Commercial Optimal Layout Model of Stadium based on Utility Function Theory

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Abstract

Modern stadiums undertake large sports events and meet people's demand for spiritual civilization. However, to sustain their development, these arenas should also conduct affiliated business activities and guarantee their normal business revenue. Therefore, the manner of setting up commercial outlets (i.e., restaurants and supermarkets) in stadiums is a topic that merits investigation. Three studies conducted in different time periods analyzed the Beijing National Stadium, popularly known as the Bird's Nest. In particular, the stadium structure, people stream, characteristics of sports events, parking facilities, audience demands, and other factors were examined. The basic data were collected, and a mathematical model of the commercial optimal layout was constructed based on the production and utility function of the stadium. This model could be used in determining the quantity of the best commercial outlets in different areas, achieving the effect of "minimum cost and maximum utilization," helping improve the operation of the stadium, and providing references for relevant institutions.

Key words: ASYMMETRIC FINGERPRINTING, OBLIVIOUS TRANSFER, MULTICAST COMMUNICATION

1. Introduction

Stadiums play an important societal role by holding large events and enriching the cultural life of people. However, to sustain their normal operations, stadiums must establish supporting commercial services within their premises to obtain sufficient profit and reflect a certain economic property [1]. Through these commercial outlets, stadiums can increase their operating income and satisfy the demands of their audience [2]. At present, relevant research is primarily concentrated on as follows: Chen and Li analyzed the non-optimal layout of the regional commercial combination and proposed the optimal countermeasures [3]. Cai and Chen discussed the green land system layout method to achieve a "win–win" situation between ecological environment protection and tourism development and construction by implementing ecological optimization in the green land system and designing a hot spring resort in China [4]. Xiao applied the influential relation between the dynamic transfer of comparative advantages and the industrial structure upgrading of processing trade to analyze the optimal layout of the geographical position of processing trade in Shandong [5]. The majority of the domestic stadiums present the characteristics of a comprehensive operation. When they hold sports events, these stadiums rapidly organize several temporary parking areas, house utilities, and catering service outlets to meet the demand of their

growing audience; by contrast, during daily operations, stadiums effectively utilize their spare spaces for sports accessories stores, supermarkets, and clubs [6]. The establishments that successfully administer such a system include the Regal Shanghai East Asia Hotel, Huanglong Sports Center Trust-Mart Supermarket, and Hongkou Gymnasium. In this type of a setup, all of the premises of a stadium can be maximized. However, the commercial optimal layout cannot be stereotyped among all stadiums because in actual operations, the structures and audience demand of arenas generally vary. [7] Hence, the process of setting up business outlets and determining the reasonable number of such outlets that can be established in stadiums should be investigated. The commercial layout and operations of stadium can only be improved when their actual conditions are fully considered. The significance and key points of the study are presented in this paper.

2. Modeling basis

2.1 Modeling factors

There are many factors influencing the commercial layout of the stadiums, the differences in stadium capacity, number of audiences, way of transportation, consumption ability and consumption habit etc. all can change the layout of business outlets [8], characterized by the diversity and complication of customer group.

During the holding of events, the main customers are players, judges, coaches, audiences and journalists etc.; during the holding of performance activity, the main customers are performance companies, actors, audiences and journalists; in case of commercial activities the customer groups are transformed into businessmen and consumers etc., while in the ordinary daily operations, the public are the main customers [9]. These customers have a large preference difference, with a significant influence on the commercial layout of the stadiums. In addition, the capacity and positioning of the stadiums themselves also directly decide the size and type of business outlets.

Therefore, during research on the optimal commercial layout, it is required to fully consider the factors of the stadium structure and audience demands. Meanwhile, it is required to propose corresponding hypothesis to ensure that the model is scientific and feasible.

2.2 Modeling theory

It can be known from the utility theory that the audiences' consumption desire value is related with the utility value of business outlets. Different sizes of outlets play different utilizations [10]. At the same time, with the increasing of people stream in the business district, the utility generated will be even greater. Business outlets should be aimed at meeting the maximum utility of people group and it is required to determine their quantity and position [11]. Therefore, during the calculation of layout, by introducing the thought of production function in economics and with the Cobb-Douglas production function theory, the production function can be obtained:

$$Q = f(K,L) = AL^{\alpha}K^{\beta} \quad (1)$$

Where Q is output, K is capital, L is labor, A、α、β are all constants, and $\alpha+\beta=1$, and there is a replaceable relation between the production elements L and K, i.e., under the precondition of remaining the production unchanged, increasing L can decrease K accordingly, and it can be obtained that the utility function is:

$$Q_i = f(m_i,n_i) = Am_i^{\alpha}n_i^{\beta} \quad (2)$$

$$\text{s.t}\begin{cases}\alpha+\beta=1\\ m_iu_{mi}+n_iu_{ni}=U_i\end{cases} \quad (3)$$

Its equilibrium is:

$$\frac{MP_{mi}}{u_{mi}} = \frac{MP_{ni}}{u_{ni}} \quad (4)$$

According to this model, it is possible to calculate the size and quantity of optimal business outlets in area i, i.e. the optimal layout model of the stadium, where Q_i: the utility output of business district i ($i=1,2,3\cdots20$); m_i: number of large business outlets in business district i; n_i: number of small business outlets in business district i; u_{mi}: utility value that can be provided by a large business outlet in business district i; u_{ni}: utility value that can be provided by a small business outlet in business district i; α_i: correspondence of coefficient contribution of large MS in business district i for Q_i to the necessity of large business outlets to be set; β_i: correspondence of coefficient contribution of small MS in business district i for Q_i to the

necessity of large business outlets to be set; U_i : correspondence of total desire value of all people to the utility output value provided by business district i; MP_{mi}: marginal output of utility that can be provided by a large business outlet in business district i; and MP_{ni}: marginal output of utility that can be provided by a small business outlet in business district i

3 Modeling case analysis

3.1 Analysis on stadium base data

With the Bird Nest as an example, the utility function theory is used to construct its commercial optimal layout model, and the commercial layout structure most appropriate for the Bird Nest is obtained through calculation. Bird Nest belongs to a stadium with strong comprehensive functionality and with relatively reasonable business layout, so the validity of this model can be verified in combination with the actual condition of the Bird Nest. First, according to the stadium condition of the Bird Nest, the basic modeling hypothesis is proposed. The Bird Nest is located in the south of central area of the Olympic Park, with a total floor area of 21ha, and number of audience seats up to 91,000. During the Olympic Games, it undertook the opening and closing ceremonies, football and track and field events final of the Olympic Games and Paralympic Games. After the Olympic Games, it became a large venue for the Beijing citizens to participate in sports and entertainment activities and a landmark building, as shown in Figure 1.

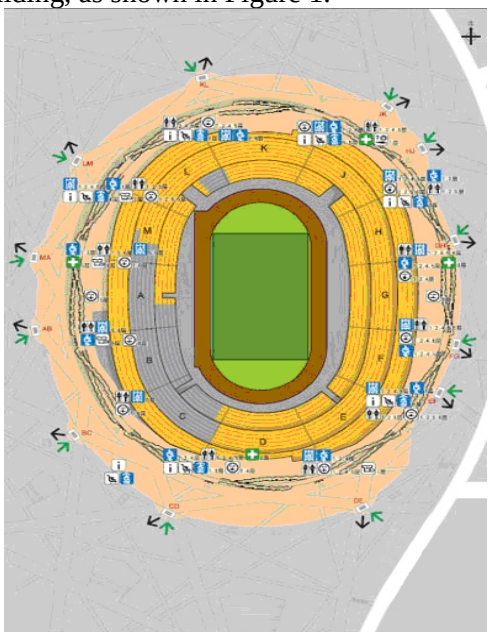

Figure 1. Structural diagram of Bird Nest Stadium

As shown in figure 1, the Bird Nest has a large area, with more seats, comprehensive function, complicated personnel structure and high cost of construction and operation, and flexible financing and operation mechanism were adopted in construction and operation. During operation, in the form of 58% government funding and 42% funding by other companies, the problem of difficulty in financing was solved. During operation, with PDP model, CITIC consortia with 42% shares operate the stadium for 30 years proprietarily. With the operation of professional companies, the Bird Nest has successfully held a lot of sports events, art performances, national fitness entertainment and other projects, whether during the game or in spare time, the people stream is maintained at a higher level, and the requirements for the commercial layout are also relatively high. Based on this, a reasonable hypothesis can be proposed to the audiences of the Bird Nest. Assume that (1) each audience has meal after watching the competition or playing in the stadium; (2) assume that the people stream in each grandstand area is similar; (3) assume that the population proportion and personal preference in the stadium have the same structure. Second, observe the people stream in Bird Nest in a period of time, and their activity law and personal trend, and accumulate the basic data for modeling. In order to ensure the objective and fair modeling, three observation periods are respectively established: sports events, art performance and mass entertainment, and the contents mainly include audience gender, way of traveling, consumption habit and law of people stream in the stadium. The investigation result is as shown in Table 1.

Table 1. Number of male and female audiences in the Bird Nest

sports events	Male	19395	Female	18691
art performance	Male	18683	Female	17254
mass entertainment	Male	8679	Female	7924

It can be known from Table 1 that although the items are different in setting, in the three investigations, the proportion of male and female audiences in the Bird Nest is basically approximate to 10:9, so the male and female proportion in the Bird Nest is set to be 10:9, and the age structure is set as 4 intervals:

below 20 years old, 20-30 years old, 30-50 years old and above 50 years old. Through the three investigations, the population structure can be obtained, as shown in Table 2.

Table 2. Distribution of population in the Bird Nest

Male				Female			
Age group 1	Age group 2	Age group 3	Age group 4	Age group 1	Age group 2	Age group 3	Age group 4
0.070	0.288	0.104	0.092	0.045	0.262	0.076	0.063

On this basis, the people groups of different genders and age intervals in the Bird Nest are analyzed in terms of difference in way of traveling and consumption habit accordingly, so as to reflect the basic structure of "personal trend".

Through the comprehensive analysis on the investigation data in three times, it is found that such group has extremely similar personal trend. In consideration of different total numbers of people in the three investigations, the investigation result is subject to weighted average processing, so as to improve the reliability of data statistics and the comparative diagram of personal trend, as shown in Figure 2.

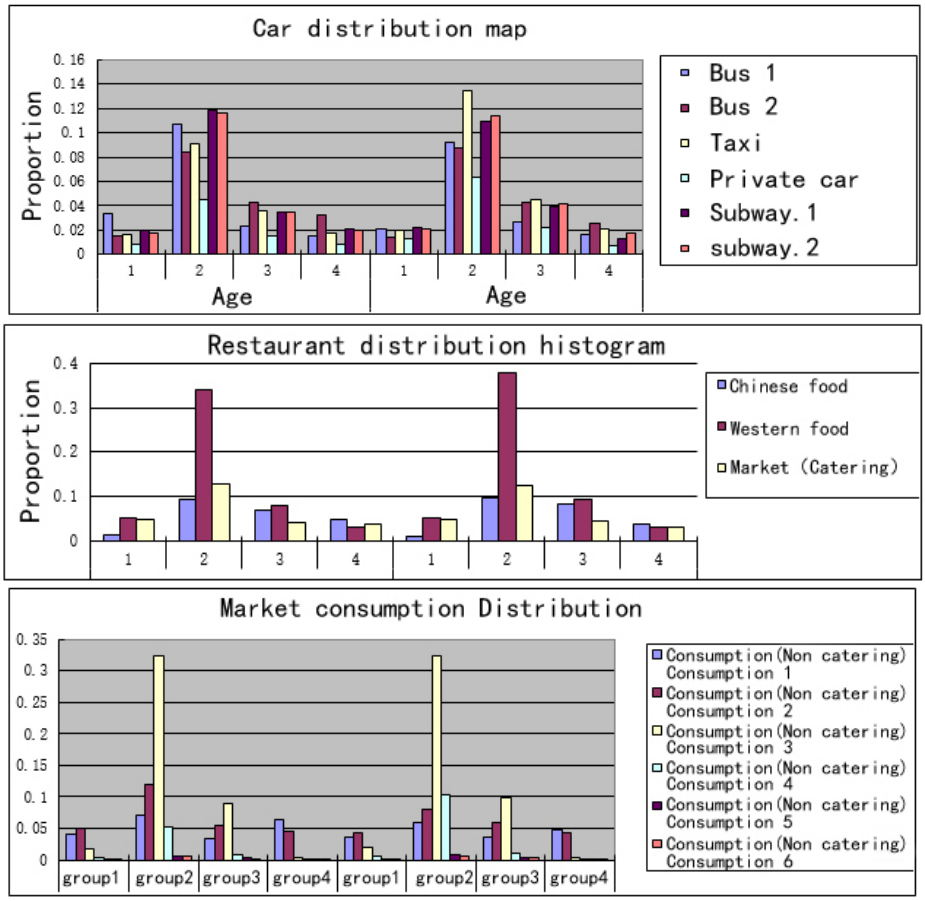

Figure 2. Distribution diagram of personal trend of population in the Bird Nest

The data in the figure show that in the population structure, the population at stage 2 has the highest proportion, and the populations of various age groups are relatively balanced in structure of way of traveling, presenting the characteristics that "most select metro and bus for traveling and few drive private cars", and this might be related with the fact that the Bird Nest is far from the surrounding parking spaces and that the public transportation is developed. In terms of diet habit, the populations of different tenders and age groups all present the characteristics of "less Chinese food and more western food", this might be

related with the rapid and convenient characteristics of western food and strong timeliness of events in the Bird Nest. In the age of the second stage, the proportion of male and female having western food is about 41.2% and 39.5%. It is worth noting that the diet structure of old men is relatively balanced. In terms of consumption habit, due to large base number of population in the second age group, such group is the main force of commercial consumption. According to the result of the three investigations, whether in terms of age or age group, the consumption quota in the stadium is mainly concentrated on Grade 3, i.e. RMB 300-300, and the number of people exceeding Grade 3 presents a decrease trend. Based on this, it is possible to preliminarily master the basic information such as population structure and personal trend in the Bird Nest, so as to provide convenience for the follow-up analysis and modeling.

3.2 Analysis on law of people steam in the stadium

In this experimental study, AUS, average ranking score (RS) and precision are used to evaluate the effect of user-level personalized recommendation algorithm. These three evaluation indicators are very common and representative in bipartite graph network recommendation.

The setting of business outlets in the stadium is closely related with the distribution of stream of audiences. Setting the corresponding outlets according to the change law of people stream can maximally play its commercial effect. Therefore, in combination with the three hypotheses of audience property and stadium structure of the Bird Nest, the mobility law of audiences in the Bird Nest can be obtained.

It can be known from the statistical investigation result that there are mainly 10 exists in the Bird Next in total, which are corresponding to 10 grandstands. The consumption proportions of the audiences in the stadium are respectively: 72.8% for diet, including about 53.6% for western food and about 19.2% for Chinese food and 27.2% for shopping consumption. According to the principle of shortest path, the traveling route of the audiences during consumption is obtained, as shown in Table 3.

Table 3. Distribution of traveling route of audiences in the Bird Nest

Grandstand	Go out from exit B1	Go out from exit B6
B1	B1	B1 B2 B3 B4 B5 B6
B2	B1B2	B2 B3 B4 B5 B6
B3	B1 B2 B3	B3 B4 B5 B6
B4	B1 B2 B3 B4	B4 B5 B6
B5	B1 B2 B3 B4 B5	B5 B6
B6	B1 B2 B3 B4 B5 B6	B6
B7	B7 B8 B9 B10 B1	B7 B6
B8	B8 B9 B10 B1	B8 B7 B6
B9	B9 B10 B1	B9 B8 B7 B6
B10	B10 B1	B10 B9 B8 B7 B6

It can be known from the table above that take exists B1 and B6 as an example, designed based on the shortest path, during the traveling of audiences of various grandstands, some areas are the inevitable road to take, and the proportion of people stream in these areas decide the layout scale of business outlets. Therefore, they should be further calculated. Assume that the proportion stream proportion in B_i business districts is p_i, take Table 3 as an example, the following can be obtained:

$$p_i = \frac{\alpha B_{1i} + \beta B_{6i}}{\sum_{i=1}^{6} (\alpha B_{1i} + \beta B_{6i})} \quad (5)$$

Where, B_{ij} represents the times of people B_i stream coming out of area B_j, α and β respectively represent the proporiton of people stream in different exits. It can be seen from Table 3 and the consumption proportion that the proportions of people coming out of exits B_1, B_6 are respectively 27.2% and 72.8%; similarly, the proportion of people stream in various business districts of the Bird Nest can be obtained, as shown in Table 4.

Table 4. Proportion of people stream in various business districts of the Bird Nest during consumption

Business districts	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
Proportion	0.289	0.052	0.086	0.07	0.11	0.17	0.062	0.057	0.049	0.055

Statistics have shown that during the consumption period, there is a difference of people stream in various business districts, with the most people in area B1 and the fewest in area B9. In addition, after activity in the stadium, the distribution of people stream leaving the stadium also will influence the business profit. For example, generally some audiences of the Bird Nest are used to purchasing relevant products when leaving the stadium. Therefore, according to formula (5) and the traveling structure, it can be known that the it is relatively convenient for the audiences taking bus and metro to select the exit opposite to exist B_6; and it is relatively convenient for the audiences taking private cars and taxies to select the exit opposite to B_1. In combination with the investigation data, the proportion of people stream in various business districts during the traveling period can be obtained, as shown in Table 5.

Table 5. Proportion of people stream in various business districts of the Bird Nest during traveling

Business districts	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
Proportion	0.258	0.096	0.082	0.098	0.091	0.116	0.04	0.052	0.082	0.085

In consideration that the activity periods of audiences of the Bird Nest in the stadium are between the intervals of consumption and traveling, so the overall distribution of people stream on that day should be the average value of two stages, and the proportion of people stream in various business districts of the Bird Nest on that day is obtained, as shown in Table 6.

Table 6. Proportion of people stream in various business districts of the Bird Nest

Business districts	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
Proportion	0.274	0.074	0.084	0.084	0.101	0.143	0.051	0.055	0.066	0.070

It can be known from the table that in the Bird Nest, the people stream in area B1 is higher than in other areas, which meets the basis of establishing a large business outlet. The business potential in other areas is less than area B1, and it is considered to establish a small business outlet. Mastering such law of people stream helps to preliminarily determine which place is most suitable to establish a business outlet, but for the specific construction quantity and which size of business outlet is relatively reasonable, it is required to further construct an optimal layout model for solution.

3.3 Analysis on optimal layout model

The size of people stream decides the size of business potential, while the purchase desire of the population decides the size of business profit. According to the utility theory, the purchase desire value of the population is corresponding to the specific utility of the business outlets, and different sizes of business outlets provide different utilities, and the utility of business outlet expands with the increasing of people stream.

In combination with the production (1), first the necessary proportion of different sizes of business outlets is proposed.

$$\frac{\beta_i}{\alpha_i} = \frac{0.05}{The\ size\ of\ \ people\ stream\ of\ \ i} \tag{6}$$

This reflects the necessity to construct the small business outlet and large business outlet. Then, according to the area proportion of the large and small outlets, the utility ratio is obtained.

$$\frac{u_m}{u_n} = \frac{60}{\frac{(20+50)}{2}} = \frac{12}{7} \tag{7}$$

The area of large outlet is 60, and that

of small one is 20-50.

Where, the utility value of the small business outlets is as follows:

$$u_{ni} = 200000\, p_i \frac{(q_1 + 2q_2 + 3q_3)}{6} \quad (8)$$

In combination with formulas (1),(8), the utility value of small outlets in various business districts of the Bird Nest can be obtained, as shown in Table 7.

Table 7. Utility value of small business outlets of Bird Nest

Business districts	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
Proportion	0.836	0.279	0.312	0.308	0.324	0.553	0.246	0.229	0.196	0.183

The statistics in the figure show that when all areas in the Bird Nest are set as the small business outlets, the utility value in various area is basically consistent with the size of people stream, with the highest value still in area B1; the utility value of the large business outlets is $\frac{12}{7}$ times of that of the small business outlet, so the utility value also presents the same relation with people stream, indicating that the people stream has an important influence on the setting of business outlets. Based on the utility functions (2)-(4) and utility ratio (7), the following can be obtained:

$$\begin{cases} MP_{mi} = \dfrac{\partial Q_i}{\partial m_i} = \alpha A n_i^{\beta} m_i^{\alpha-1} \\ MP_{ni} = \dfrac{\partial Q_i}{\partial n_i} = \beta A n_i^{\beta-1} m_i^{\alpha} \end{cases}$$

$$\Rightarrow \frac{MP_{mi}}{MP_{ni}} = \frac{n_i \alpha_i}{m_i \beta_i} = \frac{u_{mi}}{u_{ni}} \Rightarrow \frac{n_i}{m_i} = \frac{u_{mi}\beta_i}{u_{ni}\alpha_i} \quad (9)$$

When i=1, it can be obtained that the total desire value of population passing through the first business district is U_1 =104288.6; it can be known from (6) and Table 6 that $\frac{\beta_1}{\alpha_1} = \frac{0.05}{0.274}$, and the following can be obtained from formulas (3) (7) and production equilibrium condition:

$$\frac{n_1}{m_1} = \frac{u_{m1}\beta_1}{u_{n1}\alpha_1} = \frac{12}{7} \times \frac{0.05}{0.274} = 0.313$$

$$\frac{12}{7} m_1 u_{n1} + 0.313 m_1 u_{n1} = 104288.6$$

Namely $2.027 m_1 u_{n1} = 104288.6$

therefore, $m_1 = \frac{104288.6}{2.027 u_{n1}} = \frac{104288.6}{2.027 \times 8270} = 5.899$

$n_1 = 0.313 \times 5.899 = 1.847$

After taking the integer, it can be verified from formula (7) that when $m_1 = 3$, $n_1 = 2$, i.e. when the number of large business outlets is 3 and that of the small ones is 2, the utility value in area B_1 is the largest and the business profit is the highest. In a similar fashion, the business layout table of other areas can be obtained, as shown in Table 8.

Table 8. Layout table of various business outlets in the Bird Nest

Business districts	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
(large, small)	(3, 2)	(3, 4)	(2, 4)	(2, 4)	(3, 4)	(3, 3)	(2, 4)	(2, 5)	(2, 5)	(2, 5)

It can be seen from the table above that in areas B1 and B2 with more population, it is possible to set more large business outlets, and in areas B1 and B2 with less population, it is possible to set more small business outlets. These business outlets have certain substitution effect. When maintaining the total desire value of each business district unchanged, it is possible to use more small business outlets to replace a large business outlet. Because the small business outlets are characterized by low cost and flexible setting, they can provide higher unit supply level. Through the comparison of layout of actual business outlets in the Bird Nest, it is found that the modeling calculation result is basically consistent with the setting of the actual area. In the areas with more people stream, there are indeed large supermarkets and restaurants, and licensed merchandise retailing stores and wax work museum are also introduced, which fully meet the different demands of audiences. In the areas with less population, only scattered fast food and retailing pavilions are set, which

reflect the flexibility of operation.

4. Conclusions

Through the investigation of the statistics of the audience population in the Bird's Nest, the population structure and people stream law in this stadium are analyzed in terms of travel mode, consumption behavior, dietary habit, and audience structure. Research has indicated that people of different ages and genders have significantly diverse consumption behaviors, but the population of the same age has relatively similar behavior and psychology. Therefore, a certain law in the population action exists. Mastering the mobility law of the population of different periods is beneficial for establishing a reasonable business layout of stadiums. Large supermarkets must be set up in areas with a large stream of people, whereas small supermarkets should be constructed in areas with a small stream of people. However, the number of these supermarkets should be controlled because their overabundance could strengthen competition, waste resources, and engender population congestion; by contrast, the scarcity of supermarkets could fail to satisfy the demands of the population. In combination with the optimal layout model, the optimal number and type of business outlets in different areas are obtained. Upon actual inspection, the results are generally consistent. This model may be applied in creating a reasonable business layout that considers the actual conditions of the stadium, and thus increase business profit and comprehensively benefit the stadium.

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