Stress calculation of moment transmitting roll with profile on the base of Reuleaux triangle

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Abstract
Detailed calculation of polar moment of inertia for cross section of Reuleaux triangle form is considered. With the help of received dependences, it is possible to determine to a high precision concerning stress, appearing in profile connection during torque transmission. Also software module for strength prediction of profiling rolls is described. It allows to fulfill strength check and adjustment of overall parameters of profile moment transmitting connection on the base of Reuleaux triangle.
Key words: POLAR MOMENT OF INERTIA, AXIAL MOMENT OF INERTIA, TWISTING, REULEAUX TRIANGLE, PROFILE CONNECTION, TANGENTIAL STRESS.

Torsion analysis of moment transmitting connections lies mainly in determination of maximum tangential stress in cross section. Basic calculating formula for determination of tangential stress in the point y

$$\tau = \frac{T_{\max} \cdot y}{I_\rho},$$

where $T_{\max}$ – maximum torque, transmitted by the roll,
I_ρ – polar second moment o f area;
y – coordinate, where stress must be determined.
When cross section is of simple form, calculation of I_ρ is easy to perform. In general terms it may be presented as the sum of two axial moments of inertia

$$I_\rho = I_{Xc} + I_{Yc} \quad (1)$$

where I_{Xc} and I_{Yc} - axial moments of inertia.

Moment transmitting connection, performed on the base of Reuleaux triangle (figure 1) has rather complex profile in cross section. It consists of three circular segments (1, fig. 1) and a triangle (2, fig.1). Centers of circles, describing circular segments, lie in the corners of triangle.

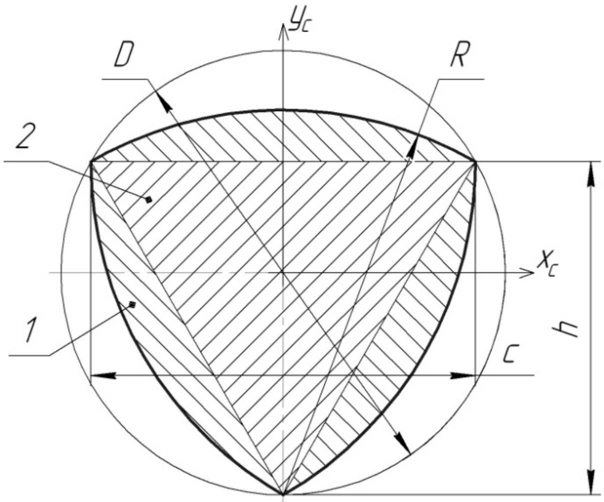

Figure 1 Determination of polar moment of inertia of Reuleaux triangle

Triangle side *c* (parameter R in the fig. 1) may be determined from the formula

$$c = R = \frac{D\sqrt{3}}{2}$$

Altitude of the triangle *h* may be determined as follows

$$h = \frac{c\sqrt{3}}{2} = \frac{3D}{4}$$

Triangle polar moment of inertia is equal to

$$I_{\rho\,tr} = \frac{ch^3}{36} + \frac{hc^3}{48} = \frac{3\sqrt{3}D^4}{256} = 0{,}0203D^4 \quad (2)$$

Polar moment of inertia of circular segment may be determined from the formula (fig.2):

$$I_{\rho segm} = I_{Xc1 segm} + I_{Yc segm} - A \cdot y_c^{\,2} + A \cdot \left(y_c - \frac{D}{2}\right)^2 \quad (3)$$

where $I_{Xc1 segm}$ and $I_{Yc segm}$ - axial moments of segment;
A – circular segment area;
y_c – coordinates of segment gravitational center.

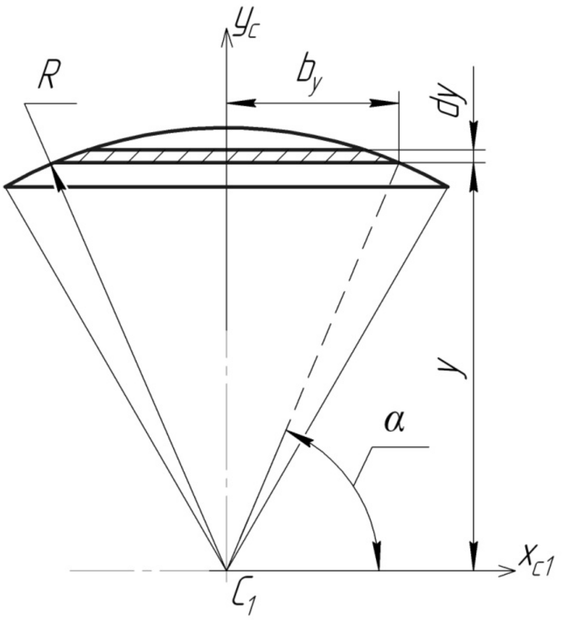

Figure 2 a Determination of polar moment of inertia of circular segment about the axis X_{C1}

Now let us determine axial moment of inertia $I_{Xc1segm}$, $I_{Yc segm}$ and circular segment area (fig. 2, b).

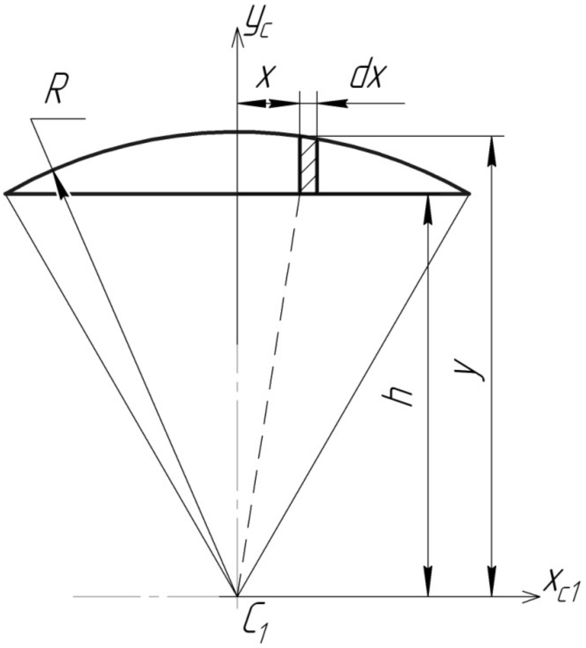

Figure 2 b Determination of polar moment of inertia of circular segment about the axis Y_C.

As one knows from mechanics of solid deformable body:

$$I_{Xc1segm} = \int_A y^2 \mathrm{d}A$$ and $$I_{Yc} = \int_A x^2 \mathrm{d}A$$

Calculating $I_{Xc1segm}$, surface element is located at a distance $y = R \cdot \sin\alpha$. Correspondingly $\mathrm{d}y = R\cos\alpha \mathrm{d}\alpha$. Value $b = R \cdot \cos\alpha$. Area of surface element equals $\mathrm{d}A = 2b \cdot \mathrm{d}y$. The angle may change within $\frac{\pi}{3} \le \alpha \le \frac{\pi}{2}$. Circular segment area is equal to

$$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \mathrm{d}A = D^2\left(\frac{\pi}{8} - \frac{3\sqrt{3}}{16}\right) = 0{,}06794D^4 \quad (4)$$

Moment of inertia concerning X_{C1} axis equals

$$I_{Xc1_{segm}} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} R^2 \cdot \sin^2\alpha \cdot 2b\mathrm{d}y = \frac{R^4}{4}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right) = 0{,}04319D^4$$

(5)

Let us calculate $I_{Yc segm}$

$$I_{Yc segm} = 2\int_0^{\frac{R}{2}} x^2(\sqrt{R^2 - x^2} - h)\mathrm{d}x,$$

where $x = R\cos t$ - distance from Y axis to surface element.

To take this integral in parametric form is the most preferable. Let us introduce the parameter t as follows:

$$\cos t = \frac{x}{R}$$

Correspondingly $t = \arccos\frac{x}{R}$, $\mathrm{d}x = -R\sin t\mathrm{d}t$. Consequently, limits on integral will be changed: at $x_1 = 0$, $t_1 = \arccos 0 = \frac{\pi}{2}$, at $x_2 = \frac{R}{2}$, $t_2 = \arccos\frac{R}{2 \cdot R} = \arccos\frac{1}{2} = \frac{\pi}{3}$. As a result we will get

$$I_{Yc segm} = 2\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} R^2\cos^2 t(\sqrt{R^2 - R^2\cos^2 t} - h)(-R\sin t\mathrm{d}t) =$$

$$= R^4\left(\frac{\pi}{24} - \frac{7\sqrt{3}}{96}\right) = 0{,}002590D^4$$

(6)

First moment of area of segment in x-axis equals

$$S_{Xc} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} y\,dA = \frac{R^3}{12}$$

Coordinate of gravitational center may be determined from the formula

$$y_c = \frac{S_{Xc}}{A} = 0{,}7967D \quad (7)$$

Finally, inserting (4), (5), (6) and (7) into (3), we will get polar moment of inertia of circular segment:

$$I_{\rho_{segm}} = 0{,}0432D^4 + 0{,}00259D^4 - 0{,}0679D^2\cdot(0{,}7967D)^2 + $$
$$+ 0{,}0679D^2\cdot(0{,}7967D - 0{,}5D)^2 = 0{,}008633D^4.$$

(8)

Let us assume polar moment of inertia of Reuleaux triangle in the form of the sum of triangle polar moment of inertia and tripled polar moment of inertia of circular segment:

$$I_\rho = 0{,}0203D^4 + 3\cdot 0{,}008633D^4 = 0{,}04619D^4 \quad (9)$$

On the base of these calculations, computer program was developed. Width across corners and maximum torque, being transmitted by profile connection, are the input data. Program interface is shown in the figure 3.

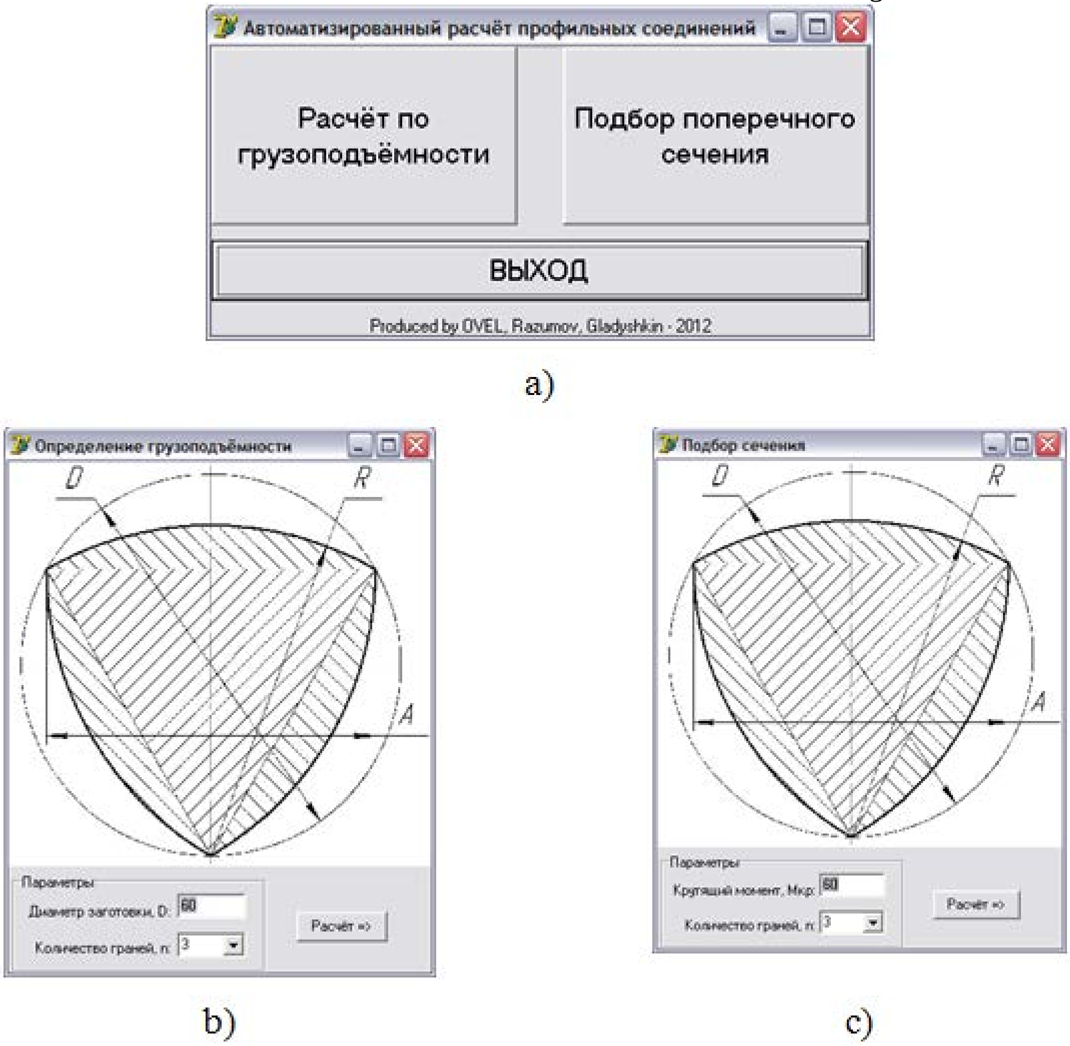

Figure 3 Program interface a) calculation route selection b) calculation in accordance with allowable carrier power c) selection of minimum size of cross section

The given above mathematical tool may be reprocessed and realized for calculation of five-cornered connection. However Reuleaux pentagon is impractical among machine-building profiles.

In such a way, this methodology will allow to shorten time, spent for design of profile moment transmitting connections, received on the base of Reuleaux triangle.

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