UDK 622.625.6

Mathematical modeling of end carriage motion on the overhead monorail

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Abstract
Correlation between parameters of carriage motion and overhead monorail was determined. The dependences of formation of extra dynamic loads on the monorail motion were defined. Theoretical researches of carriage and monorail interaction taking into account the elastic strains were carried out. Amplitudes of forced oscillations at monorail joints were found. Received dependences will allow to determine reasonably the parameters of end carriages and monorail.
Keywords: MONORAIL TRACK, JOINT, CARRIAGE, FORCED OSCILLATIONS, AMPLITUDE

Problem statement

Motion of end carriages along the overhead monorail is inseparably connected with force impacts, which arise because of elastic strains, overtravel and inertial forces. Force impacts lead to dynamic oscillations and extra loads affecting the wheels of end carriages and support hanger of the monorail, which increase the wear rate and reduces the resource of interfacing elements.

Analysis of latest research and publication

The correlation of moving transport and railroad track is investigated in the works [1, 2]. Modeling of dynamic processes of transport means is fulfilled in the works [2, 4]. Behavior of monorail carriage is explored in the work [5]. This article is continuing of the mentioned above works. The aim of research is determination of correlation between the parameters of end carriage and underslung monorail track taking into account elastic strain of monorail and vertical inertial force.

Statement of basic material

Let us consider the motion of single carriage on the overhead monorail track, sections of which have the length L and are pivotally connected with each other (fig.1). At the beginning of the track the monorail is fixed in order to exclude its longitudinal swaying. The carriage may be represented as single mass model with wheels. Under the influence of mass long axis of monorail track is bent and its progressive motion is accompanied by vertical shifts, which depend not only on the static load but also on the vertical inertia force.

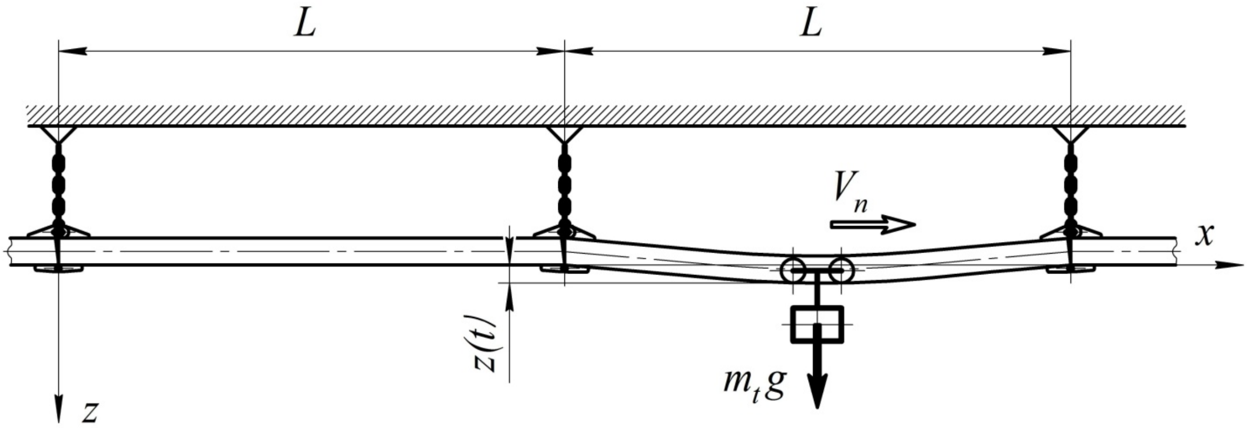

Figure 1. The scheme of deformation of overhead monorail track under the moving carriage.

Let us consider the monorail as unweighable bar, which are bent by concentrated force F_μ, the line of action of which crosses through the gravitational center of carriage. Differential equation of defected axis of monorail will be [6]

$$\frac{\partial^4 z}{dx^4} EJ + k_\mu z = 0, \qquad (1)$$

where $k_\mu -$ hardness of elastic foundation, which determines knife-edge load $k_\mu z$, causing the bending of monorail z.

Let us denote

$$\alpha_\mu = \sqrt[4]{\frac{k_\mu}{4EJ}}, \; \varphi_z = \alpha_\mu x. \qquad (2)$$

In respect with values (2) let us find the integral of the expression (1)

$$z = \frac{F_\mu \alpha_\mu}{2k_\mu} e^{-\varphi_z} \left(\cos\varphi_z + \sin\varphi_z\right). \qquad (3)$$

It follows that the maximum deflection of monorail, which arises under the load, will be $z_f = F_\mu \alpha_\mu / (2k_\mu)$. The value k_μ being included into this expression may be obtained as $k_\mu = D_z / L$, where $D_z -$ is the force, which must be applied to the monorail track in order to deform its support hanger per unit length.

Then

$$\alpha_\mu = \frac{1}{L} \sqrt[4]{\frac{3}{2\gamma_z}}, \qquad (4)$$

where $\gamma_z -$ coefficient, which considers relative stiffness of monorail and its support hanger, equals $\gamma_z = 6EJ / (D_z L^3)$.

During moving of the carriage with constant velocity, vertical reaction in the place of contact of the wheel with monorail will be

$$R_z = \frac{2k_\mu z}{\alpha_\mu} = 2D_z \sqrt[4]{\frac{2}{3}\gamma_z} z.$$

Motion equation of the carriage will be

$$\frac{d^2 z}{dt^2} m_t + \frac{2k_\mu}{\alpha_\mu} z = F_\mu. \qquad (5)$$

In case when $F_\mu = 0$, expression (5) describes natural oscillations of the carriage moving across the monorail. Bending of monorail under action of gravity of the carriage is equal to

$$z = z_o \cos\sqrt{\frac{2k_\mu}{m_t \alpha_\mu}} t + z'_o \sqrt{\frac{m_t \alpha_\mu}{2k_\mu}} \sin\sqrt{\frac{2k_\mu}{m_t \alpha_\mu}} t, \qquad (6)$$

where $z_o -$ initial vertical displacement of the carriage wheel towards the original position of the balance ; $z'_o -$ vertical velocity of the wheel at initial time.

In case when $F_\mu \neq 0$, the expression (5) describes forced oscillations, which arise under the action of gravity of the carriage, forces of wheel tightness to monorails, inertial forces caused due to disalignment of wheel rotation with its gravity center.

During motion of the carriage along the elastic monorail, due to arising oscillations, clamping force of the wheel may vary within wide limits. Let us denote $\beta_m -$ as the correlation of clamping force of carriage wheel to its gravity force. Herein we will have $F_\mu = m_t g(1+\beta_m)$.

Then the solution of equation (5) will be as follows

$$z = A_\mu \cos\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t + B_\mu \sin\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t + \frac{\alpha_\mu}{2k_\mu} m_t g(1+\beta_m), \quad (7)$$

where $A_\mu, B_\mu -$ are the arbitraty constant integrations setting the range and phase of natural oscillations, corresponding to the initial conditions of carriage motion.

If there is disalignment of wheel rotation with its gravity center, then the expression F_μ may be represented as $F_\mu = q_o \cos\omega_k t,$ where $q_o -$ is the value of centrifugal force; $\omega_k -$ the velocity of wheel rotation during carriage motion along the monorail.

Herein

$$z = A_\mu \cos\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t + B_\mu \sin\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t + \frac{1}{1-\frac{\alpha_\mu m_t \omega_k^2}{2k_\mu}} \frac{\alpha_\mu}{2k_\mu} q_o \cos\omega_k t \quad (8)$$

In the expressions (7) and (8) the first two summands consider natural oscillations of the carriage with monorail, the third one is forced oscillation. Constants A_μ, B_μ entering this expression should be chosen in such a way that the initial conditions are fulfilled. Thus if under the gravity forces at the initial moment of the motion monorail bending is equal to static bending, and the initial velocity equals zero, then the value of bending is

$$z = \frac{\alpha_\mu}{2k_\mu} m_t g(1+\beta_m) + \frac{1}{1-\frac{\alpha_\mu m_t \omega_k^2}{2k_\mu}} \frac{\alpha_\mu}{2k_\mu} q_o \left(\cos\omega_k t - \cos\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t\right) \quad (9)$$

It should be marked that the period of natural oscillations of the considered system does not depend on the initial conditions and may be found as

$$T = 2\pi\sqrt{\frac{\alpha_\mu m_t}{2k_\mu}}. \quad (10)$$

Practically the velocity of carriage motion varies gradually. Thus when the angular velocity ω_k takes the maximum value, the influence of centrifugal forces q_o reduces. Herein it is effective to consider only the following forced oscillations

$$\frac{1}{1-\frac{\alpha_\mu m_t \omega_k^2}{2k_\mu}} \frac{\alpha_\mu}{2k_\mu} q_o \cos\omega_k t.$$

Amplitude of oscillation will differ from monorail static deflection, determined as $\alpha_\mu q_o/(2k_\mu)$, only by multiplier, so called dynamic coefficient of centrifugal force

$$\mu_o = \frac{1}{1-\frac{\alpha_\mu m_t \omega_k^2}{2k_\mu}}.$$

Practically this coefficient takes on the value more than one and is determined by angular velocity of rotation ω_k, and also by natural period T. Using turn-around time of the wheel T_{tw}, we may find

$$\mu_o = \frac{1}{1-\left(\frac{T}{T_{ок}}\right)^2}$$

The given above dependences refer to the monorail and wheels of ideal shape, without dimples and knobs. If the monorail or a wheel has the dimple from the level of ideal form with the depth Δz, then vertical displacement of the carriage $z+\Delta z$ corresponds the monorail bending z. The equation of carriage vertical displacements may be represented as follows

$$\frac{d^2(z+\Delta z)}{dt^2} m_t + \frac{2k_\mu}{\alpha_\mu} z = F_\mu. \quad (11)$$

From (11) it follows

$$\frac{d^2z}{dt^2}m_t + \frac{2k_\mu}{\alpha_\mu}z = F_\mu - \frac{d^2\Delta z}{dt^2}m_t. \qquad (12)$$

Analysis of expression (12) shows that the following force corresponds to the displacement Δz

$$f_z(t) = \frac{d^2\Delta z}{dt^2}m_t.$$

During motion across the monorail there appears the range of successive strikes. Hold that at one time t_0 there appears the force $f_z(t_0)$. Within the time dt_0 this force will change the velocity of carriage, which corresponds to displacement

$$z_t = \sqrt{\frac{\alpha_\mu m_t}{2k_\mu}} f_z(t_o)\sin\left(\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}(t-t_0)\right)dt_o$$

In respect with (6) full movement of the carriage within t time will be

$$z = \sqrt{\frac{\alpha_\mu m_t}{2k_\mu}}\int_0^t f_z(t_o)\sin\left(\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}(t-t_0)\right)dt_o \qquad (13)$$

For unloaded area of monorail track, connected from bent pieces (fig. 2a), the equation of long axis may be represented as

$$\xi(x) = \frac{\Delta z}{2}\left(1-\cos\frac{2\pi x}{L_{vz}}\right),$$

where L_{vz} – is the length of rounding area, fulfilled from bent pieces of monorail.

For this case the expression (12) will be as follows

$$\frac{d^2z}{dt^2}m_t + \frac{2k_\mu}{\alpha_\mu}z = F_\mu - m_t\frac{4\pi^2V_n^2\Delta z}{2L_{vz}}\cos\frac{2\pi x}{L_{vz}}. \qquad (14)$$

where V_n – is the velocity of carriage motion.

Considering (13), the solution of the equation (14) will be

$$z = \frac{2\pi^2V_n^2\Delta z}{L_{vz}\left(\frac{4\pi^2V_n^2}{L_{vz}} - \frac{2k_\mu}{\alpha_\mu m_t}\right)}\left(\cos\frac{2\pi V_n t}{L_{vz}} - \cos\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t\right). \qquad (15)$$

Let us determine the time of carriage moving across the dimple $T_{ov} = L_{vz}/V_n$.

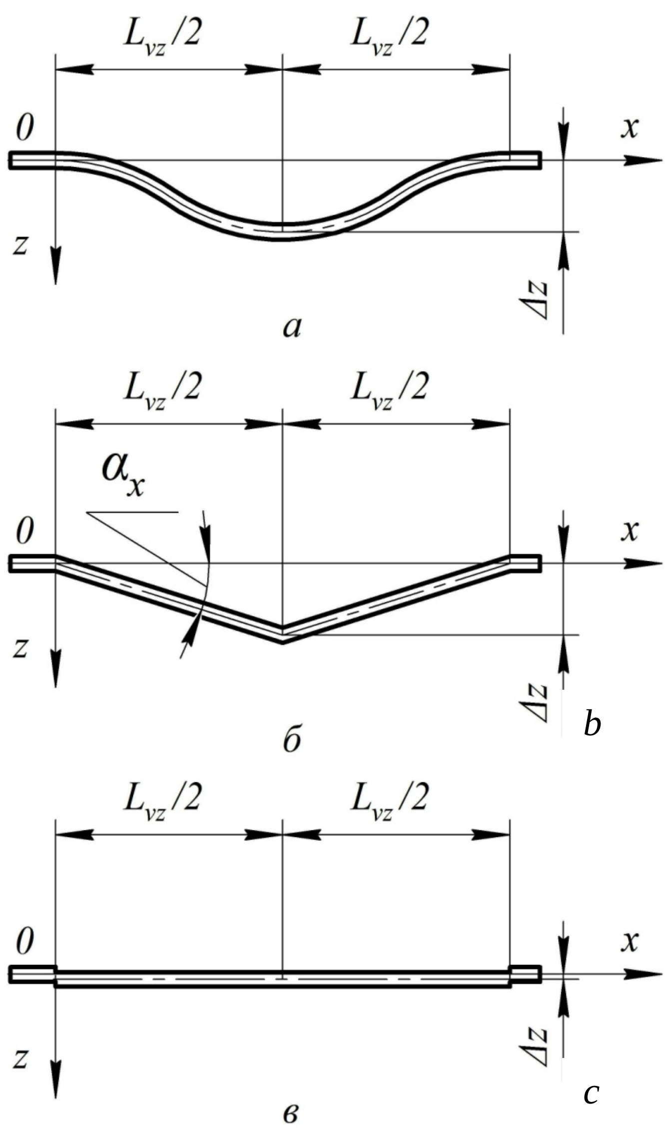

Figure 2. The scheme of coupling of monorail track sections:a – from bent sections; b – from linear sections; c – biased

Then using the expression (10), equation (15) for any moment of time t, taking on the value $0 < t < T_{ov}$, one may write

$$z = \frac{\Delta z}{2\left(1-\frac{T_{ov}^2}{T^2}\right)}\left(\cos\frac{2\pi t}{T_{ov}} - \cos\frac{2\pi t}{T}\right). \qquad (16)$$

From (16) one may see that the deformation of monorail track at deviation Δz depends foremost on the T_{ov}. Considering the correlation between deformation and acting loads, which are determined by monorail hardness, it follows that for harder track similar deviations Δz will lead to more high loads affecting the carriage and monorail support hanger.

For monorail rounding, which consists of linear pieces (sections), there appear oscillations during passing from horizontal area to the inclined. If one defines $\alpha_x -$ as the tilt angle of route section (fig. 2b), then the displacement may be found as $\xi(x) = \alpha_x x = \alpha_x V_n t$.

During motion across the horizontal piece, vertical displacements of the carriage may be determined by the expression (5). Then, after passing the bending point, when vertical velocity is still equal to zero, we have

$$\frac{d(z + \xi(x))}{dt} = 0.$$

Further during carriage moving on the inclined area, we have the following

$$\frac{d\xi(x)}{dt} = a_x V_n, \quad \left(\frac{dz}{dt}\right)_{t=0} = -a_x V_n.$$

Amplitude of forced oscillations on this area is

$$z_x = -\frac{2\Delta z V_n}{L_x}\sqrt{\frac{\alpha_\mu m_t}{2k_\mu}}\sin\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t. \qquad (17)$$

Expression (17) remains correct until $0 \le t \le \frac{T_{ov}}{2}$. As $x = \frac{L_x}{2}$ (fig. 2b) the area of monorail track changes the incline sign. Then, till $\frac{T_{ov}}{2} \le t \le T_{ov}$ there appear new oscillations

$$z_x = -\frac{2\Delta z V_n}{L_x}\sqrt{\frac{\alpha_\mu m_t}{2k_\mu}}\sin\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}t + \frac{4\Delta z V_n}{L_x}\sqrt{\frac{\alpha_\mu m_t}{2k_\mu}}\sin\sqrt{\frac{2k_\mu}{\alpha_\mu m_t}}\left(t - \frac{T_{ov}}{2}\right).$$

(18)

During changing to horizontal area, there are added new oscillations, which are in respect with (17) and (18)

$$z_x = \frac{\Delta z T}{T_0}\left(-\sin\frac{2\pi t}{T} + 2\sin\left(\frac{2\pi t}{T} - \frac{\pi T_{ov}}{T}\right) - \sin\left(\frac{2\pi t}{T} - \frac{2\pi T_{ov}}{T}\right)\right).$$

If the monorail track has the dimples with the depth Δz (fig. 2c), then at the initial time when the carriage gets into the dimple, there appear oscillations with an amplitude $\Delta z \cos\left(\frac{2\pi t}{T}\right)$ and when it gets out - $\Delta z \cos\left(\frac{2\pi t}{T} - T_{ov}\right)$.

Conclusions and development prospects

The obtained dependences are the base for development of space and dynamic model of overhead monorail necessary for sound choice of its parameters.

Hereafter it is planned to fulfill experimental researches of oscillations of monorail, conditioned by nosing motion of rolling stock and action of dithering from overhead monorail track.

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