HAM-FMD: Mining Functional Modules in Protein–protein Interaction Networks using Ant Colony Optimization and Multi-agent Evolution. *Neurocomputing*, 121(9), p.p.453-469.
6. Rui Xu, Huaping Chen, Xueping Li (2013) A Bi-objective Scheduling Problem on Batch Machines via a Pareto-based Ant Colony System. *International Journal of Production Economics*, 145(1), p.p.371-386.
7. A.A. Mousa, I.M. El_Desoky (2013) Stability of Pareto Optimal Allocation of Land Reclamation by Multistage Decision-based Multipheromone Ant Colony Optimization. *Swarm and Evolutionary Computatio*, 13(12), p.p.13-21.
8. Chengming Qi (2013) Vehicle Routing Optimization in Logistics Distribution Using Hybrid Ant Colony Algorithm. *TELKOMNIKA Indonesian Journal of Electrical Engineering*, 11(9), p.p.5308-5315.
9. Haijiang Wang, Shanlin Yang (2013) Electricity Consumption Prediction Based on SVR with Ant Colony Optimization. *TELKOMNIKA Indonesian Journal of Electrical Engineering*, 11(11), p.p.6928-6934.
10. T.A. Sebaey, C.S. Lopes, N. Blanco, J.A. Mayugo, J. Costa (2013) Two-pheromone Ant Colony Optimization to Design Dispersed Laminates for Aeronautical Structural Applications. *Advances in Engineering Software*, 66(12), p.p.10-18.
11. Thirachit Saenphon, Suphakant Phimoltares, Chidchanok Lursinsap (2014) Combining New Fast Opposite Gradient Search with Ant Colony Optimization for Solving Travelling Salesman Problem. *Engineering Applications of Artificial Intelligence*, 35(10), p.p.324-334.
12. R.F. Tavares Neto, M. Godinho Filho (2013) Literature Review Regarding Ant Colony Optimization Applied to Scheduling Problems: Guidelines for Implementation and Directions for Future Research. *Engineering Applications of Artificial Intelligence*, 26(1), p.p.150-161.
13. Koushik Venkata Narasimha, Elad Kivelevitch, Balaji Sharma, Manish Kumar (2013) An Ant Colony Optimization Technique for Solving Min–max Multi-Depot Vehicle Routing Problem. *Swarm and Evolutionary Computation*, 13(12), p.p.63-73.
14. Li Roozbeh Nia, Mohammad Hemmati Far, Seyed Taghi Akhavan Niaki (2014) A Fuzzy Vendor Managed Inventory of Multi-item Economic Order Quantity Model under Shortage: An Ant Colony Optimization Algorithm. *International Journal of Production Economics*, 155(9), p.p.259-271.