On Attaining the Multiple Solutions of SHEPWM for Bipolar Waveform Based on Improved Trust-region Algorithm

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Abstract
There exist multiple sets of solutions when solving the selective harmonic elimination PWM (SHEPWM) equations of voltage source inverter control. It is a very difficult but necessary to get multiple sets of solutions for SHEPWM problem, for different set of solutions have different harmonic loss and pulse-width characteristics. An improved trust-region algorithm to obtain multiple groups of solutions for SHEPWM equations is presented in the paper. The proposed algorithm avoids the disadvantage of singular or ill-conditioned matrix and effectively overcomes the strict requirements of the iterative initial values of N-R method. The key factors of choosing set of solutions s for specific engineering application are discussed to obtain suitable overall harmonic performance and switching characteristics. Taking odd number of switching angles in quarter-cycle of bipolar switching patterns as example, multiple sets of solutions are acquired for wide modulation index. Finally, experimental results from a single-phase laboratory prototype confirm the operation of the calculation results.
Key words: INVERTER, SELECTIVE HARMONIC ELIMINATION, MULTIPLE SOLUTIONS, IMPROVED TRUST-REGION ALGORITHM, HARMONIC PERFORMANCE; SWITCHING CHARACTERISTICS

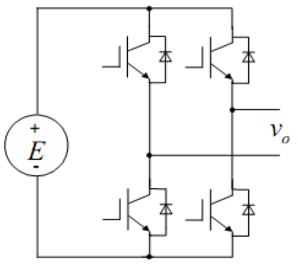
1. Introduction

High-power AC variable-frequency drives have found widespread applications in industry in the last decades. They can be used for steel rolling mills[1], locomotive traction applications[2], fans and pumps[3,4], and other applications[5]. In such applications, the frequency of pulse-width modulation (PWM) is often limited by switching losses, voltage stress, minimum pulse-width and harmonic distortion factor (HDF). In order to overcome these problems, the selective harmonic elimination PWM (SHEPWM) offers enhanced operations at low switching frequency[5-9]. The main challenge associated with SHEPWM equations is to obtain the numerical solutions of nonlinear equations in arbitrary modulation index which naturally exhibits multiple solutions[6,10-15]. For different independent solutions have different harmonic loss due to the uncontrolled low-order harmonics, and switching characteristics for the proper setting of minimum pulse width, much effort has been made to get multiple solutions for SHEPWM in the past few decades and several algorithms have been done to deal with the SHEPWM problem. These algorithms include the well-known Newton-type iterative methods such as the Newton–Raphson(N-R) method[6,9,10-12]. The main disadvantage of Newton-type algorithms is that the solutions deeply depend on the selection of initial values. It is very difficult in most cases to provide a good guess. Only one set of solutions is obtained for a good guess even there may exist multiple solution sets. Some sets of solutions may have merit relative to harmonic performance, switching characteristics, or some other system properties. Other suggested approaches such as introducing transformation steps to ensure convergence and finding all solutions, for instance, homotopy algorithm[13], polynomial resultants theory[15], Walsh functions [16], Vandermonde matrix [17]. The homotopy algorithm to solve the SHE problem is long and cumbersome. The Walsh functions and resultants theory are used to convert SHEPWM equations into an equivalent set of polynomial equations or a matrix equation. Nevertheless, the calculation of high-order matrix equation is severe numerical difficulties and the existence of singular or ill-conditioned matrix, when the number of switching angles is high[6,18]. Vandermonde matrix method is only suitable for single-phase system. The technique of approximate SHEPWM based on sampling regulation strategy has been developed to avoid the problem of solving SHEPWM equations[19].It is difficult to ensure the accuracy of approximate method in wide modulation range. The formula of obtaining the initial solution is only suitable for modulation index of 0 [12].

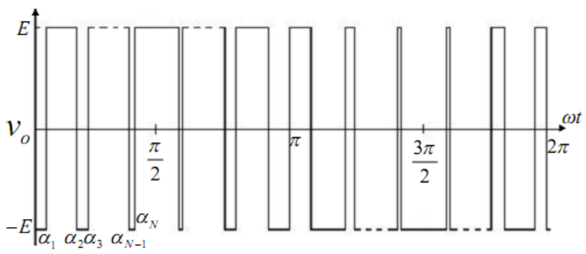
This paper proposes an improved trust-region algorithm to obtain multiple sets of solutions for SHEPWM equations. By introducing the damping factor in the inverse matrix and adaptively adjusting the relaxation parameter, the proposed method avoids the disadvantage of singular or ill-conditioned of Jacobi matrix and overcomes the strict requirements of the iterative initial values in N-R method, so that the trial-and-error processes to calculate multiple sets of solutions can be significantly reduced. The bipolar switching pattern of odd switching angles as example is used to illustrate the proposed method, which confirming its ability to find multiple sets of solutions.

These key factors of choosing solutions are given in this paper to obtain suitable overall harmonic performance and switching characteristics for specific engineering application. The multiple solution trajectories, minimum pulse-width and HDF for the nine switching angles case are discussed. For simplicity and to show practicality, the schemes developed in this paper are applied to a single-phase experimental system. Experimental results prove the validity of multiple solutions.

2. The Disadvantage of the Newton-type Method for Solving SHEPWM Equations



(a) Circuit of voltage source inverter



(b) Bipolar PWM waveform of inverter

Figure 1. PWM inverter for bipolar switching patterns

The PWM inverter is shown in Figure1(a), where E is the voltage of the DC side and ω is angular frequency. Conventional SHEPWM for inverter assumes that the PWM waveform is quarter-wave symmetric, as shown in Figure1(b). N is the switching number in the first quarter period. The constraint $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N < \pi / 2$ is enforced. The resulting equations based in the Fourier series for Figure1(b) is described by

$$v_o(t) = \sum_{n=1}^{\infty}(a_n \sin n\omega t + b_n \cos n\omega t) \quad (1)$$

The waveform in Figure1(b) is observed that b_n are absent due to quarter-wave symmetry. a_n is also zero, when n is even. a_n is the amplitude of the nth harmonic. The odd numbers of switching angles in quarter-waveform guarantees that the PWM signal conforms to the law of sine, and PWM wave in 90° is high level.

For N is odd, a_n is a function of the switching angles:

$$a_n = \frac{4E}{n\pi}[-1-2\sum_{k=1}^{N}(-1)^k \cos(n\alpha_k)] \qquad n \text{ is odd} \quad (2)$$

m is the modulation index used to set the fundamental, which is defined as:

$$m = a_1 / E \quad (3)$$

The traditional formulation of the SHEPWM problem can be conceptualized as the appropriate distribution of the switching angles to turn the inverter bridge switches on and off. Ideally, the SHEPWM of inverter is to choose the switching angles $\alpha_1, \alpha_2, \ldots, \alpha_N$ to synthesize a desired sinusoidal voltage waveform, which implies that we will have N degrees of freedom to control the fundamental and N-1 other low-order harmonics from the output of an inverter.

For three-phase inverter, since triple harmonics are absent in output voltage, only the N - 1 lowest odd-order nontriplen harmonics are to be eliminated. Thus, SHEPWM equations may be written as

$$\begin{cases} -1-2\sum_{k=1}^{N}(-1)^k \cos(\alpha_k) = m \\ -1-2\sum_{k=1}^{N}(-1)^k \cos(x\alpha_k) = 0 \quad x = 5, 7, \cdots n \end{cases} \quad (4)$$

where n=3N−2 for three-phase system and N is odd.

These equations of Eq.(4) may be formally denoted as

$$\mathrm{F}(\boldsymbol{\alpha}) = \mathbf{0} \quad (5)$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_N]^T$。

The Eq.(5) to be solved to find the switching angles are nonlinear transcendental equations and several solutions may exist depending on modulation index. Newton-type iterative processes such as the N-R method are usually used to obtain the solutions. It needs a good initial guess that should be very close to the exact solution in N-R method. In each iteration step, it computes the trial step:

$$\begin{cases} \Delta\boldsymbol{\alpha}^k = -\mathrm{F}'(\boldsymbol{\alpha}^k)^{-1}\mathrm{F}(\boldsymbol{\alpha}^k) \\ \boldsymbol{\alpha}^{k+1} = \boldsymbol{\alpha}^k - \mathrm{F}'(\boldsymbol{\alpha}^k)^{-1}\mathrm{F}(\boldsymbol{\alpha}^k) \end{cases} \quad (6)$$

where $\mathrm{F}'(\boldsymbol{\alpha}^k)$ is the Jacobi of $\mathrm{F}(\boldsymbol{\alpha})$ at $\boldsymbol{\alpha}^k$,and $\Delta\boldsymbol{\alpha}^k$ is Newton direction.

N-R method is mathematically the most preferable because of its quadratic convergence under the condition that the Jacobi $\mathrm{F}'(\boldsymbol{\alpha}^k)$ is nonsingular at the solution of Eq.(5). The conventional N-R method can be generalized to find one of the solutions of a nonlinear problem $\mathrm{F}(\boldsymbol{\alpha})$. Apparently, successful application of the N-R method depends on whether the initial guess is desirable or not.

Because the search space of the SHE problem is unknown for arbitrary r modulation index, and one does not know whether the existence of the solutions of the equation, providing a good guess is very difficult in most cases. If an undesirable initial guess, $\mathrm{F}'(\boldsymbol{\alpha}^k)$ is nearly singular or ill-conditioned, and $\Delta\boldsymbol{\alpha}^k$ will be very large. As the result, the iterative numerical solutions are not suitable.

3. Improved Trust-region Algorithm

3.1. Levenberg-Marquardt(L-M) method

For the general nonlinear system, [20] suggests that the optimum value of $\boldsymbol{\alpha}^k$ is the one which minimizes or causes a decrease in the L_2 norm of the residual $\mathrm{F}(\boldsymbol{\alpha}^{k+1})$ at each iteration step. So, $\boldsymbol{\alpha}^k$ is chosen as the value of $\boldsymbol{\alpha}$ which ensures that it be norm-reducing in the sense that

$$\|\mathrm{F}(\boldsymbol{\alpha}^{k+1})\| < \|\mathrm{F}(\boldsymbol{\alpha}^k)\| \quad (7)$$

The N-R method with exact line searches is attracted to a point of no interest at which the Jacobian is singular, which does not necessarily satisfy the requirement of Eq.(7). Since Eq.(6) often contains singular or ill-conditioned points, these behaviors give cause for concern. To avoid these behaviors of

$F'(\boldsymbol{\alpha}^k)$ in N-R method, we may have to modify the Newton direction. One possibility is to add some multiple λI of the identity to $\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k)$, and define the step $\Delta\boldsymbol{\alpha}^k$ to be[18,21]

$$\Delta\boldsymbol{\alpha}^k = -\left\{\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I\right\}^{-1} \left[F'(\boldsymbol{\alpha}^k)\right]^T F(\boldsymbol{\alpha}^k) \tag{8}$$

where λ is the damping factor. This approach is the classical Levenberg-Marquardt(L-M) algorithm.

For any $\lambda>0$ in Eq.(8), the matrix in parentheses is nonsingular. Therefore, some practical algorithms choose λ adaptively to ensure that the matrix in Eq.(8) does not approach singularity, which involves a large computational cost. The L-M method is sometimes considered to be the progenitor of the trust-region approach. The N-R method uses the line search to receive the approximation. The L-M method can be obtained the same approximation by replacing the line search with a trust-region strategy.

3.2. Improved Trust-region Algorithm based on L-M Method

The major disadvantage of L-M method to choose λ is the matrix $\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I$ inversion involves a large computational cost when λ is been adjusted. In order to reduce the difficulty of choose the initial value in every iteration, a new approximation can be obtained by taking a weighted average of the calculation result in Eq.(8) and the approximations made in the previous step.

$$\overline{\boldsymbol{\alpha}}^{k+1} = (1-\mu)\boldsymbol{\alpha}^k + \mu\boldsymbol{\alpha}^{k+1} = (1-\mu)\boldsymbol{\alpha}^k + \mu\left[\boldsymbol{\alpha}^k - \left\{\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I\right\}^{-1} \left[F'(\boldsymbol{\alpha}^k)\right]^T F(\boldsymbol{\alpha}^k)\right]$$

Finally, an improved trust-region method is obtained.

$$\boldsymbol{\alpha}^{k+1} = \boldsymbol{\alpha}^k + \Delta\boldsymbol{\alpha}^k = \boldsymbol{\alpha}^k - \mu\left\{\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I\right\}^{-1} \left[F'(\boldsymbol{\alpha}^k)\right]^T F(\boldsymbol{\alpha}^k) \tag{9}$$

where μ （$0<\mu<1$） is the relaxation factor. The $\Delta\boldsymbol{\alpha}^k$ of improved trust-region method can be varied by adjusting the λ and μ, therefore, the search area is changed in next iteration step and its convergence properties is improved.

4. Practical Search Procedure of Improved Trust-region Algorithm

As above mentioned, the key of improved trust-region method is the selection of rational λ and μ. Since λ larger, the condition number of $\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I$ becomes more reasonable, which can effectively weaken the ill-conditioned property of $\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I$. Nevertheless, the speed of convergence will slow down. Meanwhile, the smaller μ, the wider region of convergence, which can reduce the difficulty to select the initial value. However, it will also reduce the speed of convergence and increase the amount of computation.

As in Eq.(9), the parameter μ may be chosen to ensure that Eq.(7) holds, which needs less amount of computation than that by adjusting λ. An appropriate way for choosing λ and μ is presented to reduce the amount of computation in this paper, which includes the following two aspects:

1) Choosing the damping factor λ

$\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I$ will become positive definite for $\lambda > 0$. In order to reduce the amount of calculation, an empirical value of $\lambda = 0.001$ is found after a lot of calculations in this paper. If the results can not meet the norm-reducing requirements, a growth factor β ($\beta > 1$) is used for λ multiplied to $\beta\lambda$. β will usually take 5 or 10. Thus, it need not calculate the matrix condition number of $\left[F'(\boldsymbol{\alpha}^k)\right]^T F'(\boldsymbol{\alpha}^k) + \lambda I$.

2) Calculating the relaxation parameter μ

μ should be as large as possible to reduce the amount of computation. In this paper, the initial value of μ is 1. If these results are improper, ω should decrease to increase convergence range. ω is usually divided by W, and W will usually take 2 or 5.

For an arbitrary modulation index, the calculation procedure of the improved trust-region method can be shown as following:

(1) Generating a set of initial guess for $\boldsymbol{\alpha}^1$, $\beta > 1$. Iteration limit of M, ε_α, ε_f.

(2) Set $\mu = 1$, compute $\mathrm{F}(\boldsymbol{\alpha}^k)$, $\left\{\left[\mathrm{F}'(\boldsymbol{\alpha}^k)\right]^T \mathrm{F}'(\boldsymbol{\alpha}^k) + \lambda I\right\}^{-1}$ and $\left\|\mathrm{F}(\boldsymbol{\alpha}^k)\right\|$.

(3) Compute $\boldsymbol{\alpha}^{k+1}$ and $\left\|\mathrm{F}(\boldsymbol{\alpha}^{k+1})\right\|$ in Eq.(9). If $k > M$, go to step (7).

(4) If $\left\|\mathrm{F}(\boldsymbol{\alpha}^{k+1})\right\| < \left\|\mathrm{F}(\boldsymbol{\alpha}^k)\right\|$, go to step (5). Otherwise, go to step (6).

(5) If $\left\|\Delta\boldsymbol{\alpha}^k\right\| < \varepsilon_\alpha$, $\boldsymbol{\alpha}^* = \boldsymbol{\alpha}^{k+1}$, then stop. Otherwise, set $\boldsymbol{\alpha}^k = \boldsymbol{\alpha}^{k+1}$, $k = k+1$, go to step (2).

(6) If $\left\|\mathrm{F}(\boldsymbol{\alpha}^{k+1})\right\| < \varepsilon_f$, $\boldsymbol{\alpha}^* = \boldsymbol{\alpha}^{k+1}$, then stop. Otherwise, set $\mu' = \mu/W$, go to step (3).

(7) If k=M and solutions are improper, set $\lambda' = \beta\lambda$ and k=1, go to step (2).

The above procedure is less computationally complex by adjusting μ instead of changing λ.

5. Optimal Selection of SHEPWM Solutions

It is known from above that there may exist numerous valid solutions to SHEPWM. Though each of them can eliminate the low-order harmonic, their performances, such as harmonic distortion factor (HDF), THD and minimum pulse-width are different. The key factors of choosing solutions should

1) Avoid switching transition near current peak

In order to reduce the switching losses**,** switching transition avoids near current peak. This implies the last switching angle should be far from π/2.

2) Have few low-order zero-sequence harmonics for simpler filter design and lower loss due to current ripple

These low-order zero-sequence harmonics affect the insulation of motors or grid interacting transformers. An important criterion to evaluate the difference between these solutions is the amplitude of the non-eliminated nontriplen harmonics, especially the first and second non-eliminated harmonics in the spectrum. In this paper, the HDF (defined as $\sqrt{(a_{3N+2}^2 + a_{3N+4}^2)}/a_1$) and the amplitude of the most significant nontriplen harmonic is used.

3) Fulfill the requirement of dead-time

An important constraint imposed upon the solutions deals with the limitations of the semiconductor switching devices of medium and high-power adjustable speed drives(ASD) regarding dead-time and minimum conduction time[21]. A typical dead-time is in the range of a few us and a minimum on-time is 10 us. Thus, these solutions must not exist narrow pulse phenomenon.

The feasible solutions will be identified based on the criteria mentioned above. The common engineering practice is that a combination of different sets for different modulation index must be selected to optimize the performance of the SHEPWM technique for all values of modulation index.

6. Illustrative Examples

To illustrate the effect of using the improved trust-region method to solve Eq.(5)**,** the multiple solutions for modulation index in the range of 0.3<m<1.15 is considered in there. For N is odd, the number of switching angles is varied between three and nine and m is set to 0.85 in this case. Table 1 shows the solutions in the calculation results.

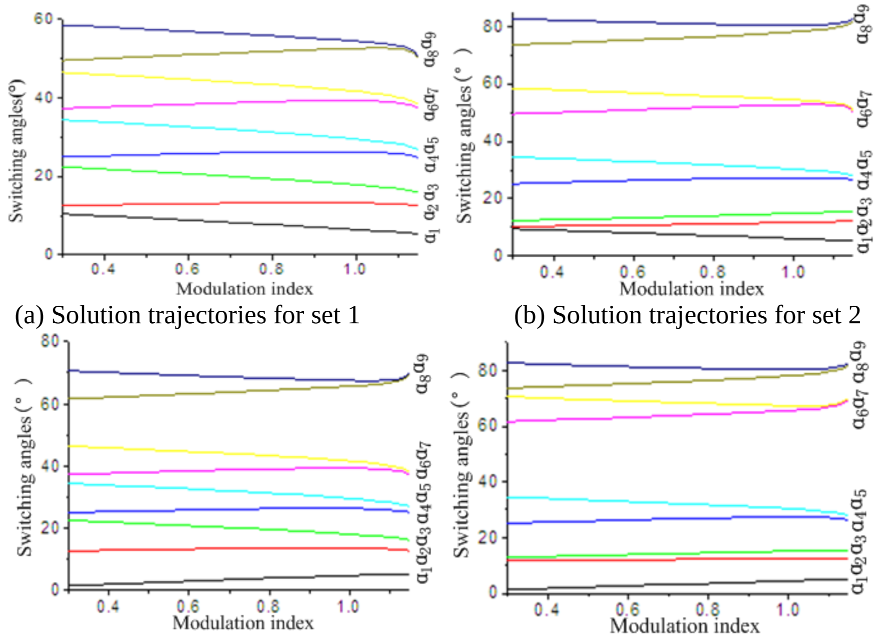
Table1. Complete solution sets for m=0.85 (degree)

The number of switching angles	Switching angles								
	α1	α2	α3	α4	α5	α6	α7	α8	α9
3	17.516	37.335	47.525						
	7.530	71.686	80.988						
5	12.018	23.265	31.294	45.884	51.976				
	6.070	24.295	31.903	67.868	73.821				
7	9.156	16.881	23.600	33.340	38.513	49.736	54.101		
	4.901	17.469	23.958	33.587	38.688	65.853	70.181		
	8.052	13.113	16.458	50.078	54.289	80.729	85.544		

	4.332	14.554	17.081	65.698	69.899	80.695	85.513		
9	7.394	13.234	18.998	26.175	30.787	39.070	42.889	51.931	55.340
	6.833	11.299	14.259	27.056	31.387	52.064	55.414	76.823	80.541
	4.066	13.600	19.222	26.346	30.913	39.159	42.956	64.642	68.037
	3.748	12.139	14.649	27.087	31.425	64.577	67.921	76.793	80.505

It is clear from table1 that the SHEPWM problem exhibits multiple solutions. After obtaining the initial solutions, the solution trajectories as a function of m, at least in the performance range of modulation index, can be easily obtained because of the continuity of the nonlinear equations.

The relationship between the switching frequency and the number of harmonics to be eliminated is that the higher the number of harmonics to be eliminated, the higher the switching frequency needed for the inverter to operate. The multiple solution trajectories for the nine switching angles case are shown in Figure 2.



(a) Solution trajectories for set 1

(b) Solution trajectories for set 2

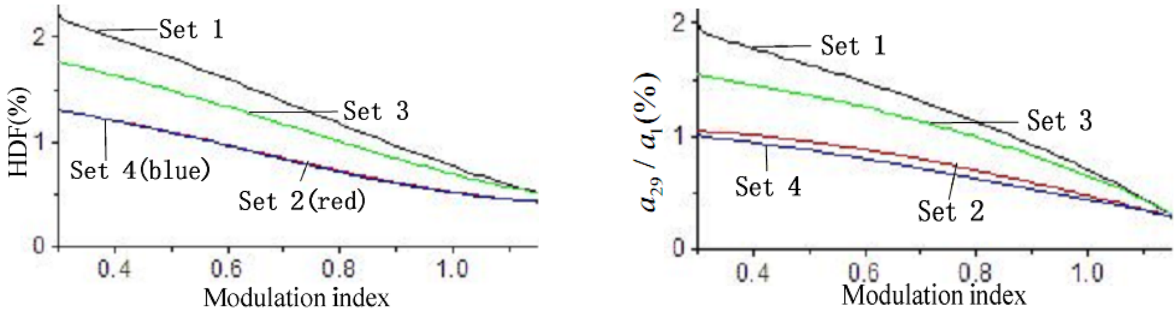
(c) Solution trajectories for set 3

(b) Solution trajectories for set 4

Figure 2. Solution trajectories versus modulation index

Switching transition avoids near current peak to reduce the switching losses. The Figure2(a) is the best solution in the four sets of solutions, for α_{max} is less than 60°. α_8 and α_9 in Figure2(b) and Figure2(c) are greater than 60°, but four switching are greater than 60°in Figure2(d). The switching loss of Figure2(b) and Figure2(c) is slightly larger than that of Figure2(a), but that of Figure2(d) is the biggest in Figure2.

The multiple solutions in Figure2 are compared based on the HDF and the amplitude of the most significant nontriplen harmonic(the 29th harmonic in this case). Since there are four clear sets, the respective HDF factor and the amplitude of 29th harmonic are plotted in Figure3.



(a) HDF versus *m* for all four sets (b) Amplitude of 29th harmonic versus *m* for all four sets

Figure3. Harmonic performance corresponding to solutions in Figure. 2

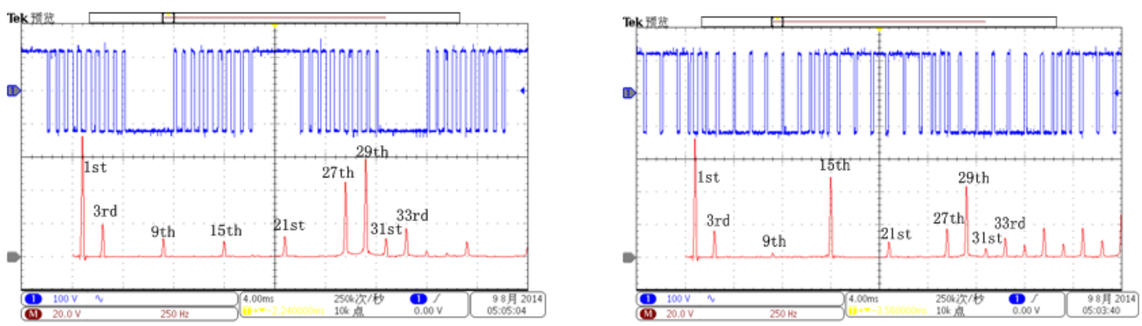
Figure3 shows that, for all common values of *m*, set2 and set4 produce less harmonic error than that of set1 and set3, and the amplitude of 29th harmonic clearly showing that set 4 is slightly superior to set 2. So, if all the possible solutions of SHEPWM equations have been found, an optimal solution with the lowest HDF or the lowest amplitude of 29th harmonic can be identified easily.

The constraint imposed upon the solutions deals with the limitations of dead-time and minimum conduction time. For this case, considering a typical dead-time of a few us and a minimum on-time of 10 us, an angle separation of 20us(1/3 of a degree for a 50 Hz system) is considered as the minimum angle separation between the solutions. Set2 and set4 exist obvious narrow pulse phenomenon in low modulation index.

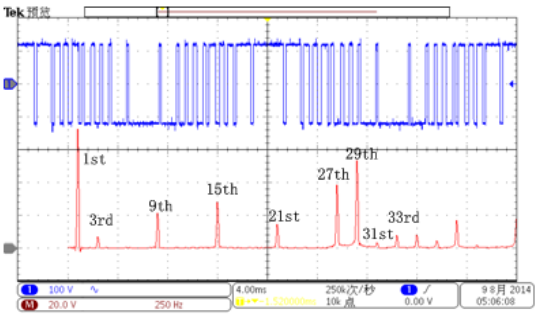
Four sets of solutions and corresponding solution trajectories, minimum pulse-width, HDF and the amplitude of 29th harmonic for the nine switching angles case are discussed. The final solutions will be made by considering the less harmonic power loss and no narrow pulse phenomenon. Therefore, the set1 and set3 are the optimal section of SHEPWM solutions in low modulation index for avoiding narrow pulse phenomenon, but the set4 are the optimal section of SHEPWM solutions in high modulation index for the lowest HDF and lower amplitude of 29th harmonic.

7. Experimental Results

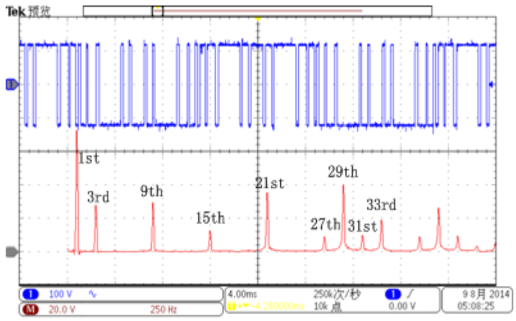
Experiments on single-phase inverter have been carried out to verify the correctness of the switching angles computed by the proposed method. The prototype consists of power circuit and control units. The main power circuit is a single-phase H-bridge inverter, the power stage is a 500V/20A Power MOSFET IRF460, and it is controlled by microcontroller TMS320F28335. TMS320F28335 provides the PWM pulse generation and dead-band logics. The input DC voltage is 120V and the fundamental frequency is 50Hz. For the nine switching angles in Table1, Figure4 illustrates the four sets of phase output voltage waveform and their associated spectrum.



(a) Set 1. (b) Sset 2.



(c) Set 3.



(d) Set 4.

Figure4. Experimental results of waveform and associated harmonic spectrum

The experiment results of PWM waveform and associated harmonic spectrum show that all the four solutions can eliminate all nontriplen odd low-order harmonics components. For N=9, the aimed 5th, 7th, 11th, 13th, 17th, 19th, 23rd, and 25th harmonics are eliminated very well as expected, and the amplitude of the first nontriplen odd harmonic, i.e., the 29th is also plotted for all values of all four sets in Figure. 4. The experimental results and the previous theoretical analysis are consistent.
The difference between these solutions is the amplitude of the non-eliminated triplen harmonics and nontriplen odd low-order harmonic voltage components, especially the 29th and 31st harmonics in the spectrum. Although the amplitude of the 31st in all four sets is very small. The amplitude of the 29th in set1 and set3 is much larger than that in set2 and set4, which is consistent with Figure3.

8. Conclusion

In this paper, multiple sets of solution to the SHEPWM problem for any performance feasible fundamental amplitude can be obtained using an improved trust-region algorithm. By introducing the damping factor in the inverse matrix, the proposed method avoids the disadvantage of singular or ill-conditioned in Jacobi matrix of Newton-type method. In order to effectively overcome the strict requirements of the iterative initial values in Newton-type method, relaxation parameter adaptively adjusted, so that the trial-and-error processes can be significantly reduced. To illustrate the effect of using the improved trust-region method to solve the SHEPWM equation, the multiple solutions for modulation index in the range of 0.3<M<1.15 and the number of switching angles varying from three to nine is considered in there. For different solutions have different harmonic performance and switching characteristics, the key factors of choosing solutions for specific engineering application are discussed in there. The output PWM waveforms of four sets of solutions case have been investigated with respect to different low-order harmonic, HDF, and minimum pulse-width. Experimental waveforms have been presented to confirm the theoretical analysis and calculation results.

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