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Unsteady oscillations of a mandrel and holdup mechanism of piercer plug

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Abstract
The solution of a task on unsteady oscillations of system "pipe –mandrel - core" of piercing mill of pipe-rolling plant is specified. Differential equation of forced longitudinal oscillations of a mandrel with a core, taking into account unsteady influence of deformation zone and variability in time of pipe mass is worked out. Exact solution of the task concerning non-steady longitudinal oscillations of the system "pipe –mandrel - core" of piercing mill in Bessel's and Neumann's fundamental function is given. Analysis of dynamics of the system "pipe –mandrel - core" is represented by dynamic factor. Some peculiarities of functioning of the mechanism of mandrel holdup taking into account variability of mass of mill exit end and non-stationary influence from the side of deformation zone are established
Keywords: PIERCING MILL, PIPE, DYNAMICS, VIBRATION, MANDREL, CORE, DIFFERENTIAL EQUATION, VARIABLE WEIGHT, LONGITUDINAL RIGIDITY, FORCED OSCILLATIONS, CENTERING UNIT, CAUCHY PROBLEM, LAGRANGE'S TASK, BESSEL FUNCTION, NEUMANN FUNCTION

Introduction

For formation of stable geometry of pipes rolled on the modern piercing mills of pipe-rolling plants (PRP), besides practical importance has stable dynamics of mechanisms of exit end. For realization of stable technological operations of piercing of blind pipe shell and stabilization of vibroactivity of the core of holdup mechanism for mandrel at the exit end of modern piercing mills of native and foreign manufacture with axial billet discharge there used numerous holdup, guide, centering and steady control mechanisms [1] (figure 1a).

Problem statement

Core with the mandrel takes significant dead and time-variant fluctuating dynamic loads from the side of piercing pipe shell. By virtue of the fact that the mandrel core has significant flexibility and grater mass, in the considered mechanical system there appear great in value and time-variant fluctuating dynamic loads, which condition its longitudinal oscillations along the axis of piercing under corresponding harmonic forms. Herein the mandrel together with core of its holdup mechanism fluctuates in the deformation zone. Centering band of the mandrel passes from the designed position of the groove and in the overclamping of working rolls causes unnormalized variation in wall thickness of the billet (pipe). Intensity of nonsteady impact of the deformation zone on the mandrel, change in time of the mass of rolled pipe and parameters of supporting nodes of the mandrel core, set in-line of the piercing axis, complicates greatly adequate description of the dynamic processes [1, 2].

Ways of intensification of technological process and questions of quality improving of the rolling pipes dictate fully necessary conditions of improvement of the constructions of all base mechanisms of core fixation on the rolling axis and setting of the mandrel position in deformation zone by intermediate centering units, installed along all the exit side of piercing mill [3].

The aim of the work

The aim of the work is formation of scientifically grounded propositions concerning development of the exit end construction of piercing mill and technology of pipe manufacturing on piercing mill PRP. This approach conditions deeper study of the influence of various parameters of the process of screw rolling (influence of deformation zone and time variation of the mass of piercing billet) on the mill mechanical system behavior and ready product quality.

Problem-solving procedure

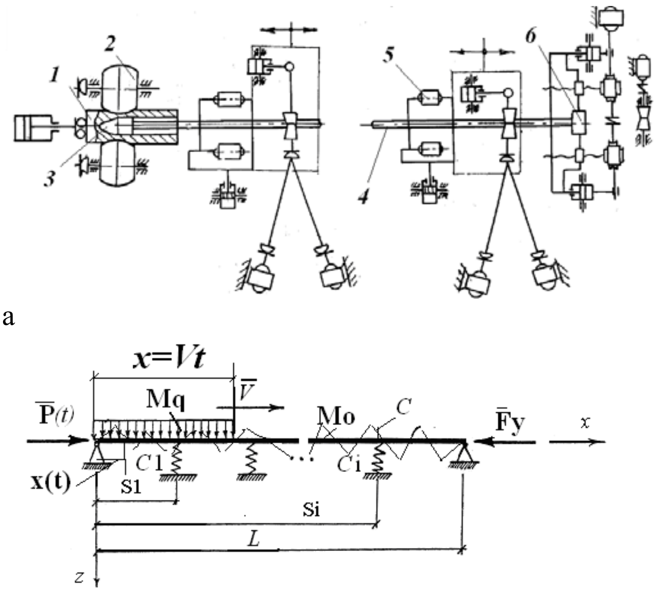
Solution of set task conditions some specification of calculation scheme (fig. 1b) and, in connection with this, further development of mathematical model of investigated system “billet-mandrel-core” of piercing mill PRP, which reflects real processes, occurring in the initial mechanical system of piercing mill, the most precisely.

In this work the developed dynamic and mathematical model “pipe-mandrel-core” of piercing mill is considered as subject of research. This work differs from the known ones [1-3] by complex approach to the investigation of non-steady dynamic processes with further consideration of time-varying parameters inactive characteristics of rolled pipe and cycling in time process loads, acting from the side of deformation zone of piercing mill.

Investigation of developed dynamic model “framed structure- pipe shell” allows to analyze nonsteady dynamical state of core system with mandrel within all the piercing process of pipe shell and on the base of above mentioned to solve the task concerning working out of sound technical solutions on the exploitation of equipment of exit end of piercing mill.

Let us assume the hollow core of uniform cross-section with hinged bearing at the ends and elastic supports (centering units) between them (fig. 1b) as the calculation scheme of forced longitudinal oscillations of the mandrel with core of mandrel holdup mechanism of piercing mill PRP. The core together with rolled billet is subjected to the impact of harmonic component of the axial force of piercing $\vec{P}_0 \sin(\omega t)$ from the side of deformation zone. Herein across the core with the mass M_0 and length l and with semi-constant velocity of piercing $\vec{V}$ (uniformly) along the rolling axis there moves rolled billet with the mass M_q.

Let us take on the statement of the specified task solution concerning forced longitudinal oscillations of the mandrel with core system of mechanism of its holdup at the exit end of piercing mill PRP taking into account variability in time of the mass of rolled billet.



a

b

Figure 1. Piercing mill PRP with axial billet discharge a) and refined calculation scheme of the mandrel with core of mechanism of its holdup b) under forced longitudinal oscillations of the system: 1 – pipe shell (billet); 2 – working rolls; 3 – mandrel; 4 – mandrel core; 5 – centering unit; 6 – steady-control mechanism

We will use corresponding differential equations of longitudinal motion of the mandrel across the rolling axis [3, 4] for building of correct mathematical model of the system and further analysis of dynamic condition of the mandrel with core during piercing of pipe shell.

$$\frac{d}{dt}\left[M(t)\frac{dx(t)}{dt}\right]=\sum_{k=1}^{n}F_{kx}, \qquad (1)$$

where $x(t)$ – longitudinal motion of the mandrel in the deformation zone across the rolling axis; $M(t)$ – rim variable in time of the mass of the system cincidering the initial mass of the core M_0.

Elastic force of mandrel core in the lengthwise direction of rolling axis we will assume within allowances under Hooke linear law

$$F(t) = cx(t). \qquad (2)$$

Here c – is axial stiffness of the mandrel with core and mechanisms of exit ends of the mill (stiffness of all elastic system of mill exit end in the direction of piercing axis).

Under the assumption that internal friction in the system as compared with cyclic technologies $P_0 \sin(\omega t)$ and nonsteady dynamic loads is inessential, according to [5-6], we will come to differential equation of mandrel longitudinal motion.

$$M(t)\frac{d^2x(t)}{dt^2} + \frac{dM(t)}{dt}\frac{dx(t)}{dt} + cx(t) = P_0 \sin(\omega t), \qquad (3)$$

where ω – activity rate of constraining force (without sliding equals angular velocity during conjoint rotation of the mandrel, core and billet around the axis of piercing).

It should be marked that variability in time of the mass of rolled billet causes proportional change of inaction of the whole mechanism of mandrel holdup, that determines the character of development of nonsteady dynamic processes in mechanical system.

Basing on the results of numerous researches [1-3], the law of system mass changing in respect with variability in time of mass of movable pipe, one may take as follows:

$$M(t) = M_0 + M_q \left.\frac{x}{l}\right|_{x=\upsilon t} = M_0(1+\gamma t), \qquad (4)$$

where $\gamma = \frac{M_q}{M_0}\frac{\upsilon}{l}$ – is the velocity of mass change (exponent index) ($\lambda > 0$ the system mass always increases); $M_q = m_q\, l$ – the mass of rolled billet; υ – velocity of motion (piercing) of the billet on the mandrel core; $M_0 = m_0\, l$ – initial mass of core system.

Basing on the researches of questions of body dynamics of variable mass, proceeding from the statement of fundamental problem of I. V. Meshcherskiy [4], in this approach of task solution in the equation (3) we do not consider reactive summand of inertial loading $\frac{dM(t)}{dt}\frac{dx(t)}{dt}$ from action of rolled pipe.

Consequently, for analysis of corresponding part of the equation (3) in respect with (4) one may form Cauchy problem and under certain initial conditions [1-7]. Then differential equation of longitudinal oscillations of the mandrel with core (3) taking into account the law of change of pipe mass (4) in time in the statement of Cauchy problem looks as follows

$$\begin{cases} M_0(1+\gamma t)\dfrac{d^2x(t)}{dt^2} + cx(t) = P_0 \sin(\omega t) H(t); \\ x(0) = x_0 = 0; \qquad \dfrac{dx(0)}{dt} = \dfrac{dx_0}{dt} = 0. \end{cases} \qquad (5).$$

Let us proceed to finding of dynamic factor during action of reactive component. Not considering the action of reactive force, we will find the solution for the differential equation (5) at set initial task conditions. Having divided the parts of equation (5) in M_0, let us write:

$$(1+\gamma t)\frac{d^2x(t)}{dt^2} + \omega_0^2 x(t) = \frac{P_0 \sin(\omega t)}{M_0} H(t), \qquad (6)$$

where $H(t)$ - is Heaviside impulse function; $\omega_0^2 = c / M_0$ – square of free-running frequency of the mandrel with core.

For solution of non-homogeneous differential equation of forced longitudinal oscillations (6) in closed species, we may insert the change of variables

$$1 + \gamma t = \xi. \qquad (7)$$

Then Cauchy problem (6) in respect with these conditions we will represent as follows:

$$\frac{d^2x(\xi)}{d\xi^2} + \frac{\omega_0^2}{\gamma^2} x(\xi) = f(\xi) \qquad (8)$$

where $f(\xi) = \frac{P_0}{M_0 \xi \gamma^2} \sin\left[\frac{\omega}{\gamma}(\xi - 1)\right]$.

Solution of homogeneous differential equation at $P_0 = 0$

$$\frac{d^2x(\xi)}{d\xi^2} + \frac{\omega_0^2}{\gamma^2} x(\xi) = 0$$

at the next approximation of WBK-method [7] we will represent in fundamental Bessel and Neumann functions

$$x_1(\eta)=\eta J_1(\eta);\ \ x_2(\eta)=\eta Y_1(\eta), \qquad (9)$$

where $J_1(\eta)$; $Y_1(\eta)$ – are fundamental Bessel and Neumann functions [8];

$$\eta=\eta(t)=\eta_0\sqrt{1+\gamma t};\ \eta_0=\frac{2\omega_0}{\gamma};$$

According to Lagrange [6], initial Cauchy problem has the solution in quadratures:

$$x(\eta)=c_1(\eta)x_1(\eta)+c_2(\eta)x_2(\eta), \qquad (10)$$

where

$$c_1(\eta)=-\frac{2}{\eta_0^2}\int_{\eta_0}^{\eta}\frac{\eta f(\eta)x_2(\eta)d\eta}{\Delta(\eta)};$$

$$c_1(\eta)=\frac{2}{\eta_0^2}\int_{\eta_0}^{\eta}\frac{\eta f(\eta)x_1(\eta)d\eta}{\Delta(\eta)}. \qquad (11)$$

where $f(\eta)=\frac{P_0\eta_0^2}{M_0\gamma^2\eta^2}\sin\left[\frac{\omega}{\gamma}\left(\frac{\eta^2}{\eta_0^2}-1\right)\right]$.

From the set initial Lagrange problem we may form Wronskian determinant for the system of equations. Revealing the main determinant of the system of equations, we may write

$$\Delta(\eta)=x_1(\eta)\frac{dx_2(\eta)}{d\eta}-x_2(\eta)\frac{dx_1(\eta)}{d\eta}. \qquad (12)$$

Here $x_1(\eta)$ and $x_2(\eta)$ – are corresponding fundamental decisions of homogeneous differential equation (8).
As the following equality is true

$$\frac{d}{d\xi}=\frac{\eta_0^2}{2\eta}\frac{d}{d\eta}, \qquad (13)$$

then differentiating the expressions (9)we will obtain

$$\frac{dx_1(\eta)}{d\eta}=\eta J_0(\eta); \qquad (14)$$

$$\frac{dx_2(\eta)}{d\eta}=\eta Y_0(\eta). \qquad (15)$$

To obtain the expressions after corresponding substitutions (9), (14) and (15) into (12) and transformations we find

$$\Delta(\eta)=\frac{\eta_0^2\eta}{2}\left[J_1(\eta)Y_0(\eta)-J_0(\eta)Y_1(\eta)\right]=\frac{\eta_0^2}{2}$$

(16)
Further substituting into (11) we find

$$c_1(\eta)=-\frac{\pi\eta_0P_0}{2c}g_1(\eta);$$

$$c_1(\eta)=\frac{\pi\eta_0P_0}{2c}g_2(\eta). \qquad (17)$$

where

$$g_1(\eta)=\int_1^{\frac{\eta}{\eta_0}}\sin\left[\frac{\omega}{\gamma}\left(y^2-1\right)\right]Y_1(y\eta_0)dy; \qquad (18)$$

$$g_1(\eta)=\int_1^{\frac{\eta}{\eta_0}}\sin\left[\frac{\omega}{\gamma}\left(y^2-1\right)\right]J_1(y\eta_0)dy. \qquad (19)$$

It is obvious that after corresponding substitutions into (10) and some transformations we will finally find dynamic motions of the mandrel with a core along the axis of pipe rolling.

$$x(\eta)=\frac{\pi P_0}{2c}\eta\eta_0\left\{b_1(\eta)\sin\left[\frac{\omega}{\gamma}\left(\frac{\eta^2}{\eta^2}-1\right)\right]+b_2(\eta)\cos\left[\frac{\omega}{\gamma}\left(\frac{\eta^2}{\eta^2}-1\right)\right]\right\}$$

(20)
where

$$b_1(\eta)=\int_1^{\frac{\eta}{\eta_0}}\left[J_1(y\eta_0)Y_1(\eta)-J_1(\eta)Y_1(y\eta_0)\right]\cos\left[\frac{\omega}{\gamma}\left(\frac{\eta^2}{\eta^2}-1\right)\right]dy;$$

(21)

$$b_2(\eta)=\int_1^{\frac{\eta}{\eta_0}}\left[J_1(y\eta_0)Y_1(\eta)-J_1(\eta)Y_1(y\eta_0)\right]\sin\left[\frac{\omega}{\gamma}\left(\frac{\eta^2}{\eta^2}-1\right)\right]dy.$$

(22)

Herein the determination of dynamic factor we will trace to the formula

$$K_\partial(\eta)=\frac{ampl[x(\eta)]}{x_c(\eta)}=\frac{c}{P_0}ampl[x(\eta)], \qquad (23)$$

where $x_c(\eta)=\frac{P_0}{c}$ – static deformation of elastic systems of mechanism of mandrel holdup along the axis of rolling.

Consequently by means of substitutions and transformations we will find the amplitude of dynamic factor under unsteady mandrel oscillations.

$$K_\partial(\eta)=\frac{\pi\eta\eta_0}{2}\sqrt{b_1^2(\eta)+b_1^2(\eta)}. \qquad (24)$$

In such a way, under unsteady impact of the deformation zone, the behavior of the system "pipe-mandrel-core" changes as appropriate. Herein maxima of dynamic factor become contrary to those, which were previously obtained without account of reactive force [3,5].

Non-homogeneous differential equation (5) was composed and represented in Cauchy form, which accurately describes forced oscillations of the mandrel with core of piercing mill. Further for assurance of developed mathematical model the solution of differential equation (5) we will make with the help of Runge-Kutta method for the most common first form of oscillation of the system "pipe-mandrel-core".

Let us make extended calculation of on the base of mathematical model of the task concerning forced oscillations of the mandrel with core for piercing mill PRP 30-120. For this purpose let us take the following initial data for piercing mill PRP 30-120: $l = 12$ m; $\upsilon = 2{,}1$ m/s; $M_0 = m_0 l$; $m_0 = 100$ kg/m; $M_q = m_q l$; $m_q = 90$ kg/m; $c = 22 \cdot 10^6$ N/m; $t \in [0;5]c$.

The results of numerous analysis of differential equation (5) of longitudinal oscillations of the mandrel together with core of mechanism of its holdup during pipe rolling with diameter of 114x12, material – steel 20 on the piercing mill PRP 140 are represented in the figure 2.

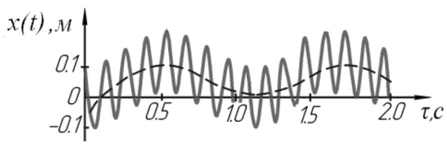


Figure 2. Forced longitudinal oscillations of the mandrel with core on the piercing mill PRP 30-120 (billet with the diameter 145x20, material –steel 20).

Calculated curve, given in the figure 2, shows extra unsatisfactory unsteady conditions of functioning of the mandrel with core within the process of pipe shell piercing on the piercing mill PRP 30-120. Amplitude of longitudinal oscillations reaches 0.137 m, which leads to the moving of centering band of the mandrel from design position on the groove overclamping and formation of increased variation in wall thickness of rolled billets.

Further we will investigate the influence of change in time of mass of mechanical system on the dynamics factor K_d on the example of piercing mill PRP 30-120.

In the table 1 there are the values of K_d, obtained with the help of numerical quadrature of differential equation (5) (the first line), approximate formula (20) (the second line) and exact formula (24) (the third line) at increase of the mass of piercing mill PRP 30-120 systems.

Table 1 Values of K_d in different time moments obtained in different ways

t, s	0,5	1	2	3	4	5	6
K_d	1.2997 1.2992 1.2994	1.8218 1.8217 1.8215	0.6037 0.6034 0.6033	1.3776 1.3778 1.3779	0.5575 0.5574 0.5572	1.1327 1.1325 1.1323	0.2757 0.2774 0.2721

The results from the table 1 represent high convergence and accuracy of asymptotic dependences in engineering analysis and exact solution of the task under the formula (24).

The figure 3 represents dependence of dynamic factor K_d on the time t, for the case of linear increase of the mass of system "pipe-mandrel-core" calculated with the help of formula (24).

Graph in the figure 3 testifies that in the system "billet (pipe) - mandrel- core" with increasing mass the first maximum $K_d < 2$ and next maxima are lower than the first one. Therefore there is some stabilization of vibroactivity in the system "billet (pipe) - mandrel- core", which coincides with the results of some experimental researches of piercing mill PRP 30-120 [2,3].

Analysis of calculation results shows that mathematical model of the processes truly describes unsteady dynamic phenomena in the system "mandrel-core" . Amplitude-frequency response characteristics of dynamic processes during longitudinal oscillations of the system within realization of all process of piercing of pipe shell on piercing mill PRP 30-120 exceed admissible level of vibroactivity.

In course of realization of technological process unsteady vibroactivity of the system implicates formation of billets with increased variation in wall thickness, which further becomes of complicated and hard-to- eliminate character. It is obvious that consideration of degree of impact of deformation zone and variability of mass of rolled billet, which moves with velocity $\vec{V}$, is constitutive parameter within the limits of chosen unsteady dynamic model of the mandrel with core system of exit end of the mill.

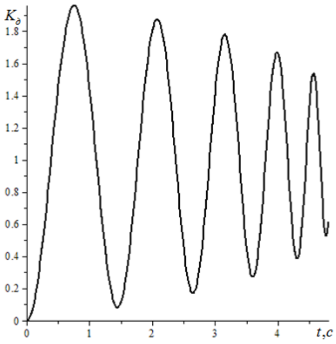


Figure 3. Dependence of dynamic factor K_d on the time *t* for piercing mill PRP 30-102 during increase of the mass of mechanical system without considering of reactive component ($\gamma > 0$)

Possibility of complex mathematical modeling of various piercing regimes for pipe shell on the design stage of pipe rolling process on the piercing mill PRP makes significant difference of obtained results from the results of known before works [1-3] in the field of researches of dynamic stability and vibroactivity of the systems of mandrel holdup mechanism.

Conclusions

1. Base computational scheme is extended and reasonable mathematical model of forced oscillations of mandrel with core of mechanism of mandrel holdup is developed within the limits of considered unsteady dynamic model of the system “pipe-mandrel-core” of piercing mill PRP. Variability in time of mass of rolled pipe, cyclic technological loads from the deformation zone and parameters of elasticity of holdup mechanism of mill mandrel are considered. The results of computational solution of differential equation of longitudinal oscillations of the mandrel with core under Runge-Kutta method, which allows to estimate longitudinal motions of the mandrel in the deformation zone along with closed solution of the task, taking into account variability in time of the mass of rolled pipe on the example of piercing PRP 300-120 (billet with diameter 145x20, material – steel 20).

2. Closed solution of the task of unsteady dynamics of mechanism of mandrel holdup in fundamental Bessel and Neumann functions is given. Herein cyclic loadings from the deformation zone of the mandrel with core by technological load and variability in time of the mass of rolled billet are considered. Unsteady processes in the system “pipe-mandrel-core” of piercing mill PRP are represented by dynamic factor of mandrel motions. It is stated that the value of dynamic factor, for example, for mechanism of holdup of mandrel of piercing mill PRP 30-102, is not equal to two, but significantly exceeds this value with time. Its values changes steadily in the course of unsteady longitudinal oscillations of the mandrel.

3. It is possible to determine effective regimes of pipe shell piercing and forecast quality indicators of produced pipes on the basis of the value of admissible level of vibroactivity of elastic system of mandrel holdup mechanism by means of complex mathematical modeling of unsteady forced oscillations of mandrel holdup mechanism on the stage of production processes setting of pipe shell piercing. For example, during manufacturing of pipes on the piercing mill PRP 30-102 (billet with diameter 145x20, material – steel 20) it is determined by calculation that piercing rate on the mill should not exceed 1.75 m/s. This value is close enough to the results of experimental researches of piercing mill PRP 30-102 [2, 3], which points at the accuracy of results obtained.

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