A Flexible Job-shop Scheduling under Deteriorating Processing Time Environment

Lin Ren[1, 2]

[1]School of Economics and Management,
Southwest Jiaotong University, Chengdu 610031, China
[2]College of Mathematics and Computer Science,
Hunan city University, Yiyang 413000, China

Guohua Zhou

School of Economics and Management,
Southwest Jiaotong University, Chengdu 610031, China

Fangli Xia, Wang Fei, Chenshun Wu

S College of Mathematics and Computer Science,
Hunan city University, Yiyang 413000, China

Abstract
In order to solve the flexible job shop scheduling problem with deterioration processing time, a model is developed based on the makespan. Improved genetic algorithm is present to explore the relationship and rule between deteriorating processing time and scheduling effect. Finely, a case proves the relationship of deteriorating processing time on the scheduling result. It provides guidance for flexible manufacturing system.
Key words: FLEXIBLE JOB-SHOP SCHEDULING PROBLEM; DETERIORATING EFFECT; GENETIC ALGORITHM; RELATIONSHIP GRAPH

1. Introduction

In traditional production scheduling, usually we assume the processing time of jobs to be a constant and the machine always available. But in actual production, the processing time of jobs is not unchangeable. It grows longer with changes in the workshop production environment or different processing sequences. For example, in iron and steel production, because of job waiting, the temperature is lower than the threshold required by processing. Reheating is needed to satisfy the demands of production conditions. As a result, processing time increases. Machine faults or regular equipment maintenance makes it necessary to change the processing sequence and increases the processing time. This phenomenon is called processing time deterioration.

This kind of issue was first put forward by Gupta [1]. Being in line with the actual production environment, it received much attention once raised. Pinedo[2] analyzed that fatigue would prolong the actual processing time. Biskup[3] attributed processing time deterioration to two reasons, i.e., different actual time caused by different positions in processing sequence and different actual time caused by different start time. Yang[4] studied single machine scheduling with deteriorating effect under complex maintenance jobs. WU[5] presented three different heuristic algorithms, to study single machine scheduling with linear increasing deterioration function. Lee[6] studied single machine sequencing with sectional deterioration in jobs, using branch and bound method and solved it with simulated annealing algorithm. NG [7] adopted priority sequencing rules and solved dual machine deteriorating job scheduling with the minimum completion time, in combination with branch and bound method. Mazdeh[8] built a parallel machine scheduling model to minimize the total delay and deterioration costs of machine and solved it with tabu search algorithm. Ma Ying et al.[9] studied single machine scheduling with unavailable time periods and proved the polynomial solvability of such problems using inference method. Zhao et al. [10] discussed scheduling with linear deteriorating processing time in jobs and optimized using Johnson rules, with the minimization of the maximum weighted completion time, maximum delay and maximum costs as the objective functions. Guo Peng et. al [11] studied job sequencing with processing time as the step function and solved single machine scheduling of 40 jobs using an improved genetic algorithm, aimed at minimizing the maximum completion time. Numerical examples proved to be more effective than simulated annealing algorithm.

The above studies only focus on single or dual machine scheduling and lack discussions on flexible job scheduling under deteriorating processing time environment. Heuristic algorithm and branch and bound method adopted in the above studies are subject to a large calculation when solving the processing time deterioration scheduling of a large number of jobs. In addition, a large majority of models with deteriorating effect often use specific deterioration functions, but the application scope of models based on specific functions is small and they cannot clarify the relationship between processing time and scheduling effect. Under this background, this paper draws lessons from the existing flexible job scheduling findings [12-17], studies flexible job scheduling under deteriorating processing time environment using an improved genetic algorithm with global search ability, analyzes the interactive relationship and rules between deteriorating processing time and scheduling effect under different deteriorating effects, to provide a theoretical basis for the optimum design and implementation of job-shop scheduling.

2. Flexible Job-shop Scheduling Model with Deteriorating Processing Time

Assume that there are n jobs, $J=\{J_1,J_2,\dots J_n\}$, $\forall J_i \in J$ $\forall J_i \in J$ and n_i n_i operations $O_i=\{O_{i,1},O_{i,2},\dots,O_{i,j},\dots O_{i,n_i}\}$ in need of processing. The machine set is $A=\{M_1,M_2,\dots M_m\}$. Each operation has one or more optional processing machines. Different machines may have different processing time. If the start time of Operation O_{ij} of Job i is t, then the processing time is $P_{ij}=P_{ij}^{'}+\alpha t$. $P_{ij}^{'}$ is the basic processing time of Operation O_{ij}. α is a constant greater than zero, known as the deterioration(growth) rate of jobs (unless particularly stated, assume the start time of the first job to be 0). In order to improve the universality of the present study, assume Operation O_{ij} to be processed in the rth place, then the actual processing time can be represented as $P_{ijr}(t)=P_{ij}^{'}f(d(t,r))$. $d(t,r)$ signifies the deterioration function and grows bigger as the start time or the processing position is postponed. For Scheduling Sequence $\pi=(J_1,J_2,\cdots,J_n)$, $C_i=C_i(\pi)$ denotes the end time of Job J_i in Scheduling Sequence π. The corresponding objective function and constraints are as follows:

This paper takes the minimization of the maximum completion time as a performance index to evaluate the effect of deteriorating processing time on scheduling, to shorten the manufacture cycle and improve the utilization ratio of resources. The mathematical model is as follows:

$$C = Min\{max(C_i)\} \quad (1)$$

$$\sum_{j'=1}^{n_i} Z_{i,j,j'} = 1, \quad \forall i \in (1,n), \forall j \in (1,n_i) \quad (2)$$

Where

$$Z_{i,j,j'} = \begin{cases} 1, & O_{ij} \text{ is the pre-operation of } O_{ij} \\ 0, & \text{others} \end{cases}$$

$$C_i \geq \sum_k C_{ijk}, \ \forall i \in (1,n), \forall j \in (1,n_i), k \in A \quad (3)$$

$$C_{ijk} \geq S_{ijk} + P_{ijk} - (1 - V_{ijk}) * M$$

$$\forall i \in (1,n), \forall j \in (1,n_i), k \in A \quad (4)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n_i} X_{ijk}^t = 1 \quad (5)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n_i} X_{ijk}^t = 1 \quad (6)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n_i} X_{ijk}^t = 1 \quad (7)$$

Where

$$X_{ijk}^t = \begin{cases} 1, & \text{At } t, \text{Operation } O_{ij} \text{ is processed on Machine k} \\ 0, & \text{Others} \end{cases}$$

Where Formula (1) means that the objective of the problem is to minimize the maximum completion time. Formula (2) means that a process can only choose one pre-operation along the process route. Formula (3) determines the completion time of jobs and k is the machine index. Formula (4) ensures that the difference between start time and completion time is at least equal to the processing time of Machine k. S_{ijk}, P_{ijk} and C_{ijk} are the start time, processing time and completion time of Operation O_{ij} on Machine k. V_{ijk} represents whether Operation O_{ij} can be done on Machine k. 1 stands for yes and 0 for no. M is a very large number. Formula (5) means that the post-operation can only be processed after the pre-operation is finished. Formula (6) means that Job $t=0$ can only be processed on one machine at $t=0$. Formula (7) means that Machine k can only perform one operation at $t=0$.

The processing process also needs to satisfy the following constraints:

(1) A machine can only process one job at a time;

(2) There is no interruption or queue-jumping in processing;

(3) At $t=0$, the processing probabilities of all jobs are the same;

(4) There are constraints between operations for the same job. There are no constraints between operations for different jobs;

(5) Different jobs have the same priority.

Assume there are two different scheduling sequences: $\pi_1 = (J_x, J_i, J_{i+1}, J_y)$ and $\pi_2 = (J_x, J_{i+1}, J_i, J_y)$, in which J_i is located in the r th place of Sequence π_1, J_{i+1} is located in the $r+1$th place of Sequence π_1. Meanwhile, J_i is located in the $r+1$th place of Sequence π_2 and J_{i+1} is located in the r th place of Sequence π_2. J_x contains r-1 jobs and J_y contains n-r-1 jobs, as shown in Figure 1.

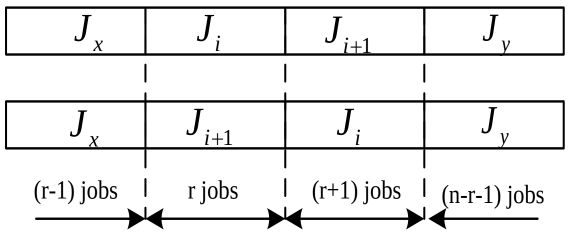


Figure 1. To Exchange the Scheduling Sequence of Two Adjacent Jobs

Assume that the completion time of all jobs in J_x is t_0 and the hypothesis $P_i \leq P_{i+1}$ is met. Thus, the completion time of J_i and J_{i+1} in Sequence π_1 is:

$$C_i(\pi_1) = t_0 + P_i f(d(t,r)) = t_0 + P_i f(t_0, \prod_{l=0}^{r} r_l), \quad (8)$$

$$C_{i+1}(\pi_1) = t_0 + P_i f(d(t,r)) + P_{i+1} f(t_0 + P_i f(d(t,r)), \prod_{l=0}^{r+1} r_l)$$

(9)

Accordingly, the completion time of J_i and J_{i+1} in Sequence π_2 is:

$$C_{i+1}(\pi_2) = t_0 + P_{i+1} f(d(t,r)) = t_0 + P_{i+1} f(t_0, \prod_{l=0}^{r} r_l) \quad (10)$$

$$C_i(\pi_2) = t_0 + P_{i+1} f(d(t,r)) + P_i f(t_0 + P_{i+1} f(d(t,r)), \prod_{l=0}^{r+1} r_l)$$

(11)

In this formula, $\prod_{l=0}^{r} r_l$ denotes the deterioration function of jobs in the r_l th place. The above expression can reflect the impact of deterioration caused by different processing positions on scheduling distinctly. The impact of deterioration caused by different start time on production scheduling can be reflected from the formula below. In flexible job scheduling, assume that the processing positions of Jobs J_i

and J_{i+1} along different flexible processing routes are the same, only the start time is different, then the corresponding completion time of jobs can be represented as:

$$C_i(\pi_1) = t_0 + P_i f(d(t,r)) = t_0 + P_i f(t_0, \prod_{l=0}^{r} r_l) \quad (12)$$

$$C_i(\pi_2) = t_1 + P_i f(d(t,r)) = t_1 + P_i f(t_1, \prod_{l=0}^{r} r_l) \quad (13)$$

From Formulas (12) and (13), we can summarize that different start time will result in different completion time and have an impact on scheduling.

3.An Improved Genetic Algorithm

3.1. Encoding

This paper adopts two-layer encoding, as shown in Figure 2. One is sequence codes based on operation, which determine the processing sequence of each operation. The other is machine allocation codes based on machine, which determine the processing machine of each operation. The layered encoding used in this article can directly reflect the constraints of this problem and ensure individual effectiveness after encoding.

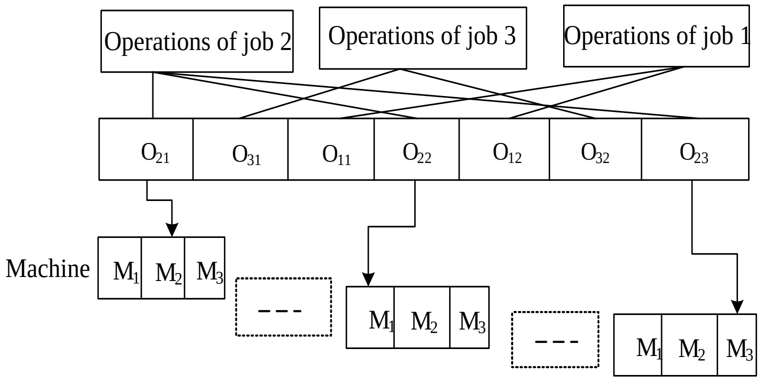


Figure 2. Two-layer Encoding

Operation-based encoding: the length is the total number of operations. Each code is an array of all operations. The operation of each job is represented with a corresponding serial number of jobs. If the serial number appears for j times, it means the j[th] operation of jobs.

Machine-based encoding: the length is the same as operation-based encoding. Each code is corresponding to a processing machine of each operation. Each position in the machine-based encoding represents a serial number of machine selected by the operation in the optional machine set.

3.2 Decoding

Select a processing machine according to machine-based encoding and arrange all operations to corresponding machines, according to the processing sequence of operation-based encoding layer, to determine the processing sequence of each operation on corresponding machines.

Assume that the basic processing time of any operation O_{ij} is $P_{ij}^{'}$ and the start time of operation is S_{ij}. It is located in the r[th] place of the processing scheme and the completion time of operation is $P_{ijr}(t) = P_{ij}^{'} f(d(S_{ij}, r))$. Assume that the immediate predecessors of machine and operation are $O_{ij[MP]}$ and $O_{ij[JP]}$, respectively, then the start time of Operation O_{ij} is determined by the maximum completion time of immediate predecessors of machine and process. Then the completion time of operation can be represented as:

$$P_{ijr}(t) = P_{ij}^{'} f(d(\max(P_{ij[JP]}, P_{ij[MP]}), r)) \quad (14)$$

3.3 Simulated annealing genetic algorithm

When the average fitness value of an early excellent individual in genetic algorithm is much higher than that of a group, according to pro rata selection, the selection probability of this individual will increase and lead to “prematurity”. In the later stages of search, as individual fitness values are close to each other, it is difficult for genetic operators to select an optimized individual gene and the convergence rate is low.

Simulated annealing algorithm is a random search algorithm. During search, not only optimal solutions are accepted, deteriorating solutions are accepted to a limited extent, too. Thus, it allows us to jump out of local optimal solutions. Combining simulated annealing algorithm and genetic algorithm enables us to overcome their respective shortcomings, exert their advantages and improve optimizing performance.

For flexible job-shop scheduling under deteriorating processing time environment, the procedures of simulated annealing genetic algorithm is as follows:

Step 1: To initialize the parameters of algorithm (population size: popsize, maximum number of iterations: $T_{\max}$, initial acceptance probability: p_r, crossover probability: pc, mutation probability: pm and annealing rate: λ).

Step 2: To generate initial populations randomly, using two-layer encoding: operation-based and machine-based encoding. Calculate the fitness value of each individual. Define f_{best} as the best solution in current population and f_{worst} as the worst solution. Calculate the initial temperature: $T_0 = -\left| f_{best} - f_{worst} \right| / \ln\left(p_r\right) = -\left|\Delta f\right| / \ln p_r$.

Step 3: To determine the termination condition: when the maximum number of iterations MM is achieved or no better solution occurs after a specified number of annealing, the algorithm ends. If conditions are met, turn to Step7. Otherwise, turn to Step4.

Step 4: To perform genetic selection, crossover and mutation in populations and calculate the fitness value of new individuals. If their fitness value is better than that of optimal individuals in the previous generation, then replace parent individuals with filial individuals and meanwhile renew f_{best} to produce the best scheduling result.

Step 5: To conduct annealing algorithm operations among the best individuals in current population. Calculate the fitness value of new individuals and compare the variation of fitness value caused by two positions $\Delta f = f(k+1) - f(k)$. If $\Delta f < 0$, accept the new position. If $\exp(-\Delta f / T_{k+1}) > rand$, accept the new position, as well. Otherwise, remain the old position.

Step 6: k=k+1 and perform annealing $T_{k+1} = \lambda T_k$, $\lambda \in (0,1)$. Return to Step 3.

Step 7: To output the optimal solution derived in this calculation.

4. Application and Analysis

This paper uses Matlab R2009b as programming language and conducts a simulation experiment on an Intel Core2 CPU/ 2.00 GHz/ 2.00G computer. The algorithm parameter settings are as follows: number of iterations 100, population size 50, crossover probability 0.8, mutation probability 0.1, annealing rate 0.98 and initial acceptance probability p_r=0.7.

In order to verify the effect of different deterioration rates on scheduling, analyze the differences in scheduling effect when some equipment components have deteriorating effect, others don't or deteriorating effect is not considered at all and further clarify the salient role of key equipment in scheduling effect. Below the author will discuss from three aspects:

(1) The flexible scheduling of deteriorating effect is not considered at all. Here the problem is converted to basic flexible job scheduling. The corresponding processing information is shown in Table 1.

Table 1. Flexible Processing Information of Machine

Job	Operation	M_1	M_2	M_3	M_4	M_5	M_6	M_7
J_1	O_{11}	50	-	-	-	-	-	40
	O_{12}	50	-	-	-	-	-	-
	O_{13}	-	45	-	-	-	-	-
	O_{14}	-	-	-	-	40	-	-
	O_{15}	-	-	38	-	-	-	-
J_2	O_{21}	-	-	-	29	-	-	50
	O_{22}	18	-	-	-	-	-	-
	O_{23}	-	28	-	-	-	-	-
	O_{24}	-	-	25	-	-	-	-
	O_{25}	-	-	-	-	26	-	-
J_3	O_{31}	-	-	-	-	19	-	-
	O_{32}	-	-	-	-	-	-	14
	O_{33}	-	-	3	-	-	-	-

				8				
	O_{34}	-	-	-	19	-	-	-
	O_{35}	-	-	-	-	18	-	-
J_4	O_{41}	19	-	-	-	-	-	-
	O_{42}	-	-	-	-	-	-	26
	O_{43}	-	-	19	-	-	-	-
	O_{44}	-	-	-	17	-	-	-
	O_{45}	-	16	-	-	-	-	-
J_5	O_{51}	-	-	-	-	-	23	-
	O_{52}	33	-	-	-	-	-	-
	O_{53}	21	-	-	-	-	-	-
	O_{54}	-	-	-	-	-	-	45
	O_{55}	-	-	-	-	33	-	-

At this point, the maximum completion time is minimized to 229 units. The scheduling results and convergence curve are shown in Figure 3 and Figure 4, respectively.

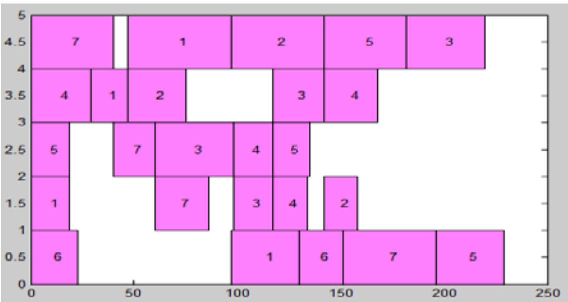


Figure 3. Flexible Scheduling Resul

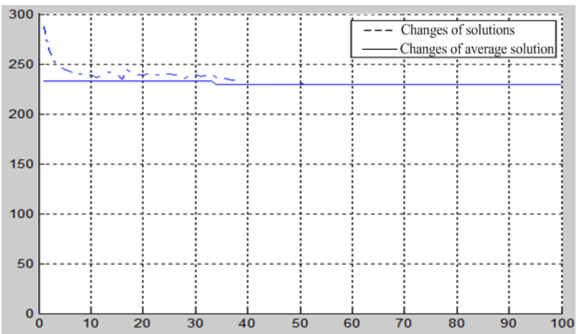


Figure 4. Convergence Curve of Algorithm

(2) The machine has a complete deteriorating effect. In order to detect the effect of deterioration rate changes on scheduling, assume that the deterioration rate changes with a step of 0.05 and analyze the relationship between deterioration rate (in actual use, the deterioration rate should not be greater than 0.5. This paper takes the upper limit 0.5 as an endpoint of study) and scheduling effect, as shown in figure 5.

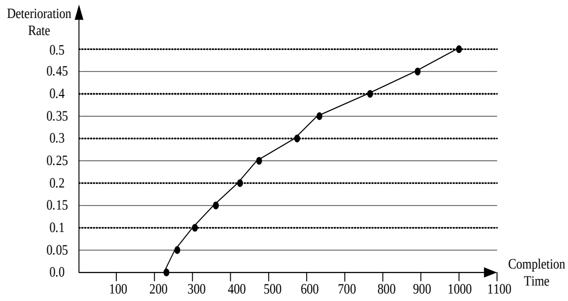


Figure 5. The Relationship between Deterioration Rate and Scheduling Effect under a Complete Deteriorating Effect

The coordinates of each point in the figure are (0; 229);(0.05; 264.2153);(0.1;305.4024);(0.15; 365.7729);(0.2; 425.0233);(0.25; 474.1438);(0.3; 572.4802);(0.35; 635.5527);(0.4; 767.3249);(0.45; 886.3180);(0.5; 1.0021e+003). From Figure 5, it can be observed that when the machine has a complete deteriorating effect, as the deterioration rate constantly increases, on the premise of optimal scheduling, the minimization of the maximum completion time tends to increase, too. If intelligent scheduling cannot be achieved, a random arrangement of job processing positions may lead to further increase in processing time. It is thus visible that to realize intelligent scheduling and maintain a low deterioration rate is the basic condition of efficient production and provides important guidance for production operation and planning. When the deterioration efficiency is 0.15 and 0.35, the output charts are shown in Figures 6 and 7 respectively.

Figure 6. Scheduling Results When the Deterioration Rate is 0.15

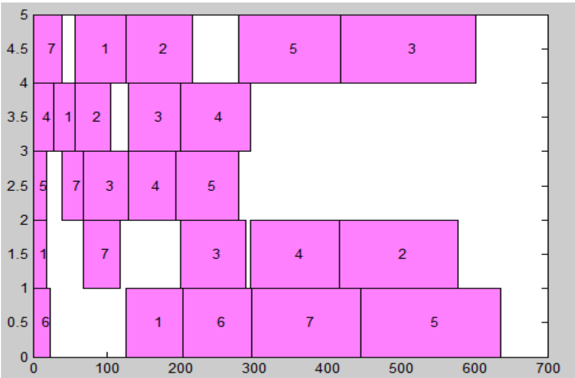
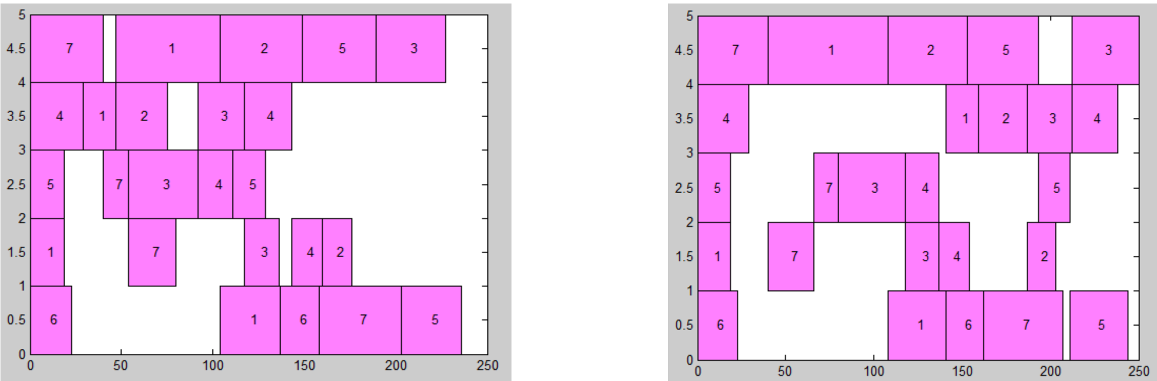


Figure 7. Scheduling Results When the Deterioration Rate is 0.35

To make a comparative analysis of Figures 6 and 7, it can be seen that the processing time of post-operations obviously increases as a result of deteriorating effect, and the minimization of the maximum completion time keeps increasing.

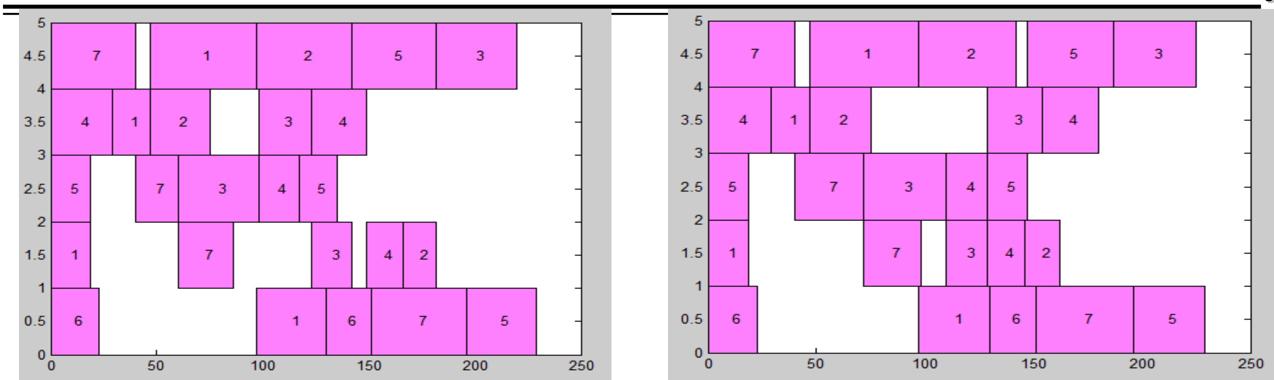
(3) The machine has a partial deteriorating effect. We study three circumstances, i.e., the machine selected by operation with the maximum processing time deteriorates, the machine selected by operation with the minimum processing time deteriorates and the machine selected by the last operation in the optimal sequence deteriorates. From Table 1, operations with the longest processing time are O_{11}, O_{12} and O_{21}, respectively. The operation with the shortest processing time is O_{32}. As O_{11} has a flexible processing route, the processing machine is flexible. For ease of research, we choose O_{12} and O_{32} as the research object, corresponding to Machines M1 and M7, respectively. Referring to the optimal solution without deteriorating effect, the last operation is O_{55}. The processing machine is M_5. The scheduling results are shown in Figures 8, 9 and 10.



(a)The deterioration rate is 0.15. The maximum completion time is minimized to 236.0500.

(b) The deterioration rate is 0.45. The maximum completion time is minimized to 250.

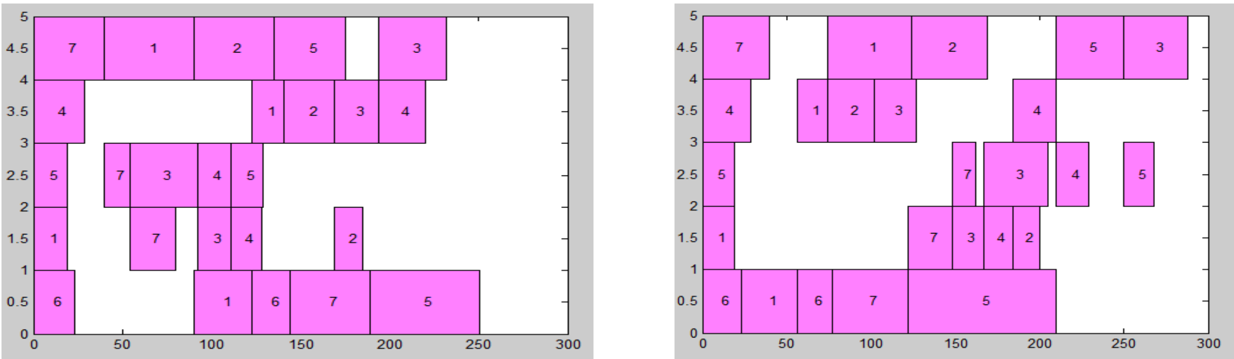
Figure 8. Scheduling Results under Different Deteriorating Effects on M_1



(a)The deterioration rate is 0.15. The maximum completion time is minimized to 229.

(b) The deterioration rate is 0.45. The maximum completion time is minimized to 229.

Figure 9. Scheduling Results under Different Deteriorating Effects on M_7



(a)The deterioration rate is 0.15. The maximum completion time is minimized to 250.3500.

(b)The deterioration rate is 0.45. The maximum completion time is minimized to 287.9000.

Figure 10. Scheduling Results under Different Deteriorating Effects on M_5

From Figure 8, if the machine with the longest processing time has deteriorating effect, then it has a large impact on completion time. From Figure 9, if the machine with the shortest processing time has deteriorating effect, such kind of deteriorating effect can be offset by a flexible processing route, to produce the same scheduling effect as without deteriorating effect. From Figure 10, when the machine selected by the last operation has deteriorating effect, due to the accumulation of previous processing time, the start time of the last operation is late and the deteriorating effect is large.

Through above analysis, we can draw the following conclusions:

(1) With the constant increase of deterioration rate, the impact on production scheduling will increase, too.

(2) When the machine selected by operation with the maximum processing time deteriorates, the resulting deteriorating effect will be larger than that of the machine selected by operation with the minimum processing time. In addition, the deterioration of machine selected by operation with the minimum processing time can be offset by a flexible processing route and produce the same scheduling results as without deteriorating effect.

(3) When the machine selected by the last operation has deteriorating effect, in most cases, due to the accumulation of previous processing time, the last operation will start late and the deteriorating effect will be large.

The above three points play an important role in production activities, especially for small manufacturing enterprises with limited funds. It is of great significance to find out the best equipment maintenance scheme and achieve the optimal performance of system.

5.Conclusion

This paper studies the flexible scheduling of deteriorating jobs. Aimed at minimizing the maximum completion time of scheduling, it proposes an improved genetic algorithm to solve this problem. It carries out classified studies on different deterioration rates and different deteriorations. Results show that there is an interactive relationship between deteriorating effect and scheduling effect. These conclusions are of important guidance value for enterprises to improve their production efficiency and plan equipment maintenance. Next, the research group will study the learning effect of humans on flexible production scheduling, to lower costs and achieve the best flexibility of system.

Acknowledgements

This work was supported by Key program of Hunan City University grants: 2015XJ02.

References

1. Gupta J N D, Gupta S K. (1988) Single facility scheduling with nonlinear processing times. *Computers and Industrial Engineering*,14(4), p.p.387-393
2. Pinedo M L. (2012) Scheduling: theory, algorithms, and system. Berlin: Springer.
3. Biskup D. (2008) A state of the art review on scheduling with learning effects. *European Journal of Operational Research*, 188(2), p.p.315-329
4. Yang S-J.(2012) Single machine scheduling simultaneously with deterioration and learning effects under deteriorating multi-maintenance activities consideration. *Computers and Industrial Engineering*, 62(1), p.p.271-275
5. W U G G , LEE W G, SHIAU Y R. (2007) Minimizing the total weighted completion time on a single machine under linear deterioration. *The International Journal of Advanced Manufacturing Technology*, 33(11), p.p.1237-1243
6. LEE L. (2007) Minimizing the makespan on a single machine with piecewise-deteriorating. Taichung Feng Chia University.
7. NG G T, Wang J B, Cheng T, et al. (2010) A branch and-bound algorithm for solving a two-machine flow shop problem with deteriorating jobs. *Computers & Operations Research*, 37(1), p.p.83-90
8. Mazdehmm, Zaerpourf, Zareei A, et al.(2010) Parallel machines scheduling to minimize job tardiness and machine deteriorating cost with deteriorating jobs. *Applied Mathematical Modeling*, 34(6), p.p.1498-1510
9. Ma, Y. and Zuo, C.R. (2009) Several polynomial solvable problems with deteriorating jobs and an availability constraint. *Journal of Hefei University of Technology (Natural Science)*, 32(3), p.p.378-382.
10. Guo, P., Cheng, W.M. and Zhang, Z.Q. (2011) Improved Genetic Algorithm for Single Machine Scheduling Problems with Deteriorating Jobs. *Journal of Southwest Jiaotong University*, 46(3), p.p.506-511.
11. Liu, X.X., Xie, L.Y., Tao, Z. and Hao, C.Z. (2008) Research on Multi-objective Scheduling Optimization for Flexible Job Shop. *Journal of Northeastern University (Natural Science)*, 29(3), p.p.362-365.
12. Liu, A.J., Yang, Y. and Cheng, W.M. et al. (2012) Improved NSGA for complex manufacturing environment. *Computer Integrated Manufacturing Systems*, 18(11), p.p.2446-2458.
13. Zhang, C.Y., Dong, X. and Wang, X.J. et al. (2010) Improved NSGA-Ⅱ for the Multi-objective Flexible Job-shop Scheduling Problem. *Journal of Mechanical Engineering*, 46(11), p.p.156-164.