
RAIL VEHICLES: THE RESISTANCE TO THE MOVEMENT AND THE CONTROLLABILITY

monograph

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C 19 Rail Vehicles: The Resistance to the Movement and the Controllability: Monograph.


The monograph substantiates the existence and determines the origin of the constituent ele-
ment of the resistance to the movement within rail carriages; the constituent is determined by the
control of the wheel pairs within the railway track. In this book, we suggest the method to analyze
closed power circuit in mechanical power transmission applied to rolling stock. The method of
mathematical modeling for two-point contact of the wheel with the rail has also been developed.
The characteristics of the kinematic resistance to the movement for a number of types of rolling
stock have been obtained. There are power factors which control the rail carriages and their analysis
is very important, therefore we address to it in the book as well. Based on the decrease in the circu-
lation ratio within the closed power circuits of the control system created for carriages by the rail-
way track, we also suggest the principles of designing the rolling stock truck arrangement with low
resistance to the movement.

The monograph is intended for scientists, engineers and technicians working in the field of
design and research for railway transport as well as for masters and postgraduate students of spe-
cialty 273 - "Railway transport".

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INTRODUCTION

For more than 30 years, one of the main and most resource-intensive problems of railway transport is the problem of heavy wear of wheels rolling surfaces and rails in curved sections of tracks. Significantly greater wear of wheels and rails in curves in comparison with straight lines is explained by the fact that the rolling stock in the curves is guided by the horizontal forces, which are necessary for doing work against the frictional forces between wheels and rails. The track curved sections make up more than 40% of the total length of the main Ukrainian railways, and 60-70% in the field of industrial and municipal transport. Taking this into account it becomes obvious that the urgency of the research studying for the processes of carriages guiding by railway tracks is beyond doubt. The main reason for the resistance to the movement associated with the control of wheel pairs by railway tracks is the frictional interaction in the contacts of wheels with rails caused by sliding. Moreover, the contact sliding is the main cause of the wheel-rail rolling surfaces wear in curved sections of tracks.

The resistance to the movement is one of the most important technical and economic characteristics of the railway carriages. The practical experience of the railway transport operation shows that from 1/3 to 1/4 of the total energy consumed by the railway rolling stock traction is used for doing work against the friction.

The forces of the resistance to the movement of trains or separate units of the rail transport are regarded as the external forces acting on it and directed, as a rule, against their movement. As the traction forces, they are conditionally reduced to the contact points of wheels with rails. The forces of the resistance to the movement are subdivided into the main ones acting constantly during the movement and the additional ones emerging only when moving along the certain sections of the track or at certain periods of time.

The sum of the forces of the main resistance ($W_o$) and the additional resistance ($W_d$) is called the total resistance of the train (W). The train movement resistance consists of the resistance to the locomotive movement ($W'$) and the resistance to the train carriages movement ($W''$). In calculations, the specific forces of
the resistance to the movement are used, i.e. the absolute resistance forces taken relative to the weight of the corresponding unit of the rolling stock. They are measured differently across the countries, for example: N/kN, kg/ton, lb1/ton, etc.

The main resistance to the movement \( (W_o) \) acts in case of the movement in straight horizontals of tracks when there is no wind and it consists of the following components associated with the following processes:

- friction in the box bearings of the rolling stock;
- friction and sliding wheels on rails;
- impacts on rails irregularities;
- aerodynamic resistance of the air medium.

The forces of the additional resistance to the movement are of the resistance forces operating under certain conditions. Moreover, the additional resistance to the movement can be composed by the components associated with the following movement conditions:

- slopes gradients of the track;
- curved sections of the track;
- operation of undercar generator in passenger carriages;
- resistance forces that emerge at low air temperatures;
- additional resistance when starting;
- contrary or cross winds.

The traction effort of the locomotive is necessary for doing work against the movement resistance of both the locomotive and the train. If the traction force is greater than resistance, the train is accelerated in accordance with Newton’s motion law. If the traction force is equal to the resistance, the train moves at a constant speed.

\[1\text{ lb} \approx 0.454 \text{ kg}\]
If the traction force is lower than the resistance, the train slows down.

The traditional design of the truck arrangement of the rolling stock and, in particular, that of the wheel pairs with the wheel rigid connection through the axle is the cause of the specific dynamic processes, which are often found with railway transport only.

Their main feature is reciprocal effect of the wheels and the wheel pairs through the axle, the truck frame and the rail track on the distribution of torque between the wheels, in case of the group drive this distribution is between the axles as well.

The common knowledge says that the control over, or guidance of railway carriages by tracks is carried out under the influence of the horizontal forces which occur when wheels contact with rails. The rigid linking of the wheels in the wheel pair and the variable radii of the wheel profiles provide classical advantages of the rigid wheel pair: ability to direct the uncontrolled movement in the transverse direction within the gap of the railway track and self-centering respectively the axis.

The concepts of the controllability are widely used in the theory of the movement of wheeled and track-type machines, ships and aircrafts, spacecraft, i.e. wherever controlling influence comes from the part of the control bodies. Based on the analysis of the controllability of these means of transport, the authors suggested the generalized term for the controllability.

In its turn, the controllability is defined as a property of a vehicle to be under trajectory and/or a course guidance, that is to preserve, or to change the magnitude and the direction of the motion speed under the influence of the control effect.

If we consider machines with control systems, their controllability is determined by the reaction of the machine to the controlling effort coming from the control body, the reaction shows itself in the form of the changes in the track parameters, in the course parameters or in the lateral kinematic parameters. For example, in the case of an automobile, this is a steering wheel turning, in the case of a ship or an aircraft, these are a helm and a control wheel respectively.

As it is known, there are no trajectory control bodies within the railway carriages because they are guided or directed by lateral reactions from the rail track.

In the current monograph, the authors have shown that the horizontal effect of the carriage on the track and the kinematic resistance to the movement are interconnected and interdependent; the carriage with a lower kinematic resistance to the
movement has a lower horizontal dynamic effect on the track. The book suggests the approach which regards the guidance of carriages by the railway track as a controlled phenomenon; this allows us to create the general approaches to the design of truck arrangements on the basis of the comprehensive methodology for evaluation of the quality indicators of the railway carriages controllability.

The monograph is intended for scientists, engineers and technicians who work in the field of design and research of the dynamics of railway carriages as well as students studying for their Master’s degrees and post-graduate students of specialty 273 "Railway transport".
1. STUDY ON THE RESISTANCE TO THE MOVEMENT OF THE ROLLING STOCK

1.1. Historical Review on the Studies of the Movement Resistance with the Railway Rolling Stock

Starting the review from the first studies on the resistance to the movement, dated back as far as the earliest ages of the railways development, and including the recent ones, one cannot help noting that almost all the researches of this kind have been experimental and conducted with the objective to obtain the formulas for the traction calculations. In general, the whole period of time devoted to the studies of the resistance to the movement can be divided into the main and the additional ones.

In 1818, George Stephenson, the inventor of the first famous steam locomotive called Rocket, conducted the first experiments to determine the resistance of wagons to the movement in England.

In 1825, Czech engineer Franz Anton Ritter von Gerstner, later built the first Tsarskosilska Railroad in the Russian Empire, revealed that the resistance of the wagon to the movement when transferring goods on the rails was seven times higher than in case of its movement on natural soil road.

In 1835, when the first railway of the Russian Empire was only being designed, P. Melnikov, the Minister of Communications of the Russian Empire in 1865-1896, wrote a book where he described the experiments on determining the rolling stock resistance to the movement.

In 1858, Russian engineer A. Dobronravov in *General Theory of Steam Engines and Theory of Steam Locomotives* [1] considered in detail the constituent elements of the resistance to the train movement. A. Dobronravov raised the question of interaction between the locomotive traction power, the train weight, the profile of the track and "ability to drive a locomotive".

In Russia, the first attempts to determine the force of traction by research were made by professor M. Okatov. In 1869, he made the experiments on "sliding" (determining the limitation of the traction force by the adhesion) on the section of Peterburg-Luban of the Mikolaiv railway.

A large series of the experimental studies on the rolling stock resistance to the
movement, the traction force, the water and fuel consumption was carried out in 1877-1879 under the supervision of Russian engineer V. Lopushinskyi on the Morskansk-Syzran railway [2].

The experiments proved the necessity to take into account the kinetic energy of the train in case of traction calculations. According to the results of the experiments, the first empirical formulas were derived for determining the basic specific resistance of passenger carriages, freight carriages and locomotives, the additional specific resistances in curved sections of tracks, as well as the dependence of the traction effort on the adhesion and the dependence of the steam flow on the train speed. The numerous studies resulted in the fundamental formula of dependence between the movement resistance and the speed:

\[ W = A + BV + CV^2, \]  

where \( W \) – the absolute movement resistance in the force units; 
\( V \)– the movement speed.

Coefficients \( A \) and \( B \) describe the influence of the mass movement on the resistance and the mechanical component of the interaction with the track. Coefficient \( C \) defines the effect of the movement aerodynamic resistance or the air resistance on the resistance.

Later, the triangular quadratic parabolas became the most common forms of describing the main resistance of the rolling stock:

\[ w''_0 = a + \frac{b + c \cdot V + d \cdot V^2}{q} \]

where \( q \) – the average weight of the unit of the rolling stock.

Even during the first experiments, the influence of "injurious or parasitic carriage movement" on the resistance was the issue of concern and was determined in a lower value for the passenger carriages than for the freight ones. This was the first mention of the kinematic resistance to the carriage movement.

In paper Determination of Fuel Consumption by Locomotives (1877), L. Ermakov, a professor of the Moscow Institute of Railway Engineers (Russian Em-
pere), scientifically developed the fundamentals of traction calculations to determine
the train weight, the movement time, the permissible speeds for trains at braking, the
consumption of water and fuel. Thus, for defining the economic benefit of the train
acceleration before the up-hill, the accumulated kinetic energy of the train was taken
into account. In his paper *Data and Calculations Relating to Locomotives Operation*
(1883), the distribution of the movement resistance forces acting in the train is con-
sidered: the resistance to the movement on the straight horizontal track, the resistance
to the movement of tenders, the resistance to the movement on up-hills and the re-
sistance in curves of tracks [3].

In 1881, Ukrainian engineer A. Borodin expressed the idea of creating artificial
conditions during experiments on locomotives and was the first who proposed to re-
locate the experiments from the railway track into the laboratory, where the locomo-
tive at any constant mode transfers its work to the transmission or through the rollers
to brakes. In 1882 in Kiev workshops, he created the world's first engine laboratory,
where compound machines were tested [4].

In 1889, on the basis of the theoretical and experimental studies of the Moscow-
Kursk and Vladikavkaz railways, professor N. Petrov and engineer V. Lopushinskyi
(Russia) suggested the calculation formulas to define the resistance of the rolling
stock [5] as a function of the motion speed:

- for two-axle freight carriages
  \[ w''_0 = 1.8 + 0.073 \cdot V + 0.001 \cdot V^2 \] , (1.3)

- for two-axle passenger carriages
  \[ w''_0 = 1.8 + 0.083 \cdot V \] . (1.4)

The forces creating the resistance to the movement have been studied both ex-
perimentally and theoretically. The theoretical work of professor N. Petrov in the
field of friction became classical and is still being studied by the specialists working
over the issues of the resistance to the movement of the rolling stock. In lecturing
course *Locomotives* and in his other papers [6-8], he systematized the researches on
the theory of the trains traction. Professor N. Petrov considered in detail the causes of
the wheels resistance to the movement at the junctions, and the cases of the air re-
sistance to the movement of the train.
The hydrodynamic theory of friction developed and published in 1883 by professor N. Petrov explained the phenomena occurring in the axle boxes of wheel pairs and helped to cast light on the problem of the movement resistance in a new approach.

N. Petrov is considered to be the founder of the theory of the train traction. He developed a formula to determine the main specific resistance to the movement of two-axle freight carriages. This formula is interesting by its structure, namely, by the presence of two main factors that affect the resistance to the movement: the speed and the loading degree of carriages.

The formula by professor N. Petrov for two-axle freight carriages was as follows:

\[ w = 1.2 + \frac{0.9 V}{q} + \frac{0.0012 V^2}{q} + \frac{0.03 V^2}{Q}, \]  

(1.5)

where \( w \) – the main specific resistance of the train (kg/t);
\( q \) – the average weight of the loaded carriage (tones);
\( Q \) – the weight of all the carriages in the train (tones).

The studies of N. Petrov and V. Lopushinskyi gave an opportunity to improve the methods of train traction calculations and proved the inaccuracy of the calculation formulas developed by Meyer, Geigard, and Franco.

In 1895, S. Smirnov, engineer and director of the Petersburg Putilov factory, outlined the foundations of the method of "minimum" for defining the directional forces of rails in the curves [9].

In 1898, Russian engineer Yu. Lomonosov began the operational tests on the stream locomotives within the trains by the order of the Traction Department of the Kharkiv-Mykolaiv railway. In paper Traction Calculations and Appendixes with Graphical Methods [10], Yu. Lomonosov substantiated the necessity to reject the purely theoretical approach in determination of the traction force and the resistance to the movement of the rolling stock with further transition to the experimental practice.

In 1904, American professor W.J. Davis published in the Street Railway Journal an article based on his experimental research of the first electric locomotives [11]. The first complex tests to determine the resistance of electric locomotives at high speeds and the influence of the carriage number on the resistance were carried out in 1900 at section Buffalo & Lockport, the maximum speed reached 60 mph. Utilizing the experimental data, W.J. Davis suggested the following formula for calculating the
resistance of trains with the electric traction:

\[ w = b = c \cdot V^2 + \frac{d \cdot V^2}{T} \cdot [A_1 + m(A_2 + A_3 + \ldots + A_n)], \quad (1.6) \]

where \( w \) – the specific resistance to the movement (lb/ton);
\( V \) – the motion speed (ml/h);
\( T \) – the train weight (ton);
\( c \) – the complex friction coefficient when sliding and rolling;
\( d \) – the wind pressure coefficient;
\( m \) – the proportionality coefficient showing the effect of each carriage on the overall aerodynamic movement resistance;
\( A_1, A_2, \ldots, A_n \) – the cross-sectional areas of the locomotive \( (A_1) \) and carriages \( (A_2, \ldots, A_n)(lb^2) \).

If all carriages in the train are the same, the formula is simplified as written:

\[ w = b = c \cdot V^2 + \frac{d \cdot A \cdot V^2}{T} \cdot [1 + m(n - 1)], \quad (1.7) \]

where \( n \) – the number of carriages in the train;
\( b = 3.5 \) – for freight trains;
\( b = 4.0 \) – for standard passenger train carriages and long electric trains;
\( b = 5.0 – 6.0 \) – for electric trains light in weight;
\( c = 0.11 \) – for heavy track constructions;
\( c = 0.13 \) – for the average construction of the track;
\( d = 0.0035 \) – for open platforms;
\( d = 0.0024 – 0.0030 \) – for the connection of carriages of electric trains;
\( m = 0.10 \).

During the period of 1908-1916, a large series of experiments was carried out in the experimental laboratory of the University of Illinois (USA) under the guidance of Professor E. Schmidt and engineer N. Dunn for determining the resistance of the freight and the passenger trains with the steam and the electric traction. The results were obtained in the range of speeds between 10-55 mph during the experimental travel. The results of the research were published in a series of the publications of the bulletin of the Engineering Experiment Station of the University of Illinois, Urbana during 1910-1927 [12-16].
In 1927, Edward C. Schmidt summarized the results of the many years of the research on the resistance of freight trains in track curved sections [17] and assumed that the additional resistance to the movement per the curve or the specific resistance could be measured in kg per ton of weight per 1 degree of the curve.

One of the issues which does not have the mainstream agreement is the authorship of the quadratic form of the movement-resistance function of the speed. It is attributed to many researchers. Sometimes it is called Petrov-Lopushinskyi formula, sometimes – as Borries Formel, sometimes – Leitzmann Formel formula, or Function de Barbier [18]. Moreover, there are references to several more author's formulas on the dedicated issue in the scientific literature. We consider it is reasonable to mention them below.

The formula by A. H. Armstrong (1910) [19]:

\[
  w = \frac{50}{\sqrt{Q}} + 0.03 \cdot V + \frac{0.002 \cdot a \cdot V^2}{Q}.
\]  

*Westinghouse* formula:

\[
  w = 4 + \frac{V}{10} + \frac{10 \cdot S^2}{36 \cdot Q}.
\]  

*Mailloux* formula (1904):

\[
  w = 3.5 + 0.15 \cdot V + \left[\frac{0.02 \cdot N + 0.25}{N \cdot Q}\right] \cdot V^2.
\]  

*Cole* formula (1909):

\[
  w = 5.4 + 0.002 \cdot (V - 15)^2 + \frac{100}{(V+2)^2}.
\]  

In formulas (1.8)–(1.11) read as follows:

- \(w\) – the main specific movement resistance (lb/ton);
- \(V\) – the motion speed (m/\(^2\)h);
- \(Q\) – the train weight (ton);
- \(a\) – the cross-section of the carriage (lb\(^2\));
- \(N\) – the number of carriages in the train.

However, the most commonly, the fundamental formula for the movement resistance is still called *Davis equation*. In 1926 in brochure [20], Davis suggested an
improved empirical formula to calculate the train resistance at straight horizontal tracks in the form as follows:

\[
w = 1.3 + \frac{29}{Q} + 0.045 \cdot V + \frac{0.0005 \cdot a \cdot V^2}{Q \cdot N}, \tag{1.12}
\]

where \(w\) – the train resistance to the movement (lb/ton);
\(Q\) – the axial loading (ton);
\(N\) – the number of the axes;
\(a\) – the cross-section of the locomotive (m²).

For many decades, Davis's original formula for the movement resistance has remained unchanged in principle, but there were developed many variants how to alter its coefficients as the new types of the rolling stock entered the scene, the modernizations of the old rolling stock occurred or the speed increase took place.

Great importance was also paid to the accuracy of the traction calculations and therefore the Davis formula has been constantly completed and improved by the efforts of hundreds of researchers.

Thus, based on long experiments in 40-50 years of the 20th century, American Association of Railway Engineers (AREA), modified the Davis equation in the form as follows:

\[
w_u = 0.6 + \frac{20}{q} + 0.01V + \frac{K \cdot V^2}{q \cdot n}, \tag{1.13}
\]

where \(w_u\) – the resistance to the movement (lb/ton);
\(q\) – the axial loading (ton);
\(n\) – the axes number;
\(V\) – the speed (ml/h);
\(K\) – the coefficient of the aerodynamic resistance.

The values of coefficient \(K\) are the following: \(K = 0.07\) for covered carriages; \(K = 0.0935\) for containers; \(K = 0.16\) for trailers on railway platforms.

The resistance to the movement in the curved sections of the track is greater than in the straight lines. This statement is beyond any doubt and confirmed by a large number of experiments. However, it is the least studied phenomenon, and the results obtained are contradictory in comparison with other components of the movement resistance. Meanwhile, it is generally accepted that the movement resistance in the
curve is inversely proportional to the radius of the curve.

In 1912, under the Ministry of Communications of the Russian Empire, *the Office of Experiments on the Types of Stream Locomotives* was established. In 1918, it was transformed into *the Experimental Institute of Railways*, and in 1935 - into the All-Union Research Institute of Railway Transport headed by Yu. V. Lomonosov. Let us consider some of the activities of this organization with long traditions in the railway industry. In 1915, they developed the document of *Rules for Comparative Tests on Types of Steam Locomotives*, which described the obligatory procedures for testing locomotives intended for operation at the railways belonging to the state. On the basis of the tests, the technical passports of the stream locomotives of almost all the series operated on the railways of Tsarist Russia were issued. Based on the experiments, they approved *Temporary Rules for Traction Calculations* [3], which were grounded on so-called *Formula of Kharkiv-Mikolaev Railway*. These rules had been used for the traction calculations of the resistance to the movement with the biaxial carriages for the long period of time.

In 1908-1916, under the guidance of G. Lebedev, Russian railway engineer, a series of experiments with four-wheel freight carriages on trucks was conducted and a formula for main specific resistance to the movement was suggested as written:

\[
\omega''_0 = 2.8 + \frac{q-12\cdot v}{700} + 0.00144 \cdot V^2. \tag{1.14}
\]

Despite the use of a large amount of the experimental data, the formula was not proved by the subsequent studies and these calculations were not applied in practice.

Based on the results of the tests with the trains composed of the four-wheeled passenger carriages and with the steam locomotive traction, Russian engineers V. Lubimov and N. Dadaev developed the dependences for the movement resistance under summer and winter operating conditions. They are respectively shown as follows:

\[
\omega''_0 = 1.2 + 0.01v + 0.003v^2, \tag{1.15}
\]

\[
\omega''_0 = 1.5 + 0.005v + 0.005v^2. \tag{1.16}
\]

Professor Yu. V. Lomonosov [21] believed that dependence (1.15) corresponds to the ideal operation conditions, and therefore for four-axle ones he proposed the
dependence, which was widely used for the calculations of the resistance to the movement until 1937.

\[ w_0'' = 1.4 + 0.02v + 0.002v^2. \]  \hspace{1cm} (1.17)

Before 1937, the main specific resistance to the movement of two-axial and three-axial carriages was determined as follows:

\[ w_0'' = 1.6 + 0.0027v + 0.0003v^2. \]  \hspace{1cm} (1.18)

However, in 1958, Russian engineer B. Karachan found the new dependence for the main specific resistance of four-wheel freight carriages [22]:

\[ w_0'' = \frac{65+v}{12+0.55q}. \]  \hspace{1cm} (1.19)

The above mentioned dependence showed a slight error between the theoretical and the experimental data.

The study of Research Institute of Railway Transport based on dependence (1.19) gave the opportunity to obtain the other formula for two-axle freight carriages; this formula has been used for traction calculations up to date and is as written:

\[ w_0'' = 1.4 + 0.2v + \frac{0.5v}{q}. \]  \hspace{1cm} (1.20)

In 1937, the Central Scientific Research Institute of the Ministry of Transport and Communications conducted the test travels on the main line of the Zhovtneva Railway with the objective to determine the main resistance of four-wheel carriages and they resulted in the dependence below:

\[ w_0'' = 1.4 + 0.012v + 0.0003v^2. \]  \hspace{1cm} (1.21)

At the same time, the test travels were carried out with the two-axle and the four-axle passenger carriages and they allowed the researches to receive the dependence of the main specific resistance on the speed in the form as given:

\[ w_0'' = 1.4 + 0.017v + 0.0003v^2. \]  \hspace{1cm} (1.22)
This dependence is also currently used in the practice of the traction calculations.

In 1947, Soviet professor P. Gurskyi conducted another series of the test travels with the four-axle tanks [23] and developed the dependence of the resistance to the movement for the loaded and the empty tanks respectively:

\[ w_0'' = 1.3 + 0.04v, \quad (1.23) \]
\[ w_0'' = 1.3 + 0.06v. \quad (1.24) \]

In 1947-1948 the Ural and Siberia railway test travels were carried out with the freight trains of different weights in order to determine the influence of low temperatures on the resistance to the movement [24]. The results were expressed in the dependence of the main specific resistance of the freight four-axle carriages:

\[ w_0'' = 0.65 + \frac{14+0.2v^2}{q_0} + 0.0002v^2. \quad (1.25) \]

The further research on the main resistance for four-wheel freight carriages were conducted by the Transport Problems Scientific Researches Section of the USSR Academy of Sciences [25]. Thus, the main specific resistance for this type of carriages was determined as the following expression based on the theoretical studies:

\[ w_0'' = 0.51 + (0.044 + 0.00004v^2)n^2 + 125\varphi + \frac{11k_cq_0}{\sqrt{Iu}} + \frac{0.0564v^2}{q_0}, \quad (1.26) \]

where \( n \) – the wheel pair running on the rolling track (cm);
\( \varphi \) – the friction coefficient for the axle sets in bearings;
\( k_c \) – the coefficient of the resistance from the elastic deflection of the track;
\( I \) – the moment of rail inertia relative to the horizontal axis passing through the center of the rail cross-section (cm\(^4\));
\( u \) – the elastic modulus of the rail base (kg/cm\(^2\)).

In 1953-1955, in Central Scientific Group of the Ministry of Internal Affairs, the comparative tests with the passenger carriages equipped with axle boxes on roller bearings and friction bearings were carried out. The experiments allowed the further development of the formula for passenger carriages with roller bearings [26], it is given below:

\[ w_0'' = 1.15 + 0.0102v + 0.0003v^2. \quad (1.27) \]
In 1956, P. Gursky [27] on the basis of the data processing obtained from the tests on the traction and the heat of the courier locomotive of 2-3-2 type (Kolomenskyi plant), determined the main resistance to the movement of the train with 10 passenger carriages equipped with the axle boxes on the roller bearings:

\[ w_0'' = 1.4 + 0.012v + 0.00026v^2. \quad (1.28) \]

The dependence developed as the result of this work was known as the most accurate one for the calculations of the passenger carriages resistance at speeds faster than 90 km/h, however, the fact that it was developed with a small number of the test travels and with the carriages and the tracks of old types could not be neglected.

The next step was made by A. Baranov [28] who suggested a generalized formula for passenger trains to be as follows:

\[ w_o = 0.53 + \frac{10.7 + b \cdot V + \left( b + \frac{8}{n_o} \right) \cdot 10^{-2} \cdot V^2}{q_o} \]

where \( b = 0.75 \cdot l \cdot 10^{-2}; \)

\( l \) – the carriage length;

\( n_o \) – the number of axles in the train.

In the Ukrainian Rules of Traction Calculations for Train Operation [29] (hereinafter referred as RTC), the formulas for determining the approximate value of the additional resistance to the movement in curves are as follows:

a) on condition that the length of the train is less than the length of the curve, the following can be applied:

\[ w_r = \frac{700}{R} \]

or

\[ w_r = 12.2 \cdot \frac{\alpha^0}{s_{cr}}, \quad (1.31) \]

where \( R \) – the radius of the curve (m);
\( \alpha^0 \) – the angle length of the curve (degree);
\( s_{cr} \) – the linear length of the curve (m);

6) if the length of the train is more than the length of the curve, they use as written:

\[
w_r = \frac{700 \cdot s_{cr}}{R \cdot l_t}
\]
or

\[
w_r = 12,2 \cdot \frac{\alpha^0}{l_t}
\]

where \( l_t \) – train length (m).

Furthermore, in several countries (USA, Italy, England and China), a formula similar to (1.30) is used for the traction calculations but with the different coefficients (refer to Table 1.1).

However, Germany, Austria, Switzerland, the Czech Republic, Slovakia, Hungary and Romania use the formulas, where instead of the radius of curve \( R \) there is the difference of \((R-b)\) in the denominator of formula (1.30) while \( b \) is considered a constant value:

\[
w_r = 650/(R - 55) \quad \text{– for curves with radius } R > 300 \text{ m};
\]
\[
w_r = 500/(R - 30) \quad \text{– for curves with radius } R < 300 \text{ m}.
\]

Table 1.1 Formulas for Approximate Estimation of the Additional Resistance to the movement in the Curve

<table>
<thead>
<tr>
<th>Countries</th>
<th>Formula for Movement Resistance in Curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>( w_r = 446/R )</td>
</tr>
<tr>
<td>Italy</td>
<td>( w_r = 800/R )</td>
</tr>
<tr>
<td>England</td>
<td>( w_r = 600/R )</td>
</tr>
<tr>
<td>China</td>
<td>( w_r = 573/R )</td>
</tr>
</tbody>
</table>

The fact that the values of the movement resistance for \( R = 300 \text{ m} \) differ by more than 30% shows that all these formulas are approximate in the best case.

In Ukraine, for more accurate calculations for the additional resistance to the
movement, RTC [29] recommends to use the following formulas:

a) if the length of the train is less than the length of the curve, there should be applied as written:

\[ w_r = \frac{200}{R} + 1.5 \cdot \tau_c \]

or

\[ w_r = 3.5 \cdot \frac{\alpha^0}{s_{cr}} + 1.5 \cdot \tau_c \]

b) if the length of the train is more than the length of the curve, there should be applied as given:

\[ w_r = \left( \frac{200}{R} + 1.5 \cdot \tau_c \right) \cdot \frac{s_{cr}}{l_t} \]

or

\[ w_r = \left( 3.5 \cdot \frac{\alpha^0}{s_{cr}} + 1.5 \cdot \tau_c \right) \cdot \frac{s_{cr}}{l_t} \]

Here, \( \tau_c \) – the unbalanced radial (centrifugal or centripetal) acceleration of the carriages movement in the curve (m/s):

\[ \tau_c = \frac{V^2}{R} - \frac{h \cdot g}{s} \]

where \( V \) – the train movement speed (m/s);
\( h \) – the height of the external rail (m);
\( g \) – acceleration of gravity (9.81 m/s^2);
\( s \) – distance between the rolling circles of the wheel pair (for tracks of 1520 mm, \( s = 1600 \) mm).

Recalling (1.38) we obtain the following:

\[ w_r = \left[ \frac{200}{R} + 1.5 \cdot \left( \frac{V^2}{R} - \frac{h \cdot g}{s} \right) \right] \]

or
\[ w_r = \left[ 3.5 \cdot \frac{\alpha^0}{s_{cr}} + 1.5 \cdot \left( \frac{V^2}{R} - \frac{h \cdot g}{s} \right) \right] \cdot \frac{s_{cr}}{l_t} \]

Figure 1.1 shows the dependencies developed with formula (1.36).

As it can be seen from Figure 1.1, according to formulas (1.31) and (1.33), the values of the resistance to the movement become negative for some curve radiiuses, the increase of the external rail obtains the negative values as well. This does not correspond to our perceptions of the physics of the process. If a carriage moves with an equilibrium speed in a curve, then the total centrifugal force in the transverse direction is to be zero.

The equilibrium speed is defined from the condition as written below:

\[ \frac{V^2}{R} - \frac{h \cdot g}{s} = 0 \]

from which we obtain:

\[ V = \sqrt{\frac{R \cdot h \cdot g}{s}} \]
Figure 1.1 – The Dependencies of the Specific Resistance to the Movement of the Rolling Stock in a Curve on the Speed. Constructed per Formulae (1.34) and per Formula by RTC (1.30).

The equilibrium speed is limited with unbalanced centripetal acceleration, which, meeting the condition of comfort for passengers, cannot exceed 0.7 m / c². At the equilibrium speed, it is possible to hope for the minimum side effect which wheel flanges produce on rails and respectively the minimum value of the component composing the resistance to the movement, it depends on flange friction, which occurs on rail lateral faces.

With the increase or the reduction of the speed, the equilibrium lateral force influencing rails increases. Moreover, when the speed increase, the force influencing the outside rail increases while the reduction of speed means the increase in the force acting on the inside rail. These causes were expressed, in particular, by E. Schmidt in 1927 in the weekly Bulletin of Engineering Experimental Laboratory of the University of Illinois [30].

The same was supported by S. Amelin and G. Andreyev, the professors of the Leningrad Institute of Railway Transport, in their book Railway Design and Operation (1986) [31].

The fact, that there exists the minimal dependency of the resistance to the movement in the curve on the speed, finds its confirmation by several experimental studies. Thus, within the researches of this kind one can name the results presented by the English researchers: A. Armstrong and J. L. Koffman. In 1910, A. Armstrong with his book Standard Handbook for Electrical Engineers (Electric Traction) provided the dependencies [32]. We show them in figure 1.2 as the scanned copies. Later, in 1961-1964, J. L. Coffman conducted a big series of experimental work on the resistance to the movement on the British railroads [77-79]. According to the research program, the analysis was carried out to reveal the dependency of the resistance to the movement on the following factors: roughness of rail lines; rolling friction of wheels; joints of rails; hunting and parasitic movements of trucks (truck hunting); friction in the flange contacts of wheels with rails; hunting and the characteristics of springs and hunting dampers. From the results of the research, J. L. Coffman drew the conclusion that the resistance to the movement needs to be predicted at the stage of the rolling
stock development. The special attention is to be paid to the issues of the interaction between the train (the locomotive and the carriages) and the track that depends on the static and the dynamic parameters of the track, the design of wheels, the profile of flanges, the characteristics of springs and spring suspension hunting dampers, such as torque transmission. In particular, J. L. Coffman notes the essential difference in the resistance of the movement with the locomotives with different type of spring suspension: laminated springs and spiral springs.

**Figure 1.2 – Dependencies of the Resistance to the Movement in a Curve on the Speed.** Provided by *Standard Handbook for Electrical Engineers* by A. Armstrong (scanned copies from book [32])

A large contribution to the experimental studies on the resistance to the movement of different types of trains belongs to Soviet researcher P. Astakhov.

In monograph *Tractive Resistance of the Rolling Stock* (1966) [39], P. Astakhov systematized the results of the long-term experimental research on the resistance to the movement with locomotives and carriages, which were conducted by Central Scientific and Research Institute of Railway Transport of the Ministry of
Railways. The monograph demonstrates technical, economic and power aspects of the problem of the resistance to the movement with the rolling stock, a short survey of the researches and the calculation formulae of the main and the added resistance, and states the methods of the experimental research with the magnitudes of the resistance to the movement. Considerable space is devoted to the analysis and balance of the main resistance components and to the clarification of nature of their patterns and rated analytical expressions.

On the experimental ring of the All-Russian Research Institute of Railway Transport in 1960-1962 P. Astakhov and P. Stromsky obtained results concerning dependency of tractive resistance in the curve from the velocity, which also confirm the existence of the minimum in the zone of equilibrium velocity. Fig. 1.3 contains scanned copies from the article by P. Astakhov and P. Stromsky [33], where the given results of researches of tractive resistance in the curve are shown.

![Fig. 5.2. The Additional Specific Resistance of Four-Axle Freight Wagons on the Test Loop Curve with the Dependence of the Speed:
1) - empty tank wagon (q=6.1 m); 2) - empty open wagons (q=6 m);
3) - loaded tank wagon (q=19.1 m); 4) - loaded open wagons (q=20.5 m)](image)

*Figure 1.3 – Dependency of tractive resistance in the curve on the velocity by results of researches on the experimental ring of the Central Research Institute of the Ministry of Railways (the scanned copy from the book [33])*

G. Astakhov in the monograph [39] on the basis of the previous research analysis suggests to improve the formula (1.31) for tractive resistance in curves and to use it in the following way:
Fig. 1.4 shows a graphic interpretation of formulae (1.43) and (1.30).

By the way, the experimental ring railroad VND_ZT has the ring form with the constant radius of 955 m and the eminence of the outside rail of 60 mm. Reference equilibrium velocity behind the formula (1.39) equals to 18.7 m/s or 67 km/h that matches results of the experiments described in article [33] and the monograph [39].

![Figure 1.4](image-url)

Figure 1.4 – The dependencies of unit resistance to the movement of the rolling stock in the curve from motion velocity constructed behind P. Astakhov’s formula (1.43) and behind the formula of RTC (1.30).

The American Railway Engineering and Maintenance-of-Way Association – AREMA suggested the following formula for more exact accounting of added resistance to the movement in curve sites of the way (2000) [34]:

$$w_r = \frac{200}{R} + 1.5 \cdot |\tau_k|$$
where $w_r$ is tractive resistance in curve (kNton/);  

$kN$ is the dimensionless parameter which depends on the type of running gear of the rolling stock: it varies from 500 to 1200 with the mean value of 800;  

$R$ is curve radius in meters (m).  

At the same time, in [35] it is noted that the following values for tractive resistance in the curve are sufficient for accuracy:  

- 0.04% (about 0.8 pounds on tone by curve degree) from the main tractive resistance,  
- 0.05% (about 1.0 pound on tone by curve degree) (for small motion velocities (1-2 pounds on tone by curve degree).  


The equivalent bias is defined from the condition of equal tractive resistance in the curve and on the “equivalent” bias which equals $800/R_{\text{eq}}$. For example, for the curve with a radius of 800 m the equivalent bias equals to $1_{\text{eq}}$.  

In [36] the following formula for calculation of tractive resistance of the train through the equivalent bias is given:

$$
Wr = m \cdot g \cdot \sin \iota \quad (1.45)
$$

where $Wr$ is the absolute resistance of the train in the curve;  
$m$ is mass of the train (t);  
$g = 9.81$;  
$\iota$ is the equivalent bias (councils): $\iota = 8001000 \cdot R$;  
$R$ is radius of the curve.  

In [36] the example of the schedule of dependency (1.45) for the French TGV-Dasye weighing 380 t is also given. (fig. 1.5).
T.J. Mlinarić and K. Ponikvar (Slovenia), in 2011 in their article [37] possess simplified formulae for determination of unit resistance to the movement in curves offered. For curves radius smaller for 300 m offers the formula

$$w = \frac{500}{R - 30}.$$ 

For curves of smaller curvature the formula is offered

$$w = \frac{12.2 \cdot \alpha}{l}$$

where $\alpha$ is the central corner of the curve;

$l$ is train length.

By the way, the last formula completely matches the formula (1.30) accepted by “Rules of traction calculations of Ukrzaliznytsia” [29].

The Canadian railroad in 1990 implemented the national version of Davis’s formula, the use of which is believed to be more reliable concerning the accuracy of results of calculations of tractive resistance and mass of trains [38]. The updated Ca-
nadian version of Davis’s formula looks as follows:

\[ w = 1.5 + \frac{18 \cdot N}{P} + 0.03 \cdot V + \frac{C \cdot a \cdot V^2}{10000 \cdot P} \]

where \( w \) – unit resistance to the movement (lbton) / ;
\( P \) – the general weight of the locomotive or ton car( );
With the empirical coefficient.

For application of Davis’s equation of Davis in the high-velocity movement, in particular, for the Japanese trains of system Shinkansen of the series 200 (Japan Shinkansen Series 200) Rochard and Schmid (B.P. Rochard, F. Schmid) [18] offered its modification in the following fashion:

\[ w = 8.202 + 0.10656 \cdot V + 0.0119322 \cdot V^2. \]  

(1.49)

In recent years, the assumptions that the initial Davis’s equations expressed a tendency to the exaggeration resistance value, and therefore the correcting coefficient is sometimes applied, were made:

\[ W_{adj} = K \cdot w_D \]

where \( W_{adj} \) is the corrected value of tractive resistance;
\( w_D \) is tractive resistance, defined behind Davis’s formula;
\( K \) is the correcting coefficient for modernization of values of resistance of Davis.

The coefficient value of the \( K \) correction is based on the testing in controlled conditions, namely:
\( K = 1,00 \) – for the rolling stock till 1950;
\( K = 0,85 \) – for the rolling stock till 1950;
\( K = 0,95 \) – for platforms with containers;
\( K = 1,05 \) – for hopper-cars;
\( K = 1,20 \) – for the empty covered cars;
\( K = 1,30 \) – for the loaded half-cars;
\( K = 1,90 \) – for empty half-cars.
D. Armstrong and P. Swift offered a specification for the aerodynamic component calculations of the railway crews tractive resistance with different features of the train architecture.

Their articles [40], in particular, have distinction explanations in the unit resistance to the movement of the empty and loaded cars. It is caused by the fact that resistance of wind emptily or poorly loaded on the train same, as well as completely loaded.

As the wind resistance becomes dominating at high velocities, the unit resistance to the movement (on the weight unit) will be higher than the train from empty cars. In the case with half-cars (the car with open top), resistance of wind of empty cars still the highest, than at full cars through increase in turbulence of air in and around the empty body to the car.

In many countries of the formula for calculations of tractive resistance significantly differ. In tab. 1.2 the formulae are demonstrated for the calculations of the rolling stock main tractive resistance, which are accepted for calculations on some countries’ railroads.

Table 1.2 – The formulae for calculating the rolling stock main tractive resistance accepted for calculations on some countries’ railroads:

<table>
<thead>
<tr>
<th>Railroad</th>
<th>Formula for the main tractive resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese national railroad</td>
<td>0.92 + 0.0048 \cdot V + 0.000125 \cdot V^2</td>
</tr>
<tr>
<td>NkN( / ):</td>
<td></td>
</tr>
<tr>
<td>loaded carriage</td>
<td>2.23 + 0.0053 \cdot V + 0.000675 \cdot V^2</td>
</tr>
<tr>
<td>empty carriage</td>
<td></td>
</tr>
<tr>
<td>Czech railroad: the loaded daN/ton car( / )</td>
<td>1.3 + 0.00015 \cdot V^2</td>
</tr>
<tr>
<td>German railroad: the loaded daN/ton car( / )</td>
<td>1.0 + 0.0001 \cdot V^2</td>
</tr>
<tr>
<td>Serbian railroad (&quot;new formula&quot;) (daN/ton/ )</td>
<td>0.0483 + 0.0183 \cdot V + 0.00001 \cdot V^2</td>
</tr>
</tbody>
</table>
French national railroad daN/ton( / ):
- loaded carriage 1,0 + V2400 /
- empty carriage 1,2 + V2400 /

Australian railroad N/ton( / ):
- loaded car 5,17 + 0.010997 ∙ V + 0.00051 ∙ V2
- empty car 18,74 + 0.1111 ∙ V + 0.00372 ∙ V2

In fig. 1.6 several dependencies the basic of unit resistance to the movement from velocities are shown and are used for traction calculations, which are used for traction calculations in the different countries of the world.

One of the last offers concerning specification of formulae for tractive resistance, it is possible to consider the formulae received in 2005 by the engineer of the Belgrade Institute of Engineers of Transport (The Institute of Transportation CIP) A. Radosavić.

Drawing 1.6 – The dependencies of unit resistance to the movement of cars on veloc-
ities which are used for traction calculations in some countries of the world.

On the basis of the experimental studies, the formula of the main tractive resistance for the cargo and mixed train with the diesel engine of the JZ 643 series was specified. Results of tests were included into article [41].

The following question deserves special attention: why is locomotive tractive resistance in the run-out mode is greater than in the draft mode? For example, RTC for locomotives (locomotives and electric locomotives) contains separate formulae for the run-out and draft mode (the jointless way):

- in the draft mode

\[ \text{wo}' = 1.9 + 0.008 \cdot V + 0.00025 \cdot V^2 \]

- on run-out

\[ \text{wx}' = 2.4 + 0.009 \cdot V + 0.00035 \cdot V^2 \]

The significant difference in values of the formulae coefficients (1.48)-(1.49) is the consequence of conditional division of the locomotive main tractive resistance of the locomotive into two parts, one of which considers locomotive tractive resistance as carts, and another – as machines. The resistance of the locomotive movement as a machine is caused by loss of friction power in the traction gear and motor-axial bearings when the torque is transmitted from the shaft of the traction motor to the driving wheel. These power losses connected with implementation of force of draft, considered in the traction characteristic. At the movement on run-out when force of draft is not implemented, the locomotive tractive resistance as machines is formed from it tractive resistance as carts.

The researchers of the University of Genoa G. Boschetti and A. Mariscotti in their report on the XX\textsuperscript{th} congress of the International confederation of measurements (XX IMEKO World Congress Metrology for Green Growth) [45] proved the need of lengthening of research in the direction of modeling, measurement and search for the ways of reducing mechanical resistance to the rolling stock movement.

As the carried-out analysis testifies, the overwhelming majority of researches of tractive resistance of the rolling stock were experimental, devoted only to establish-
ment of the dependencies of tractive resistance on velocity necessary for performance of traction calculations.

The received formulae allowed calculating the mass of inclinations, velocities and time of the movement on the site, the brake way, etc. For many decades, when creating new types of rolling stock, the characteristics of motion resistance at the design stage were not analyzed. The exception can be considered numerical researches of aerodynamic resistance to the movement which became urgent during creation of high-velocity trains.

In development of running gears of locomotives and cars there are a lot of questions of dynamics and the description in curves of small radius, often decided by introduction to vehicular parts of different elastic dissipative elements without possible deterioration in the characteristics of tractive resistance connected with the direction the railway line that, sometimes led to increase in horizontal loads at frictional contacts of wheels with rails which at the same time carried out the role of frictional dampers with the high energy dispersion level.

The data on the eminence of wheel swing surfaces wearing at operation of new large-weight locomotives types [42-44] are indirect confirmation of this. Especially, it should be noted the intensive undercut flanges and side wear of heads of rails what, preferential is result of work of resistance forces to the movement.

In some cases, at operation in curves small to radius wear so intensive that between-repairs run of locomotives on the undercut flanges make only 5... 7 thousand km.

1.2. Structure of tractive resistance of rail crews

In fig. 1.7 the structure of tractive resistance presented with the relative particle of each component though such lowland is quite conditional because the number of components can be carried both to the basic, and to added resistance.

As for the particle compound resistance of the movement, they fluctuate in considerable borders depending on motion velocity, parameters of the railway line, velocity and the relative direction of wind, meteorological conditions, design features of this or that crew.

Especially important, at the same time, there is aerodynamic resistance which
with motion velocities higher than 30 … 40 m/s make to 65% of the main tractive resistance.

1.2.1. Resistance in axle-box nodes

This component of the main tractive resistance is connected with rolling friction (at the rolling bearing) or sliding (at sliding bearings) in axial bearings wheel couples [26, 28]. Moment of force of friction on the axis neck for the case of bearings of sliding is defined from the formula

\[ M_1 = f \Pi \frac{d_{nt}}{2}, \quad (1.53) \]

where \( f \) is the friction coefficient between the neck and the insert of the bearing;
\( \Pi \) is load of the axis neck;
\( d_{nt} \) is the diameter of the neck of the axis.
Resistance force on the wheel rim

\[ F_0 = \frac{f \Pi n_{\text{III}}}{D_\kappa}, \quad (1.54) \]

where \( D_\kappa \) is the diameter of the wheel around swings.

Unit resistance to the movement from friction in axle-box bearings of sliding equals

\[ w = 1000f \frac{d_{\text{III}}}{D_\kappa}. \quad (1.55) \]

The friction coefficient in axle-box bearings of sliding is defined by the formula
where $A$ is the coefficient, which depends on the bearing deviation from central position. Its mean value for the rolling stock equals 0.005;

$n$ is the frequency of the address of the axis of wheel couple;

$\lambda$ is the relation of diameter of the neck – $d_n$ to its length;

$r_{sr}$ is the specific crush in the bearing.

Almost full replacement on the rolling stock of axle-box bearings of sliding by the rolling bearing led to reduction of energy costs by draft of trains on 4,2 ... 4,6% [24].

The sliding friction coefficient for metals, in most cases, makes 0.05 ... 0.25 while the rolling friction coefficient for the same conditions lies within 0.0001 ... 0.001. Often when calculating neglect friction losses of swing. However energy which is spent for overcoming rolling resistance is absorbed mainly in blankets of material and goes for intensive cyclic re-deformation of material.

The heating of working elements of the arising node leads to the gap redistribution, accuracy losses and course smoothness, as well as other violations of normal work of the bearing node. Generally losses of energy in roller bearings are formed from the following parts:

1) friction losses of the rotating bearing elements in the environment, including in lubricating oil, which, in addition to its main function, plays the role of the visco-plastic body [46], which prevents the relative displacement of the bearing elements and creates resistance to movement;

2) losses on effective areas of separators which result from its tensions on the directing boards of rings and friction of bodies of swing on walls of slots;

3) losses which arise when swing rollers on racetracks of rings of bearings.

The reasons of these losses are:

- relative elastic slipping of the connected surfaces which arises through dissimilarity of their curvature and according to distinction of values of elastic deformations of surface volumes - micro volumes of bodies in the contact zone which clench;

- sliding of rollers on the directing boards of rings of roller bearings;
• the general slipping of the set of bodies of swing of rather leading ring which arises at loss between them coupling;
• imperfect elasticity of material;
• molecular interaction of the contacting surfaces, which interferes with their rapprochement on the leading edge of contact and separation on back.

The moment of friction of the roller bearing can approximately be defined behind the formula

$$M = \text{fppd}^2$$, (1.57)

where \( \text{fpp} \) is the reduced friction coefficient;
\( d \) is the diameter of the landing opening of the bearing;
\( P \) is the load of the bearing

$$P = \sqrt{F_r^2 + F_a^2}$$, (1.58)

where \( F_r \), \( F_a \) are the radial and axial to structure loading.

Friction in the axial booms, seals, lubricating devices (for example, pollsters) is also related to losses in relation to the boot knot, etc. When performing an axial stroke on the basis of a slippery slip, friction losses can reach significant values.

The research data show that the main tractive resistance of rail crews decreases from use of roller bearings from 20% – at shift from the place to 5 … 6% – with average velocities of the movement [26, 28].

1.2.3. Rolling resistance of wheels on rails

Resistance to driving of the wheel on the rail has the developing nature which is still up to the end not studied. When swing the present three types of friction which differ behind the description: rolling friction, sliding friction and friction of rotation (or spin).

There are several theories which explain formation of resistance forces when swing.
Professor O. Yu. Ishlinskiya, for example, considering swing of the rigid skating rink on the visco-elastic basis, analytically investigated rolling resistance, explaining with its imperfection to elasticity of materials [48]. She proposed solutions of the task connected with swing of absolutely rigid skating rink by two types of the basis: relaxing and visco-elastic. Emergence of rolling resistance was explained by asymmetry of distribution of reactions from the basis to the skating rink therefore that agrees to the characteristics of the basis accepted in calculations (soil, according to Ishlinskaya) on the rear edge of contact residual sedimentation of the basis unlike zero draft on the leading edge of contact before the wheel always takes place. It the offered following formula for calculation of rolling resistance

\[ W = 0.2\sqrt[3]{18} \frac{\nu P^3}{V^{3/2}} \frac{1}{kbR} \]  

(1.59)

where \( V \) is the roll velocity;
\( P \) is the vertical load;
\( \nu, b, k \) are the characteristics of the swing surface (bases);
\( R \) is the radius of the surface of swing.

Other aspect of the problem of elastic imperfections connected with the elastic hysteresis of the basis was investigated by A. A. Palmgren [49]. On the basis of results of tests of persistent and spherical bearings by A. Palmgren the conclusion is drawn that losses on the elastic hysteresis in these cases represent quite insignificant part from the general losses in the bearing. The main part is represented by the losses connected with relative slipping of surfaces which contact.

D. Tabor in article [50], on the contrary, proves that the elastic hysteresis possesses the leading role in formation of resistance forces to swing. Rolling steel bags on the flat basis different hysteresis properties, D. Tabor received results which show that, depending on properties of the basis, losses of energy can increase more than by 8 times at all applied loadings. However, the author causes that these conclusions fair for rather soft materials, for example, rubber. So when pumping spheres from phosphorous bronze, aluminum and ball-bearing steel of size of power losses belonged among themselves as 10: 5: 1.

The analytical dependency received by Tabor for resistance force to swing has
where $N$ is the normal load in contact;
$a$ is the half-width contact to the site in the direction of swing;
$R$ is the radius of the cylinder or sphere;
$c, \alpha$ are the empirical coefficients [50].

J. A. Tomlinson, [51] investigating rolling resistance on samples from solid steel, defined the resistance coefficient caused by molecular interaction of surfaces at the expense of the attraction and pushing away which are shown as friction forces. Work of these forces, behind J. A. Tomlinson’s thought can be expressed as

$$N \cdot W = \mu \cdot f \cdot p \cdot n, \quad (1.61)$$

where $\mu$ is the friction coefficient;
$F$ is the relative movement of the considered sets of molecules;
$N$ is the total number of the interacting molecules during the way $x$;
$n$ is the number of cycles (rapprochements and distances) of one molecule;
$P$ is the force of interaction of one couple of molecules. Resistance coefficient to swing

$$\lambda = \frac{3qW}{4bP}, \quad (1.62)$$

where $q$ is the statistical constant which depends on structure of material;
$b$ is half-width of the tape of contact.

Values $\lambda$ were experimentally received $(2 \ldots 4) \cdot 10^{-5}$.

S. V. Pinegin, making the experiments similar to Tomlinson, drew the conclusion which the molecular component of rolling resistance on $2 \ldots 3$ orders is lower than power hysteresis losses [52, 53].

A. S. Akhatov also notes that in the practical conditions characteristic of the case of the wheel engagement with the rail, the formation of molecular bridges of the
catching, which is very developing through instant pollution of surfaces oxides and the adsorbed substances [54].

Group of researchers under the leadership of B. V. Deryagin [55] the offered tractive resistance explanation, as result of electrostatic attraction of elements of couple of swing at the expense of opposite charges which arise behind contact which continuously reveals. In the general rolling resistance many researchers applied special bench installations on which it was minimized slidings in contact of the wheel with the rail to definition of the molecular component particle. Such researches were conducted by Yu. Blokhin [56, 57], D. Tabor [50], J. A. Tomlinson [51], R.C. Druтовски [58].

Some of results, namely dependency of the relation of the molecular component of resistance – $W_m$ to damping compound resistance – $W_d$ caused by effect of the hysteresis from the contact crush is shown in fig. 1.8. For the case of engagement of the wheel with the rail the particle of the molecular component in the general rolling resistance in comparison with slipping and the hysteresis small.

\[ \frac{W_m}{W_d} \]

**Figure 1.8 –** The ratio of molecular ($W_m$) and damping ($W_d$) compound rolling resistances from the contact crush

The damping making rolling resistances was more considerable by results, received in researches of different authors. It is known that the hysteresis and the related elastic and micro plastic deformations are defined by physic-mechanical characteristics of materials, level of contact tension, the shape of the contacting surfaces and deformation velocity. Yu. G. Blokhin [56, 57], investigating swing of disks from high-strength cast iron and chromic steel, received different dependencies of rolling resistance on pumping velocity for cast iron ($W_{ch}$) and for steel ($W_{st}$). These dependencies are shown in fig. 1.9.
At researches of dependency of rolling resistance on the level of contact tension the number of authors noted, first, the sharp growth of resistance at achievement of tension determined to the threshold, secondly, unstable results which depend on number of the wheel passes on the same basis. Some explanation for these phenomena is offered K.L. Johnson [59] research according to which after several repeated contact loadings there comes the limit of fitness of material entered by K.L. Johnson, plastic deformations practically stop and there comes the new elastic state. This effect takes place under the condition if

$$po \leq 4 \cdot \tau T$$

where po is the maximum contact crush;
$$\tau T$$ is the material flowability border at shift.

![Figure 1.9](image)

*Figure 1.9 – Relative (1) and absolute (2,3) power hysteresis losses for cast iron (2) and for steel (3) depending on the velocity of driving of the wheel on the rail*

The limit of fitness depends on properties of material and NV on the condition can be approximately determined depending on hardness by Brinell

$$po \leq HB.$$  

The number of passes after which rolling resistance is stabilized also depends on the hardness of material. According to N. A. Korolyova [60] for soft steel with HRC hardness = 61... 62, 15... 20 passes are sufficient, for firmer steel – to 1000
passes. The sharp eminence of rolling resistance depending on velocity is observed at \( V > 350 \text{ km/h} \ (97 \text{ m/s}) \) that for modern rail transport to become urgent. Dependency of rolling resistance from motion velocity is shown in fig. 1.10 [60].

The Japanese researchers Tadao Ohyama and Seigo Uchida concluded that the form of contacting surfaces influences stress distribution on the contact spot, the volume of the deformed material and the contact shape [61].

The experiments made by R.C. Drutovski [62] for the sphere swing on the layer and the cylinder on the cylinder show that with identical diameters of bodies, identical contact crushes and identical roll velocities is observed essential differences in the support to swing at the expense of different volumes of the deformed metal and consequently, and different hysteresis losses. Dependencies of tractive resistance on roll velocity for cylinders (1) and the sheaf (2) shown in fig. 1.11.

For the first time, on the tiresome nature of wear at grated slidings D. V. Konvisarov in 1952 specified in the robots [63, 64].

He saw the reasons of fatigue of the blanket of details in the repeated or sign-
variable movements in mobile connections of details of machines.

One-time surfaces, scratching their different firm edges do not belong to processes of wear in full understanding of this concept. Konvisarov came to conclusion that wear of solid bodies at grated is similar with their destruction for fatigue. The spot of contact feels reusable action (thermal, mechanical) other spots of contact.

As a result in material the crack is formed, and there are its progressing and, the end the end, destruction of the surface. The inspection carried out by Konvisarov of the performed observations by performance on the pendular device of series of special tests. In the course of tests were rolled at the fair-haired steel tempered samples with cylindrical surfaces of different radiuses of curvature.

D. V. Konvisarov’s conclusions have basic value as for the first time the made attempt to explain rolling resistance with simultaneous influence of the number of real factors that did not become in other robots. Besides, samples in D. V. Konvisarov’s experiment were made of typical machine-building material which also increased the real importance of results of experience.

The examples of the complex of volume and surface impact effect accounting on rolling resistance are also the works by G. Goryacheva and L. A. Galina (1973-80) [65-67]. These and many other works allow to draw the conclusion which the main component of rolling resistance is the frictional component caused to slippings in contacts of wheels with rails.

Kinematic tractive resistance to which the special attention in this work is paid also has the nature of frictional losses.

According to the theory of longitudinal sliding of O. Reynolds difference in the zone of contact brings in extension of the basis material and that of the roller to emergence of sliding between them [68].

Deformation of surfaces during swing is followed by unequal movements, but due to sliding they have the opportunity to be deformed, being in contact. Sliding of surfaces is followed by friction which causes rolling resistance. Obviously, it is possible to carry resistance connected with sand which moves on rails for the prevention of boxing or to the skid of wheel couples to the added resistance to the movement. Its high cost are also given about the considerable eminence of tractive resistance at supply of sand in contacts of wheels with rails, speak about need of conducting theoretical and experimental studies of this problem.
1.3. Resistance connected with the direction of wheel couples the railway line

1.3.1. The nature of tractive resistance connected with the direction of wheel couples the railway line

The first railway crews had as of course, cylindrical wheels which slid on rails about contours which directed them. But already on the first engine of Stephenson (1821) rail contour regenerated in flanges of wheels.

The traditional design of wheel couples includes flanges of wheels, rigid communication of wheels in wheel couple and conic profiles of surfaces of swing of wheels. These features of the wheel design couples provide nearly 200 years the reliable direction of crews with the railway line.

The most complete work, in which the question of the crews direction was explored by the railway line, is H. Heyman’s monograph [69]. Heyman notes that the deviation from the way of swing of wheel couples (free or connected by the cart frame) can be observed only in the form of sliding. According to him, longitudinal sliding happens through rigid communication of wheels in wheel couple through the axis. Despite detailed material on the problem of the crews description in curves, Heyman did not consider the tractive resistance connected with the description.

At the movement in curves, thanks to the design of wheel couples, it is provided decrease in the directing efforts between crests of wheels rails. But, in curves of average and small to radius the shortage of obliquity of the swing surfaces causes wheel slip in the longitudinal direction. Forces of longitudinal sliding lead to increase in the distortion of the wheel couple axis in the curve and increase in the angle of crowding of the wheel at the head rails, which increases cross reactions rails and tractive resistance. Confirmation of volume is the characteristic scratch and the gnash at the movement of crew in the curve [70].

At the movement in direct sites, especially with high motion velocities, through roughnesses of the way in the plan and the conic surface of swing of wheels intensive self-oscillations of wagging of wheel couples with periodic engagement flanges wheels with rails appear [71].

With the high motion velocity, increases occur in transverse loads at rails.

Lateral forces which affect rails can reach considerable sizes [72] that the results
in intensive wear flanges and to tractive resistance eminence.

Authors suggest to call the compound support to the movement connected with the direction of wheel couples the railway line – kinematic tractive resistance.

According to the accepted classification, kinematic tractive resistance has signs both the main, and the added resistance [73, 74]. Therefore, conditionally, at the movement in direct sites of the way, it needs to be considered how the part of the basic, and at the movement in curves – as the part of added resistance to the movement.

Kinematic discrepancy of geometrical parameters of surfaces of swing of wheels and kinematic parameters of the movement is the origin of kinematic tractive resistance of crew.

Kinematic tractive resistance arises in the consequence of parasitic slippings in the closed power contours which are formed in system of the wheel couples’ direction by the railway line. The mechanical energy spent for overcoming friction at parasitic slipping is energy of kinematic tractive resistance. In systems of the direction of railway crews carts it is possible to allocate with the railway line several closed power contours.

1.3.2. The closed power contours in contacts of wheels with rails.

As shown in robots [75, 76], at two-point contact of the wheel with the rail the closed power contour with two nodal points generally and flange contacts is formed. In this contour there is the differential slipping which can be the cause additional kinematic tractive resistance through increase in rolling resistance. And related rolling resistance for the first time mentioned differential slipping Heathcote (H. L Heathcote) in the analysis of kinematics the rolling bearing [77].

In fig. 1.12 for the example the shown scheme of possible distribution of normal responses \( N_1, N_2 \) and cohesive forces \( S_1, S_2 \) in two-point flange contact of the car wheel with the rail. \( F_t \) is also the external longitudinal reaction to the wheel from the cart. Reaction \( of F_t \) is also resistance force to the movement which needs to be overcome for pumping of the wheel. Distribution between normal responses \( of N_1, N_2 \) depends on many factors: to motion velocity, curve radius, raising of the outside rail, the provision of wheel couple in the track, the cart design,
profile of the surfaces of wheel swings, etc.

Figure 1.12 – The scheme of distribution of normal responses \((N1, N2)\), cohesive forces \((S1, S2)\) at two-point flange contact of the car wheel with the rail.

On the basis of fig. 1.12 it is possible to write down balance combined equations:

\[
\begin{align*}
\sum M &= S_1 \cdot R_1 - S_2 \cdot R_2 = 0; \\
\sum F &= S_2 - S_1 + F_t = 0. 
\end{align*}
\] (1.64)

From the equations (1.64) it is possible to receive the size of kinematic tractive resistance \(W_k\)

\[W_k = F_t = S_1 (1 - R_1 / R_2).\] (1.65)

It is also possible to draw the important conclusion: resistance force to the movement \(W=F_t\) cannot equal to zero at existence of the wheel flange contact with the rail.

1.3.3. The closed power contours of wheel couples

Wheel couples together with the railway line also form the closed power con-
tours. Lack of longitudinal slippings in contacts of wheels with rails is possible only ideally. Ideal there is the case when single wheel couple is slowly rolled without a flange touch. In actual practice the movements of wheel couple as a part of the cart through existence of axle-box longitudinal reactions, in contacts of wheel couples with rails surely arise slippings. These slippings are parasitic and create additional kinematic tractive resistance, both in straight lines, and in curve sites of the way.

In fig. 1.13 the shown simplified scheme of cohesive forces $S_1$, $S_2$ and axle-box reactions of $F_{b1}$, $F_{b2}$ that affect wheel couple. Wheel couple established with transverse displacement of $\Delta y$ concerning the axis of the way also slides on rails without wagging which restrains axle-box reactions, $F_{b1}$, $F_{b2}$.

**Figure 1.13** – The simplified scheme of cohesive forces $S_1$, $S_2$ and axle-box reactions of $F_{b1}$, $F_{b2}$ that affect wheel couple

For fig. 1.13 it is possible to work out the following equilibrium equations

\[
\begin{aligned}
(S_{k1} + S_{k2}) \cdot A - (F_{b1} + F_{b2}) \cdot B &= 0; \\
S_{k1} \cdot R_1 - S_{k2} \cdot R_2 &= 0; \\
F_{b1} - F_{b2} - S_{k1} + S_{k2} &= 0.
\end{aligned}
\]  

(1.66)

From the equations (1.66) it is possible to receive value for kinematic tractive resistance
\[ W_k = S_{b1} - S_{b2} = S_{k1} \left( 1 - \frac{R_1}{R_2} \right). \] (1.67)

As the carried-out analysis testifies, the overwhelming majority of researches of tractive resistance of the rolling stock were experimental. They were devoted, only, to establishment of dependency of tractive resistance on velocity. The following characteristics (dependencies) were used for calculating train mass, speed, movement time at a section (site), brake-way etc. For many decades when making new types of moving rolling stocks characteristics of resistance to motion at the design stage haven’t been analyzed. Multiple research of aerodynamic resistance to motion, which has become of vital importance when making high-speed trains, may be considered as an exception.

In the course of process of development of locomotive and car undercarriages questions of dynamics of guiding in a curve with a small radius were often solved by introducing various springy-dissipative parts into locomotive underframes without taking into account possible deterioration of resistance to motion characteristics which are connected with directing wheel pairs by a rail track. Sometimes, this resulted in increasing horizontal stress (load) on frictional contacts of wheels with rails which fulfilled a role of frictional dampers with a high level of energy dissipation during this process.

Indirect confirmation of this process is the data concerning an increase of wear of surfaces of wheels rolling when exploiting new types of heavyweight locomotives [78]. Especially, intensive flange worn sharp and side-drift of rail heads should be taken into account which are a result of work of resistance to motion forces.

Surface form of wheels rolling determines different radii of tread circles. Spatial division of contact forces and sliding speeds leads to appearance of differential sliding motion in contacts, creating parasitic frictional forces which are a reason of additional kinematic resistance to motion. This resistance appears to be especially strong when there is a two-point flange contact.

The established classification of resistance to motion does not contribute to development of research regarding processes of guiding carriages by rail tracks. The following statement is based on the fact that frictional processes in wheels contacts with tracks are viewed separately in both basic and additional resistance to motion.

The conducted analysis allowed to define a new research area of decreasing re-
sistance to motion of rail carriages owing to factors which are not researched sufficiently in the world science and practice. The research area is based on a hypothesis that guiding of wheels pairs by a rail track is fulfilled entirely at the expense of the additional resistance to motion. The additional resistance to motion related to guiding carriages by a rail track authors propose to name a “kinematic resistance to motion”.

The offered approach to motion may become a basis for extensive research of resistance to motion with the aim of saving energy resources for haulage of trains.
2. STUDY ON CONTROLLABILITY AS THE PROCESS OF GUIDANCE FOR THE CARRIAGE BY THE RAILWAY TRACK

2.1. The Concept of Controllability in the Railway Transport

It is commonly known that pure wheel pair rolling is possible only if it rolls freely along the rails and is not tied to the truck frame. When driving the wheel pair in a real case with the wheel pair mounted in a frame, there always occurs the axle box reaction. The turning of a separate wheel pair by a truck frame, the movement of which is the result of the interaction with the other wheel pairs and with the body, is manifested in the additional sliding of the wheels, which cause opposite directed friction resistance. These resistances are overcome at the expense of the directed efforts, which are always associated with the resistance to the movement.

The horizontal impact on the track is revealed as the total effect of all horizontal reactions occurring in the contacts of wheels with rails.

A certain reduction in the impact on the track can be achieved, for example, by installing counter battens in steep curves; application of wheel flange lubricators and rail-lubricators; introduction of new profiles of wheel rolling surfaces; optimization of the rolling stock truck arrangement parameters, etc. Given the fact that the curves make up about 30% in the total length of the Ukrainian railways and that the influence of the truck arrangement on the track in the curves is greater than in the straight sections (provided that the speed was the same in both cases), the topical character of the research aiming at controlling the train guidance into the track curves becomes obvious.

The horizontal interaction of trains and tracks are described in several thousands of works, which makes it impossible to review them in detail.

Thus, the theoretical basis for the study of the horizontal dynamics of the rail transport began to be established at the end of the last century.

For geometric framing, the methods by Roy, Pouchet, Vogel, Maistre, Plass, Jakobi, later improved by K.Koroliov, I.Nikolaiev, V.Panskiy and A.Slomianskiy, were used.

The fundamentals of the dynamic guiding trains into curves were laid by S. Smirnov. They are based on the idea of finding the center of the train rotation at the intersec-
tion of its longitudinal axis and the perpendicular, lowered from the center of the curve. The principles of the least resistance to the carriage's turn in the curves and the formula for determining the force of pressing the ridge of the wheels on the rails were proposed by A. Holodetsky. Currently, to study the steady motion of a carriage in curves, the method by K. Tseglynsky modified by K. Korolev, which takes into account the elasticity of the side forces and the coefficients of horizontal dynamics, is used to calculate the lateral forces.

The grapho-analytical method by H. Heyman based on minimizing the moments of the resistance of the carriage rotation in the curve, has not been disseminated in the practice of calculations due to its complexity [69]. H. Heyman examines the behavior in the curve of the Bissell, Helmholtz, Lotter, and Eckard carriages, and notes that the steering wheel, mounted in front of the bogie, often runs to the outside rail and never has a free ride setting, since the latter is unstable. The rear wheel guides, on the contrary, almost always have a free running gear. In this connection, H. Heyman points out the important role of reversing devices for guiding wheel pairs, which should have a sufficiently large reverse moment for reliable installation without jamming in the track.

S. Kutsenko in order to clarify the calculations of the dynamic fitting of the carriage into the curve examined the specified characteristics of the profiles of wheels, the distribution of slides in the contacts and forces of resistance to motion in the curve [79].

A generalized method for determining the sideforces in curves, developed by O. Yershkov, is based on the assumption of linear dependence of the level of lateral forces on the unbalanced centrifugal acceleration and combines three important characteristics - speed, curve radius, and the external rail elevation [80].

In the analysis of the motion of the rolling stock in the curves widely used quasi-static methods with idealizations typical for them: the ideal curve and constant coefficients of friction slip in the contacts of the wheels with rails are considered.

Analyzing the design and principles of the most common reciprocal devices, D. Minov proposed their classification, according to which the characteristic of the back moment should have high stiffness with small deviations of bogies or guide wheel-pairs, which provides increased stability of motion in the straight sections of the track and fixation of the guide wheel-pair in the middle position. At significant
deviations of the bogie, the value of the back moment must sharply decrease [81].

As a rule, the requirements for the characteristics of carriages, in terms of the minimum impact on the track in the straight and curved sections of the track, are contradictory. The discussion of these issues has led to the recognition of the need for controlled orientation of the carriages by rail track. O. Kravchenko formulated the principles of minimizing the directional forces by controlling the angular moment in terms of body link with the wheel and outlined the main ways of implementing the idea of controlled motion [82].

Detailed analysis of known systems of controlled traffic of rail carriages, performed by V.M. Kashnikov [83-85], allowed to conclude that the reduction of directing efforts by optimizing the parameters of the chassis is practically exhausted and the solution of the problem is possible only through the application of controlled motion systems.

According to the classification of controlled motion systems they can be divided into active and passive ones. Passive control systems include:

- the controlled movement of the bogie center of rotation and the reduction, due to this, of the overrun axis guiding forces [86];
- radial installation of bogie axes with the help of a mechanism using unbalanced centrifugal forces [87];
- changes depending on the radius of the curve of the "guiding length" of the carriage with the help of pneumatic cylinders, which affect the running axes;
- application of bogies with turntable wheels;
- the use of a controlled connection between bogies, based on the fact that the joining forces for the front and rear bogies create a moment directed opposite the moment of the friction forces of the wheels along the rails;
- horizontal balancing of axes.

In the systems of active direction, the principles are realized, the distinguishing feature of which is the use of automatic devices for power control parameters and configuration of bogies. For example, those that create an angle in terms of linking a bogie with a body using pneumatic, hydraulic, electric or other drives.

In accordance with the direction of this work, it should be noted that the management of the direction of the carriage by the track is a prerequisite for a significant reduction in the resistance to the movement by reducing the level of horizontal reac-
tions in the contacts of the wheels with rails.

The controllability study during rail-tracked control, in the form in which it is examined by the theory of the movement of wheeled vehicles, and quantification of control is a trivial task, because the result of control is almost always (except in emergencies) known in advance, as we are dealing with the control after a rigid program. One of such quantitative characteristics is, in particular, turbability, defined by a minimum radius curve, in which it is possible to incorporate a carriage with acceptable levels of lateral pressure. The term of rotation is similar to the insertion of Tseglynsky and Koroliov [92, 93].

The authors at the same time, consider the qualitative indicators of motion controllability of rail carriages, which are not in the aforementioned theory. It is proposed to evaluate the quality of control by the level of additional influence on the carriage from the side of the track.

First of all, this is a horizontal effect on the track. It is necessary to distinguish between two modes of curvilinear movement during the control of the carriage of the track: the kinematic framing mode, in which none of the wheel pairs of the carriage has wheel flange contact with rails; and a mode of power-framing, which is characterized by the orientation of wheel pairs with wheel flange touch.

Obviously, in the mode of kinematic framing, the level of impact of the carriage on the track will be much lower than in power framing mode.

When moving the carriage in a circular curve, the main vectors of external force influence on wheel pairs, namely the main force vector \( \mathbf{F} \) and the principal vector of moments \( \mathbf{M} \), equal to zero

where

- is the main vector of the horizontal forces of inertia acting on the carriage;
- the main vector of moments of the horizontal forces of inertia acting on the carriage;
- the main vector of horizontal forces in the contacts of wheels with rails;
- the main vector of moments of horizontal forces in the contacts of wheels with rails.

In the ideal, with steady movement of the carriage in the curve at equilibrium speed, the horizontal effect on the track should be absent, but in practice it always takes place. The value of horizontal contact reactions depends on the quality of con-
trollability.

Another qualitative index of controllability is the additional resistance to the movement associated with controlling, that is, the direction of the wheel pairs by the track. In the course of the direction of the carriage by the track, there is a phenomenon such as the circulation of power flows in closed circuits, formed by elements of the chassis, drive and wheel pairs. Circulating flows of power, although related to the steering function of the wheel pairs, as a rule, are parasitic, resulting in significant additional slippage in the contacts of the wheels with rails, mechanical losses and increased resistance of the movement, especially in curved sections of the track.

In addition, any additional slippage, unrelated to the implementation of the traction effort, sharply reduces the limit values of the coefficient of adhesion, that is, worsen the traction-dynamic and brake performance of the rolling stock. The aforementioned resistance to motion, which the authors refer to as a kinematic, is closely connected to the directing forces and the frictional interaction of wheel pairs with rails.

According to the generally accepted view, bogies with wheel-pairs, installed radially in curves, have a number of advantages over carriages with conventional "rigid" wheel pairs. The latter create considerably more load on the rails. Many theoretical and experimental studies have proved that the radial installation of wheel pairs can significantly reduce the parasitic slippages in the contacts, the load on the wheel flange contacts and, consequently, reduce the wear of the wheel flanges and resistance to movement [85-91].

2.2. Investigation of the directional forces of the wheel pairs by the track

For theoretical investigation of the directional forces of the wheel pairs by the track an accurate description of tactical contact forces and forces of engagement is required.

In the first period of development of the railroad, the issue of wheel-rail adhesion was raised solely in connection with the adhesion qualities of the traction driving train. A large number of research works is devoted to increasing the maximum traction power of adhesion in the wheel-rail system for a more complete realization of its function as propulsion. However, the problem of wheel-rail adhesion is much deeper than its analysis in terms of traction. The adhesion must be considered throughout the
complex, taking into account the complete picture of the horizontal forces of interaction between the carriage and the track, given that the horizontal components of the contact forces determine the horizontal dynamics of the carriage. From the standpoint of this sense, it would be appropriate to single out neither the longitudinal phenomena nor the transversal ones as the individual problems associated only with the adhesion qualities and horizontal transversal dynamics respectively, but to consider these phenomena jointly within the scope of complex horizontal dynamics. The attempts to address the longitudinal and transversal adhesion issues separately or, at best, as partially related, often lead to the adverse effects. For example, the beneficiating activates that improve the dynamic performance may lead to deterioration in the other indicators such as the resistance to the movement, the intensive wear of the contact surfaces, etc.

2.3. Theories of Wheel-Rail Adhesion

At the beginning of the twentieth century in connection with the rapid development of the transport technology, the several scientific theories appeared at the same time reflecting the physical laws of wheel-rail adhesion. Among them the most developed are as follows:

- plastic deformation theory;
- elastic incompleteness theory;
- molecular theory;
- molecular-mechanical theory.

The pseudosliding theory or creep theory by O. Reynolds [68] was widely used in practices of adhesion force calculations. The Reynolds theory was applied by F. Carter when developing the technique for theoretical determination of the amount of wheel slip relative to the rail [94]. By representing the surface of a rail head in the form of a plane and the surface of a rolling circle in the form of a cylinder, F. Carter suggested the formula known today as the creep formula:

\[ \varepsilon = \mu \cdot \sqrt{8 \cdot \frac{P}{ER} (1 - \sigma^2)} \cdot \left(1 - \sqrt{1 - \frac{F}{F_{\text{max}}}}\right), \]  

(2.2)
where \( R \) – the wheal rolling radius;
\( \mu \) – the coefficient of the sliding friction;
\( P \) – the vertical load per a width unit of the contact surface;
\( E \) – the modulus of elasticity of the wheel material;
\( \sigma \) – Poisson's ratio of the wheel material;
\( F, F_{\text{max}} \) – the tangential force per a width unit on the contact surface and its maximum value in terms of the adhesion, respectively.

K. L. Johnson generalized Carter's plane theory to the three-dimensional case of rolling of two spheres with taking into account of longitudinal and lateral creep [95]. According to his idea, the contact surface of two bodies is divided into two asymmetric areas, namely the slide area and the adhesion area. The latter has the ellipse shape, which touches the contact ellipse by apex forward in the direction of the motion. However, Johnson's theory as a generalized Carter's theory is limited to a case study of a pure longitudinal and lateral creep, i.e. the case of the absence of a turning creep or a spin.

The strip theory by J. Halling [96], D.J. Hainess and E. Olerton [97] can also be regarded as the further development of Carter's theory. Their theory considers pure longitudinal creep with the elliptical area of the contact. In this case, the contact patch is divided into a series of strips parallel to the direction of rolling each of which was studied according to Carter's theory without any interrelation between each other.

It should be noted here that the data of the theoretical studies obtained by publications [96, 97] are in good agreement with the experimental work.

The great contribution to the development of the adhesion theory was made by J.J. Kalker, who developed a linear creep theory according to which at small creep values, the area of the contact patch where the slide occurs is so small that it can be neglected if we regard the entire contact area as an adhesion zone [98]. According to this theory, as the particles of the wheel surface fall into the contact area, they interact at the front edge of the contact area. While moving along the contact, the traction effort is generated due to the lack of sliding. J.J. Kalker suggested to express the ratio between the creep (longitudinal, lateral or angular in case of slide) and the forces and the creep moment (spin) in the form of simplified linear functions [99].

The explanations of the friction nature as a result of the deformation of certain
volumes of the contacting bodies when they penetrate each other are given by the deformation theories of friction according to which the friction arises due to the deformation wave, which travels in front of each penetrating protrusions.

This theory has been developed in Andreev’s works2.

The absorption and the dispersion of the energy in the process of formation and destruction of the frictional bonds due to the thermodynamic hysteresis are described in the energy theory of friction developed by V.D. Kuznetsov3, and P.A. Rebinde4.

Moreover, the development of this theory in the form of an entropy-energy theory of friction is presented in the work by L.I. Bershadsky5.

According to the Structural-Energy Theory of Friction, all friction processes arise and develop as a result of two influencing phenomena: on the one hand, the increase in the free energy in the friction system (activation), on the other hand, its decrease (passivation).

Furthermore, A.V. Chichinadze6 laid the foundations of the thermal dynamics of friction and further developed by M.V. Korovchinsky7 and V.S. Shchedrov8.

The aspects of the material which are within the metal science scope of studies are also taken into account for the structural phase transitions and transformations on contacting surfaces during friction. They are outlined in the works of K. L. Johnson9, B.I. Kostetskii10 and H. Krause11.

The contemporary ideas on the wheel-rail adhesion are based on an integrated approach to the above-mentioned theories of friction.

2.4. Study of Adhesion Characteristics

For the mathematical modeling of the rail vehicle movement, it is important to use the reliable external characteristics in the contacts of wheels and rails. The adhesion characteristics are commonly determined as the dependence of the adhesion coefficient on the specific sliding speed in the main contact. The presence of the longitudinal sliding of wheels relative to the rails is a necessary condition for the appearance of the adhesive forces in the contacts, in particular, the traction force. A large number of studies has been devoted to the study of adhesion characteristics. The authors of [73] proposed to represent the adhesive characteristics in the following form:

\[ k_x(\varepsilon_x, \varepsilon_y), \quad k_y(\varepsilon_x, \varepsilon_y), \]  

(2.3)

where \( k_x, k_y \) – the relative coefficients of the adhesion in the contacts, in the longitudinal and the transverse directions, respectively.

\( \varepsilon_x, \varepsilon_y \) – the relative sliding in the contacts, in the longitudinal and the transverse directions respectively.

\[ k_x = \frac{\psi_x}{\psi_o}; \quad k_y = \frac{\psi_y}{\psi_o}; \quad \varepsilon_x = \frac{V_{sx}}{V_c}; \quad \varepsilon_y = \frac{V_{sy}}{V_c}, \]  

(2.4)

where \( \psi_x, \psi_y \) – the current values of the adhesion coefficients, in the longitudinal and the transverse directions, respectively;

\( \psi_o \) – the physical adhesion coefficient;

\( V_{sx}, V_{sy} \) – the velocity of sliding in the center of the contact patch, in the longitudinal and the transverse directions, respectively;

\( V_c \) – the comparison velocity.

---

The speed of the wheel (V), or the circumferential velocity of the wheel rim (ωR), can be taken as a comparison speed.

The presence of ascending and falling branches of the coupling characteristics is confirmed by the results of most studies [100-104]. Fig. 2.2 shows the typical characteristic of the grip of the wheel with a rail. Conditionally it can be divided into three areas: 1 - the region of proportionality, in which the dependence is linear; 2 - area of elastic slipping; 3 - full slip area. With regard to numerical values, then there is a large range of them.

Barwell [100], H. Mariama and O. T. Ohyama [101] showed a significant fall in the coefficient of adhesion at elevated air humidity and in the presence of water (Fig. 2.3, 2.4).

Chap (J. Chap) showed that, with variable load, the coefficient of adhesion is less than that of constant (Fig. 2.5) [102].
Figure 2.4 - Coupling characteristics when water is brought into contact
(according to Maruyama and Okhaama [101])

The disadvantage of almost all of the experimental research conducted in the
tests of real locomotives is the questionable purity of the experiment due to the
obvious impossibility of rejecting (or taking into account) the set of side factors,
mainly dynamic, associated with fluctuations and vibration in the wheel-rail system.
Especially this refers to the area of low and middle slip characteristics of the grip.

Figure 2.5 - Coupling characteristics with dynamic vertical load (according to Chap
[102])

On the other hand, experiments on simulating bench units can not really reflect
the physical processes in the contacts of the wheels with the rails due to the practical
absence of reliable criteria for similarity of contact interaction at high normal and
tangent stresses.

Particular interest is the stand installation for experimental studies of clutch
characteristics on the basis of full wheels and rails [103]. The advantage of such a
bench unit is to provide the conditions of contact of the closest to the real ones. The
installation takes into account the low level of resistance forces compared with the
boundary forces of the clutch. The installation allows to simulate the process of
wheel grip and obtain the characteristics of the grip of the wheel with the rail, as the
dependence of the longitudinal and transverse forces of grip on the longitudinal and
transverse relative slip in contact.

In fig. 2.6 shows the installation scheme described in [103]. The main parts of the stand are: frame, drive, braking system and control panel. Frame 1 is a rigid welded profile made of profiled metal. To increase the rigidity in the plane of rotation of the wheel, as well as in the transverse vertical plane there are slopes 2, on which the magnetic rail brake is suspended 3. The frame contains two pairs of jaws: lower and upper.
Figure 2.6 – The test plant for the experimental study of the coupling of characteristics [103]
In the lower jaw there are buckets 4 supporting the axis of the support roller, in the upper part - the buckets 5 supporting the axis of the wheel. The lower jaw is reinforced on the basis of frame 6. The upper jaws are welded to the racks of the frame 1. The vertical load is created by the hydropower 8, which transmits the force through the cross member 9 and through two sets of rubber elements 10 on the buckets 5 of the axis of the wheel 11.

To create oscillations of the vertical load, the pressure cavity of the hydro-stand is connected by a conduit with a plunger driven by a cam on the shaft of the electric motor. The change in engine rotation frequency sets the required frequency of oscillations of the vertical load.

The plunger movement, which depends on the configuration of the cam, is constant, and the amplitude of the oscillations of the vertical load is varied by the selection of the number of rubber elements 10. On the transverse 9, a meszdo is established, by which the vertical loading of the wheel on the rail is measured and recorded on the oscilloscope. An example manometer is connected to the hydrostatic cavity for messing up the mess.

The boom drive includes: accelerating electric motor 12, flywheel 13, hydrotransformer 14 with filling and emptying system, cardan gear 15 and traction reducer 16. As the accelerator engine 12, a DC motor with mixed excitation was used.

Engine management is carried out from the control panel.

The electric motor 12 is connected by an elastic lamellar clutch with a flywheel 13. As a traction gear, the gear unit of the diesel locomotive TGM-23 is used, which is convenient for the layout of the stand.

Since it has a conical degree, it became possible for the entire chain of drive units (electric motor - flywheel - hydrotransformer - traction reducer) to position along the base on guide rails.

Reducer, having two stages with a gear ratio of 1: 3.46 and 1: 6.93 and a reverse, mounted on two elastic supports 17.

Measurement of the coupling force was carried out using four tenzometric dynamometers installed in the gap between the jaws of the magnetic rail brake and the stanchions of the frame of the stand.

On the stand was obtained the characteristics of grip in the longitudinal and transverse directions (Fig. 2.7) [104].
As the regression equations and the following equations were adopted:

\[
k_x = \frac{\varepsilon_x}{a \cdot \varepsilon_x^2 + b \cdot \varepsilon_x + c} \cdot \frac{1}{d \cdot \varepsilon_y^2 + 1}; \quad k_y = \frac{\varepsilon_y}{f \cdot \varepsilon_y^2 + q \cdot h \cdot \varepsilon_x + 1}.
\]

(2.5)

Figure 2.7 - Experimental clutch characteristics

2.5. Direction of crews by track and traffic safety

2.5.1. The role of combing reactions in the process of steering wheel pairs and the coefficient of resistance against rails

Guiding (up to 25%) among the main causes of disasters, accidents and serious incidents in the railway transport is occupied by stairs wheel rails. The first in the history of the railway east of the rails occurred in the United States in 1833. This happened with the passenger train Camden & Amboy at a speed of 40 km / h. Then
killed about 100 passengers. Since then and to this day in the history of rail transport, there have been thousands of casualties of rolling stock from rails, which resulted in tens of thousands of casualties. Over the past decades, the number of stairs of rolling stock from the rails has decreased markedly, but, unfortunately, they occur with fairly high regularity [105-109].

The mechanism behind the railways' rails is a complex process that is the subject of intensive research around the world. The main criterion for the safety of the movement of a rail crew is the coefficient of stability versus the rails, in the form of the ratio of transverse force to the vertical load of the wheel on the rail [110]. The safety criterion against the rails in the Nadal theory is highly simplified and based on a large number of assumptions and does not take into account many features of the wheel-rail contact [111, 112]. In particular, the Nadal's Formula does not take into account the impact on the safety parameters of the wheel angle on the rail and on the mode of motion (traction, run-off, braking) [113, 114]. The safety factor from the approach of the rails of the railcars or the stability coefficient is determined by the formula [110]

\[ k_y = \left( \frac{Y}{P} \right) \div \frac{Y}{P} \geq [k_y] \]

de \( P \) – vertykal'ne navantazhennya kolesa na reyku;
\( Y \) – bichne zysylyla v Hreb'n'ova kontakti;
– minimal'no dopustymyy koefitsiyent bezpeky vid skhodu z reyok;
– vidnosshennya bichnoyi syly do vertykal'noyi navantazhenni v hrebenevomu kontakti, pry yakomu vidbuvayet'sya vidryv poverkhni kochnennya kolesa vid reyky.
Za Nadalyem [110],

\[
\frac{Y}{P} = \frac{\tan \beta - \mu}{1 + \mu \cdot \tan \beta},
\]

(2.7)

where \( \beta \) is the angle of the crest;
μ is the coefficient of friction in the "wheel-rail" contact.

The formula (2.7) is valid only for the case where the angle of rotation is zero (ψ = 0).

Numerous attempts have been made to improve Nadal's formula for improving the safety of rail transport at the stage of design development.

The clarification of the Nadal formula is the Wagner formula [115], which takes into account the deviation of the normal load of the wheel on the rail from the vertical direction

\[
\frac{Y}{P} = \frac{\tan \beta - \mu \cdot \cos \gamma}{(1 + \mu \cdot \tan \beta) \cdot \cos \gamma},
\]

(2.8)

where γ is the angle of deviation of the normal load of the wheel on the rail from the vertical.

Based on the Nadal's Formula formula, [116] analyzes the role of the spin in the crest contact. It is argued that the data obtained are more realistic than those obtained by Nadal's formula. A new security criterion is proposed, which leads to a less conservative value than the Nadal equation.

An attempt to clarify the formula of Nadal is also made in [117]. The refinement is based on taking into account the influence of the longitudinal contact force of the creep and the angle of the wheel run on the rail. The results obtained, however, differ little from Nadal's criterion, because they do not take into account slipping in the main contact and the mode of motion of the wheel pair.

The study [118] conducted an analysis of the main types of approach from the railways of railway vehicles. Three of them are related to the characteristics of the rail track, and two - with the shape of the profile wheels and rails. It is proved that the angle of inclination of the ridge of the wheel is the main geometric parameter that affects the probability of climbing from rails. In [119], statistics are given and the causes of large-scale accidents on the railways are analyzed. Emphasizes the importance of combing contact studies as the main safety factor. In [120] an equation for estimating the load on a wheel is taken into account, taking into account the centrifugal forces, the deviation of the geometry of the trajectory and deformation of the secondary suspension of railway vehicles. A method of calculating the critical safety factor was proposed, taking into account the influence of the wheel angle on
the rail and the equivalent coefficient of friction.

The study [121] analyzes the features of contact with a wheel with a rail:

- three-dimensional distribution of reactions in the contact of a wheel with a rail;
- peculiarities of kinematics of two-point contact;
- redistribution of force and kinematic parameters between contacts when moving the contact;
- Dependence of the direction of force in the crest of contact from the angle of inclination and the mode of movement of the wheel pairs of locomotives (traction, run or braking);
- the probability of the vertical component of the frictional force in the comb.

The results of research [122] are based on numerical modeling of the movement of a railway vehicle along the track with lateral and vertical inequalities. Inequalities were random. The degree of safety was assessed through the ratio of transverse and vertical force at the point of contact of the wheel with a rail. The level of comfort was assessed by the level of lateral and vertical acceleration.

There is a general perception that the dangerous angle of the ridge of a railway wheel is the main cause of the traumatic train. The authors of the publication [123] estimate the effect of the coefficient of friction in the contact crest with a rail on the probability of an accident. It is noted that the coefficient of friction varies greatly depending on various factors: the state of the surface of the wheel and rails, atmospheric conditions, speed of the crew. The risk of approaching the rails was estimated by the Nadal criterion for various coefficients of friction. As a result, it was found that measures to reduce the friction coefficient are effective even for cases where the ramp ratio exceeds the marginal criterion of Nadal.

The paper [124] proposes a method of protection against rails, based on the coefficient of wheel unloading. This method allows you to assess the safety of the rails from the rails, whether there is a load within the security zone. Dynamic tests were conducted using the indirect method of measuring the load of the wheel on the rail for the subway. It has been found that safety indicators in the transport of a vehicle exceed the permissible limit, while in stable conditions they are within the limits of safety.

In work [125], the dynamic performance of three different types of trolleys is compared. The main dynamic parameters of the car were studied: vertical and horizontal dynamics coefficients, as well as safety coefficient from the rails' east.
The study [126] considered the process of approaching railway vehicles from rails at low speeds in curved sections of the track. The mechanism of lifting the comb for the quantification of the factors causing the east of the railway vehicles from the rails is studied. The results of tests on passenger vehicles with a real ascent from rails were analyzed. Also tested on a roller stand. The method of estimating the level of safety from the approach of the rails in the curves of the tracks is proposed.

The study [127] describes the mechanism of approaching the rails and the method of continuous control of the force between the wheel and the rail. New criteria for assessing the danger of an accident are proposed.

The study [128] developed a three-dimensional nonlinear dynamic model of the system of wheel pairs and suspension. The influence of the coefficient of friction and speed of movement east from rails was investigated. In addition, various methods of lubricating rails and their impact on the risk of climbing rails were investigated. Features of the two-point wheel-rail contact are also considered, and recommendations for improving the safety of the rails are given.

The paper [129] presents a brief overview of the individual problems of the dynamics of railway vehicles, including the analysis of the mechanism of the approach from the rails.

2.5.2. Kinematics of contact of a wheel coupling with rails

The motion of a wheeled pair is modeled in the position of the bias in the rail track with an angle of $\psi$. In fig. 2.8 shows the speed plans (blue vectors) and forces (red vectors) in the contacts of the wheelset with rails. The oncoming wheel (2) has a two-point contact with a rail at K21 (main contact) and K22 (comb attachment). The colliding wheel (1) has one-point contact at point K11. The case is considered when the incident wheel is in the state of the east of the rails. As a condition of the approach, the zero load is taken in the main contact of the incident wheel.
Figure 2.8 - Scheme of velocity vectors (blue color) and forces (red color) in the contacts of the wheelset with rails:

a - projections of velocities and forces in the contacts of the wheel pair with rails on the horizontal plane of Oxy; B is a spatial pattern of velocities and forces in the crest of contact of the incident wheel

In the simulation of the kinematic characteristics of the contact of the wheel coupling with the rails, the following parameters and their designations are considered:

V - velocity of the center of the wheel pair along the axis of the track;

Vφ11 - circumferential wheel speed at the point of contact K11, associated with the rotation of the wheel vs. its own axis;

V11 - the speed of rolling the wheel on the rail at the point of contact K11;

V11x, V11y - projections of velocity V11, respectively, on the axis Ox and Oy;

Vφ22xy is the projection of the circumferential velocity of the wheel at the point of contact K22, on the horizontal plane of the Oxy;

V22xy - projection of the speed of rolling of the wheel along the rail at the point of contact K22, on the horizontal plane of the Oxy;

V22x, V22y - projections of velocity V22, respectively, on the axis Ox and Oy;

V22xz - projection of velocity V22 on the longitudinal vertical plane Oxz;

Yf - axial reaction in a boot knot, acting on a wheel pair - a frame force;
S11y, S22y - projections of friction forces S22 in contacts K11, K22 on the axis Oy;
S22xz, S22x, S22z - projections of the friction force S22 in contact K22, respectively, on the longitudinal vertical plane Oxz, Ax axis and Oz axis;
β is the angle of the crest.
Vector equations of slip speeds in contacts K11, K22 are obtained on the basis of the principle of superposition as components of relative displacement of rolling surfaces of wheels and rails:

\[ \nabla_{11} = \nabla + \nabla_{q11}, \quad \nabla_{22xy} = \nabla + \nabla_{q22xy}. \]

(2.9)

\[ \nabla_{22} = \nabla_{22xy} + \nabla_{22xz} + \nabla_{22yz}. \]

(2.10)

In fig. 2.9 shows the scheme of contact forces in the crest contact in projections on the longitudinal vertical plane Oxz and the transverse vertical plane Oyz.
In fig. 2.9 the following designations have been adopted:
ζ is the angle that defines the position of the vector S22xz relative to the vertical axis;
ω - angular velocity of rotation of the wheel pair around its own axis;
N22 - normal load in the comb attachment K22;

![Figure 2.9 - Scheme of projections of forces in the crest contact: a - projections on the longitudinal vertical plane Oxz; b - projections on the transverse vertical plane](image-url)

2.5.3. Power interaction in the crest of the wheel contact with the rail

The total horizontal lateral load in the comb attachment $Y$ is the sum of the frame strength $Y_f$ and the discharge forces $S_{11y}, S_{22y}$ in the contacts $K_{11}, K_{22}$:

$$Y = Y_f + S_{11y} + S_{22y}.$$  \hspace{1cm} (2.11)

The frictional contact forces $S_{11y}, S_{22y}$ in the contacts $K_{11}, K_{22}$ in the theory of control of wheeled cars are called withdrawal forces. The drive forces $S_{11y}, S_{22y}$ are friction forces and are directed oppositely to the corresponding velocity vectors $V_{11y}, V_{22y}$. The forces $S_{11y}, S_{22y}$ can be approximated by the Coulomb law formulas. In this case, the most dangerous approach to the rails is considered when the angle $\zeta = 0$.

$$S_{11y} = P_1 \cdot \mu; \hspace{1cm} S_{22y} = N_{22} \cdot \mu \cdot \cos\beta,$$  \hspace{1cm} (2.12)

where - the vertical load in the contact;
- coefficient of friction of slip in the contacts of the wheels with rails.

Then

$$Y = \mu(P_1 + N_{22} \cdot \cos\beta) + Y_f.$$  \hspace{1cm} (2.13)

The equilibrium of the contact forces is represented as a sum of projections on the axis $Oy$ and $Oz$ (Fig. 2.9 b):
\[
\begin{align*}
\sum F_y = 0: & \quad Y - N_{22} \cdot \sin \beta = 0; \\
\sum F_z = 0: & \quad N_{22} \cdot \mu \cdot \sin \beta + N_{22} \cdot \cos \beta - P_2 = 0.
\end{align*}
\] (2.14)

From the second equation (2.14)

\[
N_{22} = \frac{P_2}{\mu \cdot \sin \beta + \cos \beta}.
\] (2.15)

Given that the first wheel has no comb contact, and the main contact of the second wheel is completely unloaded, we can assume that \(P_1 = P_2 = P\). Then from the first equation (2.14) we obtain:

\[
\mu \cdot P \cdot \left(1 + \frac{\cos \beta}{\mu \cdot \sin \beta + \cos \beta}\right) + Y_f - P \frac{\sin \beta}{\mu \cdot \sin \beta + \cos \beta} = 0.
\] (2.16)

2.5.4. Criterion of safety from the rails

The critical correlation of the frame strength to the vertical load, at which the complete unloading of the main contact occurs, can be found by the formula:

\[
\left[\frac{Y_f}{P}\right] = \frac{\tan \beta \left(1 - \mu^2\right) - 2\mu}{\mu \cdot \tan \beta + 1}.
\] (2.17)

In fig. 2.10 shows the relative dependences of the critical ratio \([Y_f / P] \) on the crest angle \(\beta\) and the friction coefficient \(\mu\) calculated from the classical Nadal (2.7) formula and the refined formula (2.17).
The Nadal formula for many decades remains the main criterion for assessing the stability of railcars from the rails east \([110]\). Many researchers \([111-114]\) note that it is primitive and does not take into account many factors that affect the safety of the movement of rail vehicles. Numerous attempts to correct the Nadal formula have been known to clarify it \([115-128]\). However, stairs from the rails of locomotives and cars continue to occur with sufficient regularity \([129]\).

The results presented in this section are another attempt to clarify the safety criterion from the ramp, based on the theory of Nadal.

The idea of the study is based on the hypothesis, which includes the following starting positions of contacting the wheelset with rails:

- Slipping in the rudder contact of the tangential wheel can create frictional force, the vertical component of which reduces the vertical load on the rail at the main contact. In this case, the horizontal component - increases lateral loading in the comb contact;

- Folding in the main contact of the collision wheel creates friction forces that increase the total horizontal transverse load on the crest of the incident wheel.

The combination of these factors and their impact on the safety criterion from
the rails was not considered in known studies. The refined formula for the safety coefficient for the east of the wheels of the rails has the form:

\[
k_y = \frac{P}{Y_f} \div \frac{\mu \cdot \tan \beta + 1}{\tan \beta (1 - \mu^2) - 2\mu} \geq [k_y], \quad (2.18)
\]

Ford [ky] is the minimum allowable normative coefficient of stability reserve against the rails of the rails.

As can be seen from Fig. 2.17, the value of the ratio \([Y_f / P]\) obtained according to the refined formula (2.17), depending on the values of the angle of inclination of the comb \(\beta\) and the coefficient of friction \(\mu\), is 10-50% lower than the values calculated according to the classical Nadal formula (2.7). The refinement of the safety criterion Nadal is proposed on the basis of taking into account the peculiarities of the friction interaction of the wheel coupling with rails, which had not previously been taken into account in such studies, namely:

the dependence of the vertical component of the friction force in the combing contact of the incident wheel on the incident angle is taken into account;

the influence of the friction force in the combing contact of the colliding wheelset on the increase of the lateral load on the comb is taken into account;

the influence of the frictional force in the contact of the collision wheel with the rail is taken into account - on increasing the lateral load on the crest of the incident wheel.

In addition, unlike the traditional approach, when assessing the coefficient of stability of the wheel against the rising of the rails, as the main safety factor, instead of the side load on the crest, the frame strength is used. The combined effect of these factors on the conditions of the ascent from the rails was not considered before. Dependence (2.18) raises the accuracy of determining the safety factor for approaching rails, which allows to increase the safety of vehicles at the stage of their design and operation.
3. THEORETICAL STUDY ON THE RESISTANCE TO THE MOVEMENT ASSOCIATED WITH THE CONTROL OF THE CARRIAGE BY THE RAILWAY TRACK

3.1. The Choice on the Calculation Schemes for Simulating the Carriages Control by the Railway Track

3.1.1. The System of Coordinates and the Simulation Features for the Rail Carriages

Interaction of a rolling stock and track is associated with three functions of wheel pairs: i.e. resistance, guidance and a mover. Through using coordinate system XYZΘΦΨ (fig.3.1) for each function one can distinguish kinematic links that each function applies to relative movement of a rolling stock and track on dedicated coordinates.

The resistance function is associated with the necessity to compensate gravitational forces, and it imposes some constraints on movement on the following coordinates: Z (bouncing vertical linear movement – z), Φ (pitching angular movement – Φ) i Θ (rolling angular movement – Θ).

Guidance function – on coordinates Y (lateral linear movement – y) i Ψ (angular rotating movement in a horizontal plane – Ψ).

Rolling stock movement along the track axis is done with wheel pairs as a mover, which controls jerking movements - x).

The mover and guidance functions are associated with two problematic directions of the rolling stock and track frictional interaction, i.e. the study of horizontal dynamics of rolling stock and locomotive traction features. Despite the relative inde-
pendence of the directions, they are not being studied without considering the interaction, since are united by the common problem of wheel-track adhesion.

Modelling of the "rolling stock-track" mechanical system has own peculiarities compared to the other systems.

At first, it is the availability of controlled track with own features and limits. Modelling of a railway base in some theoretical studies and in a definite movement range is the main problem to define truthfulness of the entire "rolling stock - track" system model. For many dynamics problems a rolling stock may be considered as a system of absolutely solid bodies connected with spring-dissipative links. When finding element own flexibility, coupled subsystems shall be considered as an exclusion. As an example of such subsystem is the system of torque transmission from an engine to wheels, i.e. a drive system. In many studies, in this connection, torque stiffness of wheel pair axes is considered.

Availability of system static balance several positions allowed due to non-positional element joints, for instance, frictional element joints, shall be related to "rolling stock- track" features:

- wheels and rails;
- wheel pair axes and axle boxes, if there is clearance in axial thrusts;
- body and trucks that are connected in plain view with guide blocks.

Therefore, very often, derivatives of gravitation force potential energy on corresponding general coordinates are included into a movement equation.

When studying motion with constant speed V, which happens most often, it is possible to exclude longitudinal coordinate X as cyclical one form consideration in order to see the motion in a five-dimensional coordinate system. In this case, we admit that the rolling stock mass center moves at speed V.

In theoretical studies of rolling stock controllability, two main methods of mathematical modelling were used:

- a quasi-static method, which uses controllability indexes in a fixed model constant mode;
- a time area study method or determinated method, which uses nonlinear systems of differential equations.

Results obtained with fixed models are oriented toward a special case of stabilized motion without consideration of dynamic processes.

Solutions obtained with determinated models are related to speed-movement de-
dependencies in time on corresponding general coordinates. Besides, according to calculation results, dependencies of controllability/ movement parameters on geometric/ speed features of a rolling stock were built.

Problems of the temporary area are the most painstaking from the standpoint of integration, but they allow more detailed understanding of real processes, because solutions are obtained in a customary form of dependencies in time, and the very modelling is a "numerical experiment'.

When comparing the theoretical and experimental data, and selecting mathematical model correctness, a model function was considered. In this case, the most significant variables that reflect the essence of the studied process, i.e. so called outlet data, were selected.

Intermediate values that can be easily checked, while adjusting models and computational programs, were included into selected variables as additional outlet data. Such criteria are movement trajectory parameters associated with track limits: motion, speed, acceleration and hunting frequency.

Correctness of modelling was significantly increased by introducing true external functions into mathematical models: adherence experimental features [104], contacting parameters with precise description of rail/ wheel working surface geometry, contact point coordinates and effective conicities.

Selection of criteria for rolling stock track guidance modelling is based on the suggestion that resistance increases at curved sections compared to straight ones, and under other equal conditions it is mediated with kinematic motion resistance. Kinematic motion resistance, which may be assessed by experimental features as a difference between motion resistance on a curve and motion resistance on a straight, or as additional motion resistance on a curve, is adopted as a main criterion of truthfulness. If calculation results of kinematic resistance on curves sufficiently converge with experimental data, a mathematical model may be considered as correct, and it may be used to obtain numerical dependencies on straight track sections.

Wheel pair rolling resistance is adopted as the second truthfulness criterion. In this case, theoretical results were compared with experimental results of known studies.

3.1.2. The Calculation Models of the Carriages under the Study
Selection of a computational pattern for mathematical modelling is associated with choosing of model truthfulness criteria and detalization rational degree. In the same manner as in majority of rolling stock horizontal dynamics studies, plane coordinate system $XY\Psi$ with generalized coordinates was used for mathematical models:

- $x$, $y$ – longitudinal and lateral movements of mass centers of bodies included in the system;
- $\psi$ – body angular movements in plain view;
- $\varphi$ – an additional coordinate – turning angles of wheel pairs against the symmetry lateral axis.

Coordinate $\varphi$ is added to the generalized coordinates, because it may not be considered as cyclic one even at constant movement speed.

When composing differential equations of rolling stock movement, the following general indexes of value indication were used:

- $i$ – wheel numbers of a wheel pair ($i=1, 2$);
- $j$ – wheel pair numbers (for two axle trucks $j=1, 2$; for three axle trucks $j=1, 2, 3$);
- $k$ – truck numbers ($k=1, 2$).

In Table 3.1–3.3 one can see systems of generalized coordinates for each computation pattern considered.

Moreover, the following designations of geometric parameters are adopted:
- $2B$ – the distance between thread circles of a wheel pair;
- $2A$ – the distance between longitudinal force transmission points from the wheel pair axis to the box and further from the box to the truck frame;
- $C_{jk}$ – the distance from $k$-th truck pivot to $j$-th wheel pair axis;
- $D_{k}$ – the distance from the body lateral axis to $k$-th truck pivot;
- $E$ – the distance between intermediate beam pivots.

Values $C_{jk}$, $D_{k}$, $E$ are positive, if positive force torque, whose arm they compose, is also positive.

In Fig. 3.2 one can see a general pattern of adopted designations of geometrical parameters.
**Fig. 3.2** – A diagram of rolling stock geometric parameter designations

**Table 3.1** – Generalized coordinates and quantity of motion differential equations for computational patterns of rolling stocks without an intermediate beam in trucks

<table>
<thead>
<tr>
<th># of comp./pattern</th>
<th>Type of a rolling stock</th>
<th>Body</th>
<th>Truck frame</th>
<th>Wheel pairs</th>
<th>Quan-ty of equat.</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>$y_c$</td>
<td>$\psi_c$</td>
<td>$y_1$ $\psi_1$</td>
<td></td>
</tr>
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<td>1</td>
<td>Steam loco FD 1-5O-1</td>
<td></td>
<td></td>
<td>1 $y_1$ $\psi_1$ $\phi_1$</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 $y_{r1}$ $\psi_{r1}$</td>
<td>2 $y_2$ $\psi_2$ $\phi_2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3 $y_3$ $\psi_3$ $\phi_3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 $y_4$ $\psi_4$ $\phi_4$</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td></td>
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| # of comp./pa
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**Table 3.2** – Generalized coordinates and quantity of motion differential equations for computational patterns of rolling stocks with an intermediate beam in trucks

| # comp/PA
ttern | Type of a rolling stock | Body | Interm. beam | Frame of a truck | Wheel pairs | Quan-ty of equat. |
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Table 3.3 – Generalized coordinates and quantity of motion differential equations for computational patterns of rolling stocks with longitudinal and lateral beams in trucks

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3.2. Differential Equations of the Carriage Movement

Equation systems for each rolling stock will contain several subsystems of the same type

- a wheel pair (with box bodies);
- a truck frame (with brake equipment, box and body shock absorber suspension);
- body.

3.2.1. Equations of the Wheel Pairs Movement

Generalized coordinates: $y_{jk}$, $\psi_{jk}$, $\varphi_{jk}$.

Generalized forces (Fig. 3.1):

$S_{x \ ijk}$, $S_{y \ ijk}$ – longitudinal and lateral reactions at contacts of wheels and rails;

$F_{bx \ ijk}$, $F_{by \ ijk}$ – longitudinal and lateral reactions in box units.

A movement equation on generalized coordinates $y_{jk}$, $\psi_{jk}$, $\varphi_{jk}$:

$$\ddot{y}_{jk} = \frac{1}{m} \left( \sum_{i=1}^{2} S_{y \ ijk} - F_{by \ ijk} \right);$$

$$\ddot{\psi}_{jk} = \frac{1}{I_z} \left[ (S_{x \ 1jk} - S_{x \ 2jk})A - (F_{bx \ 1jk} - F_{bx \ 2jk})B \right];$$

$$\ddot{\varphi}_{jk} = -\frac{1}{I_y} \sum_{i=1}^{2} (S_{x \ ijk} R_{ijk});$$
\[ F_{bx \ ij k} = F_{bx}(\Delta_{bx \ ij k}); \quad F_{by \ ij k} = F_{by}(\Delta_{by \ ij k}), \]

where \( S_{x \ ij k}, S_{y \ ij k} \) – longitudinal and lateral components of adhesion forces at contacts of wheel and rails of the track moving coordinate system;

\( S_{x^* \ ij k}, S_{y^* \ ij k} \) – longitudinal and lateral components of adhesion forces at contacts of wheel and rails of the wheel pair moving coordinate system;

\( F_{bx}(\Delta_{bx \ ij k}), F_{by}(\Delta_{by \ ij k}) \) – features of box connection in longitudinal and lateral directions;

\( \Delta_{bx \ ij k}, \Delta_{by \ ij k} \) – longitudinal and lateral movements in box units.

\[ \Delta_{bx \ ij k} = B(\psi_{mk} - \psi_c); \]

\[ \Delta_{by \ ij k} = y_{mk} - y_{jk}. \quad (3.2) \]

When modelling the rolling stocks with wheel pairs that rotate independently,
coordinate \( \varphi_{jk} \) shall be substituted with coordinates \( \varphi_{ijk} \) (i=1,2).

### 3.2.2. Equations of the Truck Frame Movement

Generalized coordinates: \( y_{mk}, \psi_{mk} \).

Generalized forces (Fig. 3.2):
- \( F_{bxijk}, F_{byjk} \) – box reactions;
- \( F_{mk}, M_{mk} \) – reaction force and torque at connections of the trucks and body (a pivotal assembly and supporting-and-counter device).

![Fig. 3.2 – A diagram of forces and torques that influence on two-axle truck frame on generalized coordinates \( y_{mk}, \psi_{mk} \)](image)

A movement equation for \( N \)-axle trucks

\[
\ddot{y}_k = \frac{1}{m_{mk}} \left( \sum_{j=1}^{N} F_{byjk} - F_{mk} \right)
\]

\[
\ddot{\psi}_k = \frac{1}{I_{mz}} \left[ B \sum_{j=1}^{N} (F_{bx1jk} - F_{bx2jk}) + \sum_{j=1}^{N} (F_{byjk} - M_{mk}) \right]
\]

\[
F_{mk} = F_{m}(\Delta_{my_k}); \quad M_{mk} = M_{m}(\Delta_{m\psi_k}),
\]
where \( F_m(\Delta_{my_k}) \), \( M_m(\Delta_{m\psi_k}) \) – features of the turning device at the contact of the body and truck;

\( \Delta_{my_k}, \Delta_{m\psi_k} \) – body/ truck relative longitudinal and lateral movements that depend on generalized coordinates \( y_{mk}, \psi_{mk}, y_c, \psi_c \):

\[
\Delta_{my_k} = y_{mk} - y_c;
\]

\[
\Delta_{m\psi_k} = \psi_{mk} - \psi_c.
\]

These dependencies as equations complete the differential equation system.

### 3.2.3. Equations of the Body Movement

Generalized coordinates: \( y_c, \psi_c \).

Generalized forces: \( F_{mk}, M_{mk} \).

A movement equation for a L-truck rolling stock body

\[
\ddot{y}_c = \frac{1}{m_c} \sum_{k=1}^{L} F_{mk}; \quad \ddot{\psi}_c = \frac{1}{J_c} \sum_{k=1}^{L} (F_{mk} D_k + M_{mk}).
\]

\[ (3.5) \]

*Fig. 3.3 – A diagram of forces and torques that influence on two-axle bogie frame on generalized coordinates \( y_c, \psi_c \)*
3.3. Mathematical Models of Locomotives

When selecting rolling stocks as theoretical study objects, authors pursued the following goals.

Firstly, they needed to find features of kinematic movement resistance for the most rolling stock types in the same way as it was done for the main movement resistance, for instance, in "Traction calculation rules" [29].

Secondly, based on obtained features it was necessary to make a conclusion on advantages of some truck arrangement diagrams as for the kinematic movement resistance.

Thirdly, the most rational diagrams of rolling stock controlled guidance on track should be identified.

Due to this, the attempt to cover the most of rolling stock types, including modern and old structural diagrams that were not studied from the perspective of motion resistance, was made.

Since the time of railway transport origin the rolling stock mechanical part, being unchanged in its basis, evolved a lot. Truck arrangements of hauling units are especially different. Locomotive truck arrangement enhancement is associated with combining contradictory functions of wheel pairs: support, guidance and a mover.

Problems that should be solved increased in number after transition from a truck arrangement frame diagram to a truck one. The most significant and already solved problem, i.e. enhancing vehicle guiding in curves, was substituted with some smaller ones.

Among multitudinous locomotive rolling stock dynamics studies done all over the world the most important are those that aim at decreasing the movement resistance and wheel/ rail wear, especially at curved track sections.

There are some directions to solve the problems:

- optimization of wheel rolling surface profiles;
- optimization of wheel/ rail material mechanical features;
- oiling the contact of the wheel flange and rail;
- radial positioning of wheel pairs in curves;
- using the wheel pairs with wheels that roll independently;
- control of body rocking in curves.
### 3.3.1. Mathematical Models of Frame Locomotives

Frame structured rolling stock is installed on one rigid body frame. However, very often such rolling stocks contain steering wheel pairs or trucks that turn, i.e. one/two axle wheels that either guide or support. Such structure is specific to steam locomotives, first electric and diesel locomotives. There is a number of axle configuration variants of locomotives, including ones with steering axles.

Modelling of frame structured locomotive movement was done based on the example of FD steam locomotives with 1-50-1 axle configuration and EM five axle steam locomotive.

**A FD steam locomotive movement equation**

The FD steam loco movement equation system is composed of interconnected body subsystems, guiding trucks, main and steering wheel pairs.

A body movement equation

\[
\ddot{y}_c = \frac{1}{m_c} \left( \sum_{j=1}^{5} F_{byj} + \sum_{k=1}^{2} F_{nk} \right),
\]

(3.5)

\[
\ddot{\psi}_c = \frac{1}{J_c} \left[ \sum_{k=1}^{2} \left( F_{nk} D_k + M_{nk} \right) + \sum_{j=1}^{5} F_{byj} C_j + A \sum_{j=1}^{5} \left( F_{bx1j} - F_{bx2j} \right) \right].
\]

An equation of hunting and lateral deviation of guiding trucks (k=1, 2)

\[
\ddot{y}_{nk} = \frac{1}{m_{\Box}} \left( F_{byk} - F_{nk} \right),
\]

(3.6)

\[
\ddot{\psi}_{nk} = \frac{1}{J_{\Box}} \left[ F_{byk} C_k + (F_{bx1k} - F_{bx2k}) B^* - M_{nk} \right].
\]

A main wheel pair movement equation (j=1,...,5)
\[
\ddot{y}_j = \frac{1}{m_j} \left[ \sum_{i=1}^{2} S_{yij} - F_{byj} \right],
\]

\[
\ddot{\psi}_j = \frac{1}{J_{zj}} \left[ \left( S_{x1j} - S_{x2j}^* \right) A - \left( F_{x1j} - F_{x2j} \right) B \right], \tag{3.7}
\]

\[
\ddot{\phi}_j = \frac{1}{J_{yj}} \sum_{i=1}^{2} (S_{xij}R_{ij}).
\]

A guiding wheel pair equation (k=1, 2)

\[
\ddot{y}_k = \frac{1}{m} \left( \sum_{i=1}^{2} F_{yik} - F_{ynk} \right),
\]

\[
\ddot{\psi}_k = \frac{1}{J_z} \left[ \left( S_{x1k}^* - S_{x2k}^* \right) A - \left( F_{bx1k} - F_{bx2k} \right) B \right], \tag{3.8}
\]

\[
\ddot{\phi}_k = -\frac{1}{J_y} \sum_{i=1}^{2} (S_{xik}R_{ik}).
\]

Structural features of a FD steam loco define relative movement dependencies in assemblies, whose external features are described by the following constraint equations.

Deformation in box connections of the main wheel pairs and body

\[
\Delta_{bx1j} = B(\psi_c - \psi_{1j});
\]

\[
\Delta_{bx2j} = B(\psi_{2j} - \psi_c); \tag{3.9}
\]

\[
\Delta_{by} = y_j - y_c - C_j \psi_c;
\]

Movement in box connections of guiding wheel pairs and truck-rods is absent. Relative movement in counter devices of guiding trucks
\[ \Delta_{my\ k} = y_k - C_k \psi_k - y_c - D_k \psi_c; \quad \Delta_{m\psi\ k} = \psi_k - \psi_c. \] (3.10)

**An EM steam locomotive movement equation**

The EM steam loco movement system is simpler due to absence of guiding trucks.

A body movement equation

\[ \ddot{y}_c = \frac{1}{m_c} \sum_{j=1}^{5} F_{by\ j}, \] (3.11)

\[ \ddot{\psi}_c = \frac{1}{f_c} \sum_{j=1}^{5} \left[ F_{by\ j} C_j + A (F_{bx\ 1j} - F_{bx\ 2j}) \right]. \]

A wheel pair movement equation \((j = 1, \ldots, 5)\)

\[ \ddot{x}_j = \frac{1}{m_j} \sum_{i=1}^{2} (S_{xij} - F_{bx\ j}), \]

\[ \ddot{y}_j = \frac{1}{m_j} \left[ \sum_{i=1}^{2} (S_{yij} + H_{ij}) - F_{by\ j} \right], \] (3.12)

\[ \ddot{\psi}_j = \frac{1}{f_{zj}} \left[ (S_{x1j}^* - S_{x2j}^*) A - (F_{bx\ 1j} - F_{bx\ 2j}) \right], \]

\[ \ddot{\phi}_j = -\frac{1}{f_{yj}} \sum_{i=1}^{2} (S_{xij} R_{ij}). \]

Constraint equations that describe dependencies of relative movements in box connections of pairs and body of an EM steam loco on independent variables that look like

\[ \Delta_{bx\ 1j} = B (\psi_j - \psi_c); \]
\[ \Delta_{bx\, 2j} = B(\psi_c - \psi_j); \quad (3.13) \]

\[ \Delta_{by\, j} = y_j - y_c - C_j\psi_c. \]

3.3.2. Locomotives with Linked Carriages

The first truck structured electric locos were produced with joined trucks. It was necessary to transmit traction force through them, since catching devices were installed on trucks at that time. For the first time this principle was used in electric locos General Elektric.

They were characterized by two joined 3-truck rolling stocks. The design was used further in electric locos VL19, VL22m, VL8. There is a number of rolling stock structures based on truck joining, but with force transmission through the body and partially joined spring elements, for instance, in electric locos ChS-2, Re 2/2 or Hitachi.

Motion equations for electric locos with joined trucks of VL19, VL22m, VL23-types

If there is no clearance in truck connection ball joints, structural configuration of this type rolling stocks may be described by the following relations

\[ y_{m1} = y - D\psi_{m1}; \quad y_{m2} = y + D\psi_{m2}; \]

\[ y_c = y - \frac{D}{2}(\psi_{m1} - \psi_{m2}); \quad (3.14) \]

\[ \psi_c = \frac{\psi_{m1} - \psi_{m2}}{2} \]

where \( y_{r1} \), \( y_{r2} \), \( \psi_{r1} \), \( \psi_{r2} \) – longitudinal movements and rotation angles of trucks in plain view;

\( y \) – lateral movement of the truck connection ball joint center;

\( y_c, \psi_c \) – lateral movement and rotation angle of the body in plain view.

Variation of rolling stock lateral deflection and hunting with respect to equations (3.14) may be described with three equalities in relation to variables \( y, \psi_{r1}, \psi_{r2} \) as
\[ A \times \ddot{y} = \ddot{F}; \]  
(3.15)

\[ A = \begin{bmatrix} 2a & -a & a \\ -a & b & 0 \\ a & 0 & b \end{bmatrix}; \quad \ddot{y} = \begin{bmatrix} \ddot{y} \\ \ddot{\psi}_{ml} \\ \ddot{\psi}_2 \end{bmatrix}; \quad \ddot{F} = \begin{bmatrix} F_y \\ F_1 \\ F_2 \end{bmatrix}; \]  
(3.16)

\[ a = \left(\frac{m_c + m_m}{2}\right)D; \quad b = m_c \frac{D^2}{4} + m_m D^2 + \frac{J_c}{2} + J_m; \]  
(3.17)

\[ F_y = \sum_{i=1}^{2} \sum_{j=1,3} \sum_{k=1}^{2} \left( S_{yijk} + H_{ijk} \right) + \sum_{k=1}^{2} F_{by2k}; \]  
(3.18)

\[ F_{m(k=1,2)} = \sum_{j=1,3} \left( S_{ijk} + H_{ijk} \right) C_{jk} + F_{by2k} C_{2k} + \sum_{i=1}^{2} \left( S_{x1jk} - S_{x2jk} \right) A - M_{ck}, \]

where \( M_{ck} \) – rolling torques in supporting-and-counter devices that depend on angles of relative turning of the body and truck \( \Delta \psi_k \).

\[ \Delta \psi_1 = \frac{\psi_{ml} - \psi_{m2}}{2}; \quad \Delta \psi_2 = \frac{\psi_{m2} - \psi_{ml}}{2}. \]  
(3.19)

Equations of lateral deflection variation of truck middle wheel pairs

\[ \dot{y}_{2k} = \frac{1}{m} \left( S_{y2k} + H_{2k} \right). \]  
(3.20)

Equations of wheel pair angular velocity variation

\[ \dot{\phi}_{jk} = -\frac{1}{J_y} \sum_{i=1}^{2} \left( S_{xijk} R_{ijk} \right). \]  
(3.21)
**A movement equation for eight-axle locomotive of VL8 type**

VL8 electric locos are already equipped with four 2-axle trucks. Truck joining is done through four ball joints.

After studying structural features of VL8 series electric locos, it was admitted that there is no lateral clearance in box assemblies, whereas the box longitudinal position in relation to truck frame depended on directions of box reactions \( F_{bx\ ijk} \) and clearance in truck frame box horn plates \( \Delta_{bx\ ijk} \).

The clearance-free pivotal joining of trucks is described by the following constraint equations

\[
y_{m1} = y - D(2\psi_{m2} - \psi_{m1}); \quad y_{m2} = y - D\psi_{m2};
\]

\[
y_{m3} = y + D\psi_{m3}; \quad y_{m4} = y + D(2\psi_{m3} + \psi_{m4});
\]

\[
y_{c1} = y - \frac{D}{2}(\psi_{m1} + 3\psi_{m2})
\]

\[
y_{c2} = y + \frac{D}{2}(3\psi_{m3} + \psi_{m4})
\]

\[
\psi_{c1} = \frac{\psi_{m1} - \psi_{m2}}{2};
\]

In (3.22):

\( y_{c1}, y_{c2}, \psi_{c1}, \psi_{c2} \) – movements, respectively, of first and second section bodies;

\( y \) – lateral movement of the second (middle) inter-truck ball joint;

\( y_{t1}, y_{t2}, y_{t3}, y_{t4}, \psi_{t1}, \psi_{t2}, \psi_{t3}, \psi_{t4} \) – lateral and angular movements of trucks.

Equations of loco hunting and lateral deflection variation

\[
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
    a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\
    a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\
    a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\
    a_{51} & a_{52} & a_{53} & a_{54} & a_{55}
\end{bmatrix}
\begin{bmatrix}
    y \\
    \psi_{m1} \\
    \psi_{m2} \\
    \psi_{m3} \\
    \psi_{m4}
\end{bmatrix}
= \begin{bmatrix}
    F_y \\
    M_{m1} \\
    M_{m2} \\
    M_{m3} \\
    M_{m4}
\end{bmatrix},
\]  

(3.12)
where
\[a_{11} = 4\tau_1 \quad a_{12} = -D\tau_1 \quad a_{13} = -3D\tau_1 \quad a_{14} = 3D\tau_1 \quad a_{15} = D\tau_1\]
\[a_{21} = -D\tau_1 \quad a_{22} = D^2\tau_2 + J_r \quad a_{23} = D^2\tau_3 \quad a_{24} = 0 \quad a_{25} = 0\]
\[a_{31} = -3D\tau_1 \quad a_{32} = D^2\tau_3 \quad a_{33} = D^2\tau_4 + J_r \quad a_{34} = 0 \quad a_{35} = 0\]
\[a_{41} = 3D\tau_1 \quad a_{42} = 0 \quad a_{43} = 0 \quad a_{44} = D^2\tau_4 + J_r \quad a_{45} = D^2\tau_3\]
\[a_{51} = D\tau_1 \quad a_{52} = 0 \quad a_{53} = 0 \quad a_{54} = D^2\tau_3 \quad a_{55} = D^2\tau_2 + J_r\]

\[m_1 = m_m + \frac{m_c}{2}; \quad m_2 = m_m + \frac{m_c}{4}; \quad m_3 = 2m_m + \frac{3}{4}m_c; \quad (3.24)\]
\[m_4 = 5m_m + \frac{9}{4}m_c; \quad F_y = \sum_{j=1}^{2} \sum_{k=1}^{4} F_{byjk},\]
\[M_{mk} = \sum_{i=1}^{2} (S_{yi2k} - S_{yi1k} - H_{ilik} + H_{i2k})C_{jk} + \sum_{j=1}^{2} (S_{x1jk} - S_{x2jk})A.\]

An equation of wheel pair rotation and hunting
\[\dot{\phi}_{jk} = -\frac{1}{J_y} \sum_{i=1}^{2} (S_{xijk} R_{ijk}), \quad (3.25)\]
\[\ddot{\psi}_{jk(j=1...2;k=1...4)} = \frac{1}{J_z} \left[ (S_{x1jk} - S_{x2jk})A - (F_{bx1jk} - F_{bx2jk})B \right].\]

**A motion equation for a six-axle two-truck loco with spring inter-truck connection of ChS2 type**

Schematically, the rolling stock part of passenger electric locos Skoda ChS2, ChS2T is different from other locos by its spring lateral inter-truck connection and spring connection of the body and trucks in lateral direction. The spring mass separation in lateral and angular direction led to a simple system of rolling stock motion differential equations.

A body horizontal oscillation equation
\[ \ddot{y}_c = \frac{1}{m_c} \sum_{k=1}^{2} F_{ck}; \quad \ddot{\psi}_c = \frac{1}{J_k} \sum_{k=1}^{2} M_{ck}. \] (3.26)

A truck horizontal oscillation equation

\[ \ddot{y}_{mk} = \frac{1}{m_m} \left( \sum_{j=l}^{2} \left( F_{by_{jk}} - F_{pk} - F_{ck} \right) \right), \] (3.27)

\[ \ddot{\psi}_{mk} = \frac{1}{J_m} \left\{ \sum_{j=l}^{3} \left[ F_{by_{jk}} C_{jk} + \left( F_{bx_{1jk}} - F_{bx_{2jk}} \right) B \right] - F_{pk} D - M_{ck} \right\}. \] (3.28)

where \( F_{pk} \) – spring inter-truck connection reactions that depend on counter spring deformation

\[ \Delta_p = y_{m1} - y_{m2} + D(\psi_{m1} + \psi_{m2}), \] (3.29)

\( F_{ck}, M_{ck} \) – truck supporting-and-counter device reactions that depend on dedicated lateral and angular movements of the trucks and body

\[ \Delta_{yk} = y_{mk} - y_c + \psi_c D_k; \quad \Delta_{\psi k} = \psi_c - \psi_{mk}. \] (3.30)

Equations of lateral deflection variation, hunting and rolling of wheel pairs

\[ \ddot{y}_{jk} = \frac{1}{J} \left( \sum_{i=l}^{2} \left( S_{y_{ijk}} - F_{by_{jk}} \right) \right), \]

\[ \psi_{jk} = \frac{1}{J} \left[ \left( S_{x_{1jk}} - S_{x_{2jk}} \right) A - \left( F_{bx_{1jk}} - F_{bx_{2jk}} \right) B \right], \] (3.31)

\[ \dot{\phi}_{jk} = -\frac{1}{J_y} \sum_{i=l}^{2} \left( S_{x_{ijk}} R_{ijk} \right). \]
A motion equation for a six-axle three-truck rolling stock with joined end trucks (Re 2/2)

End truck joining through a lever-torsional system of the same type as in Re 2/2 electric loco allows to the middle truck to deflect in lateral direction against the body with no dependence on other trucks. It significantly enhances features of guiding in curved track sections. Equations that relate movements of the end trucks and body look as follows

\[ y_c = \frac{y_{m1} + y_{m3}}{2}; \]

Equations of truck lateral oscillations

\[ \ddot{y}_{m1} = \frac{F_{m1}a_1 - F_{m3}a_2}{a_1^2 - a_2^2}; \]

\[ \ddot{y}_{m2} = \frac{1}{m_m} \left[ \sum_{i=1}^{2} \sum_{j=1}^{2} \left( S_{yij2} + H_{ij2} \right) - F_c \right]; \]  \hspace{1cm} (3.33)

\[ \ddot{y}_{m3} = \frac{F_{m3}a_2 - F_{m1}a_1}{a_2^2 - a_1^2}, \]

where

\[ a_1 = \frac{m_c}{2} + \frac{J_c}{16D^2} + m_m \]

\[ F_{m1} = \sum_{i=1}^{2} \sum_{j=1}^{2} \left( S_{yij1} + H_{ij1} \right) - F_p; \]  \hspace{1cm} (3.34)

\[ a_2 = -\frac{J_c}{16D^2}; \]

\[ F_{m3} = \sum_{i=1}^{2} \sum_{j=1}^{2} \left( S_{yij3} + H_{ij3} \right) + F_p. \]

Wheel pair rotation equations that describe angular velocity variation on coordinates \( \varphi_{jk} \)
\[
\dot{\phi}_{jk} = \frac{1}{J_y} \left[ M_O - \sum_{i=1}^{2} (S_{xijk} R_{ijk}) \right].
\]  

(3.35)

3.3.3. Six-Axle Carriages with the Axial Formula of 3о-3о

In all produced locos 3о – 3о axle configuration is the most often used. Depending on peculiarities of design variants of certain rolling stocks, their mathematical models will differ as well. The truck arrangement with truck of TE3 type, i.e. diesel locos TE10, 2TE10L, TE40, M62 and others, is the simplest.

Truck movement equations that consider presence of box unit longitudinal clearance look as follows

\[
\dot{y}_{m1} = m_v \left[ F_{ml} \left( \frac{m_c}{4} + \frac{J_c}{4D^2} + m_m \right) - F_{m2} \left( \frac{m_c}{4} - \frac{J_c}{4D^2} \right) \right];
\]

\[
\dot{y}_{m2} = m_v \left[ F_{ml} \left( \frac{m_c}{4} - \frac{J_c}{4D^2} \right) - F_{m2} \left( \frac{m_c}{4} + \frac{J_c}{4D^2} + m_m \right) \right];
\]

(3.36)

\[
\ddot{y}_{nk} = \frac{1}{J_m} \left[ B \sum_{j=1}^{3} (F_{bx1jk} - F_{bx2jk}) + \sum_{j=4}^{6} \sum_{i=1}^{2} (S_{yijk} + H_{ijk}) + F_{by2k} C_{2k} - M_{ck} \right];
\]

where

\[
m_v = \frac{4D^2}{4D^2 m_m^2 + m_c J_c + 2D^2 m_m m_c + 2m_m J_c};
\]

(3.37)

\[
F_{nk} = \sum_{i=1}^{2} (S_{yijk} + H_{ijk}) + F_{by2k}.
\]

Lateral deflection equations for middle wheel pairs of trucks with clearance in boxes
\[ y_{2k} = \frac{1}{m} \left( \sum_{i=1}^{2} S_{y_{i2k}} - F_{by_{2k}} \right). \]  

(3.38)

An equation of wheel pair rotation and hunting

\[ \ddot{\psi}_{jk} = \frac{1}{J_z} \left[ (S_{x_{1jk}}^* - S_{x_{2jk}}^*)A - (F_{bx_{1jk}} - F_{bx_{2jk}})B \right]. \]

(3.39)

\[ \dot{\phi}_{jk} = -\frac{1}{J_y} \sum_{i=1}^{2} (S_{x_{ijk}} R_{ijk}). \]

(3.40)

**Six-axle sections with modified trucks**

Trucks of diesel locos TE10V, TE10M, 2TE116, TE109, TE120, V300, TE114 and others with the same axial configuration underwent significant structural changes compared with TE3 trucks. Their main feature was box connection of Alstrom type without horn plates, but with spring-loaded linkages in lateral and longitudinal directions. Moreover, such diesel locos are equipped with shock absorber suspension at the body-truck connection, which is spring loaded in lateral direction.

Trucks of 2TE121 diesel locos have no box horn plates and are equipped with balanced shock absorber suspension, including coil springs, shock absorber balances, a supporting-and framing hauling drive. In the secondary suspension, in the same way as in 2TE116 diesel loco, four supports with rubber-metal elements are used.

Influenced be design ideas applied in French electric locos of F, FP, SS7100-series, trucks for electric loco VL60 and diesel locos TEP60, TEP75 were created. Lateral spring connection of the body with trucks through rubber-and-metal pendulum supports was implemented.

In case of lateral dividing of body and truck masses, and longitudinal and lateral spring connection of wheel pairs with the truck frame, which is specific to many locos, for instance, 2TE10V, 2TE10M, 2TE116, TE109, TE130, V300, TEP60, TEP70, 2TE121 and others, the oscillation system configuration may be described by the following equations.

A body oscillation equation:
\[ \ddot{y}_c = \frac{1}{m_c} (F_{y_1} + F_{y_2}); \]

\[ \dot{\psi}_c = \frac{1}{J_c} \sum_{k=1}^{2} (2D_k F_{y_{ck}} + M_{ck}); \]

\[ \ddot{y}_{mk(k=1\ldots2)} = \frac{1}{m_m} \sum_{j=1}^{3} (F_{by_{jk}} - F_{y_{ck}}); \]

\[ \dot{\psi}_{mk(k=1\ldots2)} = \frac{1}{J_m} \sum_{j=1}^{3} \left[ B(F_{bx_{1jk}} - F_{bx_{2jk}}) + C_{jk} F_{by_{jk}} - M_{ck} \right]; \]  (3.41)

\[ \ddot{x}_{jk(j=1\ldots3;k=1\ldots2)} = \frac{1}{m} \sum_{i=1}^{2} (S_{x_{ijk}} - F_{bx_{ijk}}); \]

\[ \ddot{y}_{jk(j=1\ldots3;k=1\ldots2)} = \frac{1}{m} \sum_{i=1}^{2} (S_{y_{ijk}} + H_{ijk} - F_{by_{jk}}); \]

\[ \dot{\psi}_{jk(j=1\ldots3;k=1\ldots2)} = \frac{1}{J_z} \left[ (S^{*}_{x_{1jk}} - S^{*}_{x_{2jk}})A - (F_{bx_{1jk}} - F_{bx_{2jk}})B \right]; \]

\[ \ddot{\varphi}_{jk(j=1\ldots3;k=1\ldots2)} = -\frac{1}{J_y} \sum_{i=1}^{2} (F_{\psi_{ijk}} R_{-ijk}); \]

Reactions in truck spring-dissipative connections

\[ F_{bx_{ijk}} = K_{bx} \Delta_{bx_{ijk}} + \beta_{bx} \dot{\Delta}_{bx_{ijk}}; \]

\[ \Delta_{bx_{ijk}} = (\psi_{jk} - \psi_{mk})B_i; \]

\[ \dot{\Delta}_{bx_{ijk}} = (\psi_{jk} - \psi_{mk})B_i; \]
\[ F_{by\ jk(j=1,3; k=1\ldots 2)} = \dot{\mathbf{K}}_{by\ jk} + \dot{\mathbf{b}}_{by\ jk}; \]

\[ \Delta_{by\ jk(j=1,3; k=1\ldots 2)} = y_{mk} - y_c - \psi_{mk} C_{jk} + \psi_c(D_k + C_{jk}); \]  

(3.42)

\[ \dot{\Delta}_{by\ jk(j=1,3; k=1\ldots 2)} = \dot{y}_{mk} - y_c - \dot{\psi}_{mk} C_{jk} + \dot{\psi}_c(D_k + C_{jk}); \]

\[ F_{c(k=1\ldots 2)} = \mathbf{K}_{cy\ k} + \mathbf{b}_{cy} \dot{\mathbf{A}}_{cy\ k}; \]

\[ \Delta_{cy\ k} = y_{mk} - y_c - \psi_c D_k; \]

\[ \dot{\Delta}_{cy\ k} = \dot{y}_{mk} - \dot{y}_c - \dot{\psi}_c D_k. \]

### 3.3.4. Multiaxial Locomotives Based on Two-Axle Carriages

Using the two axle module trucks allows to obtain a line of many-axle locos: four axle (in one section) – VL10, VL80; six axle – VL15, VL85; eight axle – TE13b and TEM7.

**A motion equation for eight axle locos of VL80 type**

VL10, VL80 eight-axle electric locos are equipped with four-axle sections on unified 2-axle trucks. Traction force and braking are transmitted through pivotal devices with a ball joint and box assemblies without horn plates.

The first stage suspension is combined with shock absorbers and springs, and it has lateral connection of the body with trucks.

Equations of body lateral oscillations

\[ m_c \ddot{y}_c + \beta_{cy}(2\ddot{y}_c - \ddot{y}_{m1} - \ddot{y}_{m2}) + \mathbf{K}_{cy}(2y_c - y_{m1} - y_{m2}) = 0; \]  

(3.43)

\[ J_c \dddot{\psi}_c + \beta_{cy}(2\dddot{\psi}_c - \dddot{\psi}_{m1} - \dddot{\psi}_{m2})D^2 + \mathbf{K}_{cy}(2\dddot{\psi}_c - \dddot{\psi}_{m1} - \dddot{\psi}_{m2})D^2 + \]  

\[ + \beta_{cy}(2\ddot{\psi}_c - \dot{\psi}_{m1} - \dot{\psi}_{m2}) + \mathbf{K}_{cy}(2\ddot{\psi}_c - \dot{\psi}_{m1} - \dot{\psi}_{m2}) = 0, \]
where $\mathcal{K}_{c'y}$, $\beta_{c'y}$, $\mathcal{K}_{c'\psi}$, $\beta_{c'\psi}$ – lateral and angular stiffness and a damping coefficient of the supporting and counter device at the body-truck contact, respectively.

Equations of truck lateral oscillations (k=1.2)

$$m_{m} \ddot{y}_{mk} + \beta_{c'y} (\dot{y}_{mk} - \dot{y}_{c} + \psi_{c} D) + \mathcal{K}_{c'y} (y_{mk} - y_{c} + \psi_{c} D) +$$
$$+ \beta_{by} \left( y_{mk} - \sum_{j=1}^{2} \dot{y}_{jk} \right) + \mathcal{K}_{by} \left( y_{mk} - \sum_{j=1}^{2} y_{jk} \right) = 0; \quad (3.44)$$

$$J_{m} \ddot{\psi}_{mk} + \beta_{c'\psi} (\psi_{mk} - \psi_{c}) + 2B^{2} \beta_{bx} \left( 2\psi_{mk} - \sum_{j=1}^{2} \dot{\psi}_{jk} \right) + \beta_{by} [2C^{2}\psi_{m} + C(y_{1k} - y_{2k})] +$$
$$+ \mathcal{K}_{c'\psi} (\psi_{mk} - \psi_{k}) + 2B^{2} \mathcal{K}_{bx} \left( 2\psi_{mk} - \sum_{j=1}^{2} \psi_{jk} \right) - \mathcal{K}_{by} [2C^{2}\psi_{m} + C(y_{1k} - y_{2k})] = 0;$$

where $\mathcal{K}_{bx}$, $\beta_{bx}$, $\mathcal{K}_{by}$, $\beta_{by}$ – longitudinal and lateral stiffness and damping coefficients in connections of the box and truck frame, respectively;

$\mathcal{K}_{c'y}$, $\beta_{c'y}$, $\mathcal{K}_{c'\psi}$, $\beta_{c'\psi}$ – stiffness and damping coefficients in lateral and angular directions, respectively.

An equation of hunting, lateral deflection and rotation of wheel pairs (j=1.2; k=1.2)

$$m \ddot{y}_{jk} + \beta_{by} (\dot{y}_{jk} - \dot{y}_{mk} - C\psi_{mk}) + \mathcal{K}_{by} (y_{jk} - y_{mk} - C\psi_{mk}) = \sum_{i=1}^{2} (S_{yijk} + H_{ijk}) \quad (3.45)$$

$$J_{y} \ddot{\psi}_{jk} + 2B^{2} \beta_{bx} (\psi_{jk} - \psi_{mk}) + 2B^{2} \mathcal{K}_{bx} (\psi_{jk} - \psi_{mk}) = (S_{x1jk} - S_{x2jk}) A; \quad$$

$$J_{y} \ddot{\varphi}_{jk} = M_{djk} - \sum_{i=1}^{2} S_{xkx} R_{ijk}.$$

An equation for twelve axle locos of VL85 type

The truck arrangement for 12-axle 2 section electric locos VL15, VL85 is made of 2-axle trucks: three trucks in each body section. Inclined rods that balance loading on wheel pairs in case of applying traction force and braking are used for longitudinal
connection of the body with trucks. To enhance guiding in curves, the possibility of large lateral movements of the middle truck against the body is stipulated. End trucks are equipped with cradle suspension, which similar to the suspension of electric locos VL80. Stiffness and damping coefficients of horizontal connection of end and middle trucks with the body are different and designated respectively as: $\mathcal{K}_c$, $\beta_c$, $\mathcal{K}_\psi$, $\beta_\psi$, $\mathcal{K}_{c^*}$, $\beta_{c^*}$, $\mathcal{K}_{\psi^*}$, $\beta_{\psi^*}$.

Equations of body lateral oscillations look as follows

$$
m_c \ddot{y}_c + (2 \beta_c + \beta_c^*) \dot{y}_c - \beta_c (\dot{y}_{m1} + \dot{y}_{m3}) - \beta_c^* \dot{y}_{m2} +
+ (2 \mathcal{K}_c + \mathcal{K}_c^*) y_c - \mathcal{K}_c (y_{m1} + y_{m3}) - \mathcal{K}_c^* y_{m2} = 0;
$$

$$
J_c \ddot{\psi}_c + (2 \beta_c B^2 + 2 \beta_\psi + \beta_\psi^*) \dot{\psi}_c - \beta_\psi (\dot{\psi}_{m1} + \dot{\psi}_{m2}) - \beta_\psi^* \dot{\psi}_{m2} +
+ (2 \mathcal{K}_c D^2 + 2 \mathcal{K}_\psi + \mathcal{K}_\psi^*) \psi_c - \mathcal{K}_\psi (\psi_{m1} + \psi_{m3}) - \mathcal{K}_\psi^* \psi_{m2} = 0.
$$

Equations of truck oscillations

$$
m_m \ddot{y}_{m1} - \beta_k \dot{y}_k + (\beta_k + 2 \beta_{by}) \dot{y}_{m1} + \beta_c D \dot{\psi}_c - \beta_{by} (\dot{y}_{11} + \dot{y}_{21}) -
- \mathcal{K}_c y_c + (\mathcal{K}_c + 2 \mathcal{K}_{by}) y_{m1} + \mathcal{K}_c D \dot{\psi}_c - \mathcal{K}_{by} (y_{11} + y_{21}) = 0;
$$

$$
m_m \ddot{y}_{m2} - \beta_c^* \dot{y}_c + (\beta_c^* + 2 \beta_{by}) \dot{y}_{m2} - \beta_{by} (y_{12} + y_{22}) -
- \mathcal{K}_c y_c + (\mathcal{K}_c^* + 2 \mathcal{K}_{by}) y_{m2} - \mathcal{K}_{by} (y_{12} + y_{22}) = 0;
$$

$$
m_m \ddot{y}_{m3} - \beta_c \dot{y}_c + (\beta_c + 2 \beta_{by}) \dot{y}_{m3} - \beta_c D \dot{\psi}_c - \beta_{by} (\dot{y}_{13} + \dot{y}_{23}) -
- \mathcal{K}_c y_c + (\mathcal{K}_c + 2 \mathcal{K}_{by}) y_{m3} - \mathcal{K}_c D \dot{\psi}_c - \mathcal{K}_{by} (y_{13} + y_{23}) = 0.
$$

$$
J_m \ddot{\psi}_{m1} + (\beta_\psi + 4 \beta_{bx} B^2 + 2 \beta_{by} C^2) \dot{\psi}_{m1} - \beta_\psi \dot{\psi}_c - 2 \beta_{bx} B^2 (\dot{\psi}_{11} + \dot{\psi}_{21}) -
- \beta_{by} C (\dot{\psi}_{11} - \dot{\psi}_{21}) + (\mathcal{K}_\psi + 4 \mathcal{K}_{bx} B^2 + 2 \mathcal{K}_{by} C^2) \dot{\psi}_{m1} - \mathcal{K}_\psi \dot{\psi}_c -
- 2 \mathcal{K}_{bx} B^2 (\dot{\psi}_{11} + \dot{\psi}_{21}) - \mathcal{K}_{by} C (y_{11} - y_{21}) = 0;
$$
\[ J_m \ddot{\psi}_{m2} + \left( \beta_{m2} + 4 \beta_{m2} B^2 + 2 \beta_{m2} C^2 \right) \dot{\psi}_{m2} - \beta_{m2} \psi_c - 2 \beta_{m2} B^2 (\psi_{12} + \psi_{22}) = \\
- \beta_{by} C (\dot{y}_{12} - \dot{y}_{22}) + \left( 4 \beta_{by} B^2 + 2 \beta_{by} C^2 \right) \dot{\psi}_{m2} - \beta_{by} \psi_c - \\
- 2 \beta_{bx} C^2 (\psi_{12} + \psi_{22}) - \beta_{by} C (y_{12} - y_{22}) = 0; \]

Spring rods are used in box assemblies. They divide masses of wheel/ motor assemblies and trucks in direction of considered coordinates. Equations of wheel pair oscillation (j=1...2; k=1...3)

\[ m \ddot{y}_{jk} - \beta_{by} \dot{y}_{mk} - \beta_{by} C \psi_{mk} - \beta_{by} y_{mk} - \beta_{by} C \psi_{mk} = \\
= \sum_{i=1}^{2} \left( S_{i j k y} + H_{i j k y} \right); \quad (3.48) \]

\[ J \ddot{\psi}_{jk} + 2 \beta_{by} B^2 \ddot{\psi}_{jk} - 2 \beta_{by} B^2 \psi_{mk} + \beta_{by} B^2 \psi_{jk} - 2 \beta_{by} B^2 \psi_{mk} = \\
= \left( S_{1 j k x} - S_{2 j k x} \right) A; \]

\[ J \ddot{\phi}_{jk} = \sum_{i=1}^{2} S_{i j k y} R_{i j k}; \]

**Eight-axle sections with four-axle trucks of TE136 type**

The truck arrangement of TE136 diesel loco contains two four-axle trucks composed of two 2-axle trucks with opposed arrangement of motors on each axle. The two-axle trucks are joined with each other through a low mounted balancing beam with a socket for a pivotal assembly.

The eight-axle truck structure is described by the following constraint equations

\[ y_{nk} = \frac{1}{2} \sum_{j=1}^{2} y_{jk}; \quad \psi_{nk} = \frac{1}{2C} (y_{2k} - y_{lk}); \]

\[ y_{31} = \frac{1}{4} \sum_{j=1}^{2} \sum_{k=1}^{2} y_{jk}; \quad y_{32} = \frac{1}{4} \sum_{j=1}^{2} \sum_{k=3}^{4} y_{jk}; \quad (3.49) \]
\[
\psi_{al} = \frac{1}{4E} \sum_{j=1}^{2} (y_{j2} - y_{j1}); \quad \psi_{a2} = \frac{1}{4E} \sum_{j=1}^{2} (y_{j4} - y_{j3});
\]

\[
y_c = \frac{1}{8} \sum_{j=1}^{2} \sum_{k=1}^{4} y_{jk}; \quad \psi_c = \frac{1}{8D} \left( \sum_{j=1}^{2} \sum_{k=1}^{4} y_{jk} - \sum_{j=1}^{2} \sum_{k=1}^{2} y_{jk} \right).
\]

The movement equation system is composed of 24 differential equations of the second order

\[
A \times \ddot{y} = \ddot{Q}; \quad (3.50)
\]

\[
A = \begin{bmatrix}
A_1(8 \times 8) & 0 & 0 \\
0 & A_2(8 \times 8) & 0 \\
0 & 0 & A_3(8 \times 8)
\end{bmatrix}, \quad (3.51)
\]

\[
A_1 = \begin{bmatrix}
m_{np} & m_1 & m_2 & m_2 & m_3 & m_3 & m_3 & m_3 \\
m_1 & m_{np} & m_2 & m_2 & m_3 & m_3 & m_3 & m_3 \\
m_2 & m_2 & m_{np} & m_1 & m_3 & m_3 & m_3 & m_3 \\
m_2 & m_2 & m_1 & m_{np} & m_3 & m_3 & m_3 & m_3 \\
m_3 & m_3 & m_3 & m_3 & m_{np} & m_1 & m_2 & m_2 \\
m_3 & m_3 & m_3 & m_3 & m_{np} & m_2 & m_2 & m_2 \\
m_3 & m_3 & m_3 & m_3 & m_2 & m_2 & m_{np} & m_1 \\
m_3 & m_3 & m_3 & m_2 & m_2 & m_{np} & m_1 & m_{np}
\end{bmatrix}, \quad (3.52)
\]

\[
A_2 = \begin{bmatrix}
J_z & 0 & \ldots & 0 \\
0 & J_z & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & \ldots & \ldots & J_z
\end{bmatrix}; \quad A_3 = \begin{bmatrix}
J_y & 0 & \ldots & 0 \\
0 & J_y & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & \ldots & \ldots & J_y
\end{bmatrix};
\]

\[
m_{np} = \frac{m_c}{64} + \frac{m_B}{16} + \frac{m_m}{4} + \frac{J_c}{64D^2} + \frac{J_B}{16E^2} - \frac{J_m}{4C^2} + m;
\]
\[
m_1 = \frac{m_c}{64} + \frac{m_B}{16} + \frac{m_m}{4} + \frac{J_c}{64D^2} + \frac{J_B}{16E^2} - \frac{J_m}{4C^2} \\
m_2 = \frac{m_c}{64} + \frac{m_B}{16} + \frac{J_c}{64D^2} - \frac{J_B}{16E^2} \\
m_3 = \frac{1}{64} (m_c - \frac{J_c}{D^2})
\]

\[
\ddot{y} = \begin{bmatrix}
\ddot{y}_{11} \\
\vdots \\
\ddot{y}_{24} \\
\ddot{\psi}_{11} \\
\ddot{\psi}_{24} \\
\ddot{\phi}_{11} \\
\ddot{\phi}_{24}
\end{bmatrix} ;
\ddot{Q} = \begin{bmatrix}
F_{y11} \\
\cdots \\
F_{y24} \\
M_{\psi11} \\
M_{\psi24} \\
M_{\phi11} \\
M_{\phi24}
\end{bmatrix}
\]

\[
F_{y(j=1...2; k=1...4)} = \sum_{i=1}^{2} (S_{yk} + H_{ijk}) + F_{byjk} \\
M_{\psi(j=1...2; k=1...4)} = (S_{x1jk} - S_{x2jk})A - (F_{bx1jk} - F_{bx2jk})B \\
M_{\phi(j=1...2; k=1...4)} = M_{az} - \sum_{i=1}^{2} S_{xik} R_{ijk}.
\]

In (3.54) \(F_{byjk}\) – cumulative box reactions caused by supporting-and-counter devices at the connection of the body with balancers and balancers with trucks.

The following designations are adopted:

- \(M_{c1}\), \(M_{c2}\) – reverse moments in the body/balancing device connection;
- \(M_{r1}\), \(M_{r2}\), \(M_{r3}\), \(M_{r4}\) – reverse moments in the connection of balancing devices with truck frames.

Values of indicated moments depend on relative motions and speeds of joined elements

\[
M_{cj(j=1...2)} = M_c(\Delta_{\psi cj}, \dot{\Delta}_{\psi cj}) ; \quad M_{mj(j=1...4)} = M_m(\Delta_{\psi mj}, \dot{\Delta}_{\psi mj}) ;
\]

\[
\Delta_{\psi cl} = \psi_{bl} - \psi_c ; \quad \dot{\Delta}_{\psi cl} = \dot{\psi}_{bl} - \dot{\psi}_c ;
\]

\[
\Delta_{\psi n k} = \psi_{nk} - \psi_{bl} ; \quad \dot{\Delta}_{\psi k} = \dot{\psi}_{nk} - \dot{\psi}_{bl} ;
\]
$$\psi_{bl} = \frac{1}{4E}(y_{12} + y_{22} - y_{11} - y_{21}); \quad (3.55)$$

$$\psi_{b2} = \frac{1}{4E}(y_{14} + y_{24} - y_{13} - y_{23});$$

$$\psi_{c} = \frac{1}{8D}(y_{13} + y_{23} + y_{14} + y_{24} - y_{11} - y_{21} - y_{12} - y_{22});$$

$$\psi_{mk(k=1...4)} = \frac{1}{2C}(y_{2k} - y_{1k}); \quad \psi_{nk} = \frac{1}{2C}(\dot{y}_{2k} - \dot{y}_{1k});$$

$$\psi_{bl} = \frac{1}{4E}(\dot{y}_{12} + \dot{y}_{22} - \dot{y}_{11} - \dot{y}_{21}); \quad \psi_{b2} = \frac{1}{4E}(\dot{y}_{14} + \dot{y}_{24} - \dot{y}_{13} - \dot{y}_{23});$$

$$\psi_{c} = \frac{1}{8D}(\dot{y}_{13} + \dot{y}_{23} + \dot{y}_{14} + \dot{y}_{24} - \dot{y}_{11} - \dot{y}_{21} - \dot{y}_{12} - \dot{y}_{22}); \quad (3.56)$$

$$F_{by11} = F_{m1} + \frac{F_{cl}}{2} - \frac{F_{bl}}{2} - \frac{F_{c}}{4} - F_{bl};$$

$$F_{by21} = -F_{m1} + \frac{F_{cl}}{2} - \frac{F_{bl}}{2} - \frac{F_{c}}{4} + F_{bl};$$

$$F_{by12} = F_{m2} - \frac{F_{cl}}{2} + \frac{F_{bl}}{2} - \frac{F_{c}}{4} - F_{b2}; \quad (3.56)$$

$$F_{by22} = -F_{m2} - \frac{F_{cl}}{2} + \frac{F_{bl}}{2} - \frac{F_{c}}{4} + F_{b2};$$

$$F_{by13} = F_{m3} + \frac{F_{c2}}{2} - \frac{F_{bl}}{2} + \frac{F_{c}}{4} - F_{b3};$$
\[ F_{by23} = -F_{m3} + \frac{F_{c2}}{2} - \frac{F_{b2}}{2} + \frac{F_{c}}{4} + F_{b3}; \]

\[ F_{by14} = F_{m4} - \frac{F_{c2}}{2} + \frac{F_{b2}}{2} + \frac{F_{c}}{4} - F_{b4}; \]

\[ F_{by24} = -F_{m4} - \frac{F_{c2}}{2} + \frac{F_{b2}}{2} + \frac{F_{c}}{4} + F_{b4}; \]  \( (3.57) \)

\[ F_{k} \ (k=1...4) = \frac{M_{mk}}{C}; \quad F_{cl (l=1...2)} = \frac{M_{el}}{E}; \quad F_{c} = \frac{M_{cl} + M_{c2}}{D}; \]

\[ F_{b1} = \frac{M_{m1} + M_{m2}}{E}; \quad F_{b2} = \frac{M_{m3} + M_{m4}}{E}; \]

\[ F_{bk (k=1...4)} = \frac{B}{2C} (F_{bxi1k} + F_{bx12k} - F_{bx21k} - F_{bx22k}). \]

**A TEM7 type loco with trucks and intermediate frames**

A clean up switcher TEM7 has two four-axle trucks composed of no-horn-plate-trucks joined with intermediate frames by pendulum suspensions. Traction forces from two-axle trucks are transmitted to intermediate frames through toggle traction mechanisms that include horizontal and inclined pull rods. The loco is characterized by very small minimal radius of guiding - 80 m.

Movement of the body and truck intermediate frames are constrained by the following relations

\[ y_c = \frac{y_{b1} + y_{b2}}{2}; \quad \psi_c = \frac{y_{b2} - y_{b1}}{2E}. \]  \( (3.58) \)

Equations for intermediate beams look as follows
\[
\begin{align*}
\left( \frac{m_c}{4} + m_b + \frac{J_c}{4E^2} \right) \ddot{y}_{b1} + \left( \frac{m_c}{4} - \frac{J_c}{4E^2} \right) \ddot{y}_{b2} &= F_{c1} - F_{m1} - F_{m2}, \quad (3.59) \\
\left( \frac{m_c}{4} + m_b + \frac{J_k}{4E^2} \right) \ddot{y}_{b2} + \left( \frac{m_c}{4} - \frac{J_c}{4E^2} \right) \ddot{y}_{b1} &= F_{c2} - F_{m3} - F_{m4}.
\end{align*}
\]

Reactions in the contacts of the body with intermediate beams – \( F_{c1}, F_{c2} \) and beams with trucks – \( F_{m1}, \ldots, F_{m4} \) depend on mutual movements and speeds of joined elements, and features of corresponding joints.

Truck movement equations \((k=1, \ldots, 4)\)

\[
m_n y_{nk} = F_{nk} - \sum_{j=1}^{2} F_{byjk};
\]

\[
J_{nm} \ddot{y}_{nk} = M_{nk} + \left( F_{bylk} - F_{by2k} \right) C + \left( F_{bx1jk} - F_{bx2jk} \right) A.
\]

Wheel pair movement equations

\[
m \ddot{y}_{jk} = \sum_{i=1}^{2} \left( S_{yijk} + H_{ijk} \right) - F_{byjk};
\]

\[
J \ddot{y}_{jk} = \left( S_{x1jk} - S_{x2jk} \right) - \left( F_{bx1jk} - F_{bx2jk} \right) A;
\]

\[
J \ddot{\phi}_{jk} = \sum_{i=1}^{2} S_{xijk} R_{ijk}. \quad (3.61)
\]

3.4. Models of Electric Trains, Diesel Trains, Passenger Carriages, Trams and Underground Carriages

Railway car track arrangements on one hand are much simpler due to absence of axle motors, and on the other hand, when designing them, there are more enhancement possibilities aimed at bettering dynamic features of a rolling stock. For instance, if a radially installed wheel pair is future perspective for locos, in the same time in many countries it is a serial construction for railway cars. Electric / diesel trains occupy an intermediate place in this sense.
Two axle trucks are used in truck arrangements of almost all passenger cars, electric/ diesel trains.

In Ukraine, two series of suburban trains are still used: DR1 diesel trains and ER electric trains.

A number of sophisticated technical solutions are implemented in DR1 truck arrangement. For instance, shock absorber suspension body stage springs are used in it as spring elements for not only vertical body-truck connection but for lateral connection as well. The truck frame is lowered below box assemblies of a lever type.

ER series electric trains of various modifications are very similar to each other by their truck arrangements. The shock absorber suspension is two stage suspension with frictional dampers in the box stage and hydraulic dampers in the body stage. The connection of the body with trucks is done with a cradle mechanism. ER200 electric train trucks are more perfect.

Development of passenger traffic abroad was always aimed at enhancing electric trains and increasing maximal travelling speeds. In many countries (France, Japan, Germany) electric train speeds of up to 200-300 km/h are just usual. Achievements of regular service at such speeds were preceded by extended studies of truck/ shock absorber structures. Nevertheless, it shall be noted that in foreign countries the same attention was paid to a structure, quality of track. As a rule, special lines were built for the high-speed passenger traffic. Achieving the movement high speeds without complex solving of rolling stock problems is very difficult.

The truck structure of foreign high-speed electric trains is not much different from ours by their principle diagram, however in the same time, the trains possess a number of individual peculiarities.

The two-axle motor truck of a TGC electric train (France) is characterized by installing the traction motors and gear boxes on the body, and torque is transmitted to axles through "Tripod" couplings that are well known for their operation reliability. The TGV electric train holds the record for railway high speed of 574.8 km/h, set in 2007 on Paris-Lion way. The previous record was also set by French TGV of Alstom Company.

L-GSG4 trucks of similar structure are used in German electric trains (trains BD4, RBDe4). The second stage of shock absorber suspension are coil springs and rubber-metal elements; one-linkage boxes; support-and-frame suspension of traction motors with axle gear boxes.
L-GA2T trucks are installed on suburban electric trains of Switzerland railways. Modified L-AK-M trucks are also installed on Holland suburban electric trains. Long distance electric trains *ICM* in Holland are equipped with *L-GA2* trucks, whereas diesel trains *DH2* are equipped with trucks *L-S2G4*.

High-speed passenger traffic lines of France and Finland use railway cars with trucks *L-GSG2*.

The truck arrangement of many passenger railway cars has two two-axle trucks. The railway cars used in Ukraine basically have trucks KVZ-TsNII that are unified for all passenger railway cars of long distance and inter-regional destination, as well as for suburban electric trains. The trucks are equipped with two-stage shock absorber suspension. The central suspension has a cradle structure. Box suspensions are made of coil springs and wedge type shock absorbers. This suspension stage not only functions as a first suspension, but it provides a spring joint of a wheel pair with the truck frame in horizontal plane.

Passenger railway car truck structures are various in countries of Europe, Asia and America, and they differ depending on a car/ train class.

German truck *Minden-Deutz* served as a prototype of a typical truck of long distance trains in Western Europe. It has doubled suspension. In the box stage without horn plates the box connection with the frame is done through spring loaded box flat linkages. The developed truck structures are unified for passenger railway cars and trailing cars of diesel trains with speeds of up to 200 km/h.

The truck of *DT-200* model designed for travelling at 250 km/h is the most commonly used in Japan. The linkage structure of the truck with its frame is implemented in the box assembly. Stiffness of the pneumatic suspension first stage is 1.1 kN/mm; stiffness of the central suspension in vertical direction is 0.45 kN/mm, whereas in lateral direction it is 0.36 kN/mm. Truck weight with traction motors – 10 t; base – 2.5 m; track – 1.435 m.

The worldwide practice has two more examples of commercial trucks with pneumatic suspension: *Pioneer* (USA) for suburban railway cars and R28 for long distance railways of France and Germany.

One of perspective trucks for passenger cars is TSK-1 truck designed for RT-200 train and speed of up to 250 km/h. Main differences from commercial structures are pneumatic suspension and enhanced longitudinal connection of wheel pairs with the truck frame.
The truck structure is similar to MSZhD truck, which is used in subway cars of type E (Em, Ema, Emkh, Ezh, Ezhz) and I.

Almost all trams used worldwide are equipped with two axle trucks. Sometimes they are installed on 6-axle joined cars, for instance, Austrian trams Е6 and С6, or German Tram 2000, but more often they are 4-axle trams, for instance, of CLRV type (Canada), Bt or Be 8/8.

In spite of variety of design solutions of truck arrangements with axle configuration 20-20 or 2-2, computational patterns of almost all passenger railway cars and electric/ diesel trains are similar, and they may be described by the same motion equations.

Body lateral oscillation equation

\[ m_c \ddot{y}_c = F_{c1} + F_{c2}; \]

\[ J_c \ddot{y}_c = (F_{c2} - F_{c1})D + \sum_{k=1}^{2} M_{ck}. \]

(3.62)

Reaction values in the connection of the body and trucks depend on relative movements (and speeds) of the body and trucks, and connection features in lateral and angular directions.

Equations of truck lateral oscillations (k=1.2)

\[ m_m \ddot{y}_{rk} = \sum_{j=1}^{2} F_{byjk} - F_{ck}; \]

(3.63)

\[ J_m \ddot{y}_{rk} = (F_{by2k} - F_{bylk})C - M_{ck}. \]

An equation of hunting, lateral deflection and rotation of wheel pairs (j=1.2; k=1.2)

\[ m \ddot{y}_{jk} = \sum_{i=1}^{2} (S_{yk} + H_{ijk}) - F_{byjk}; \]

\[ J\ddot{y}_{jk} = (S_{x1jk} - S_{x2jk})A - (F_{bx1jk} - F_{bx2jk})B; \]

(3.64)
$$J_y \ddot{\theta}_{jk} = \sum_{i=1}^{2} S_{x,ijk} R_{ijk}.$$ 

### 3.5. Mathematical Models of Freight Carriages

A two axle truck of 18-100 model (TsNII-HkZ-0) is the main truck type of freight cars that are operated on railways of Ukraine, Russia and CIS countries. The trucks are used in all four-axle long distance cars, as well as in four axle trucks of eight-axle open cars and tanks cars.

The model of a four axle car with 18-100 type trucks is composed of 11 masses: car body, two body bolsters, four side frames and four wheel pairs.

In the truck, two spring loaded sets are used. Each of them is composed of two row springs.

A body horizontal oscillation equation

$$m_c \ddot{y}_c = \sum_{k=1}^{2} F_{ck};$$  \hspace{1cm} (3.65)

$$J_c \ddot{\psi}_c = (F_{c2} - F_{c1})D + \sum_{k=1}^{2} M_{ck}.$$  \hspace{1cm} (3.65)

Equations of horizontal oscillations of truck body bolsters (k=1.2)

$$m_{lll} \ddot{y}_{lll} = -F_{ck} + \sum_{l=1}^{2} F_{blky};$$  \hspace{1cm} (3.66)

$$J_{lll} \ddot{\psi}_{lll} = -M_{ck} + (F_{blkx} - F_{b2kx})L + \sum_{l=1}^{2} M_{blk}.$$  \hspace{1cm} (3.66)

Truck assembled frames have comparably low parallelogram stiffness and, as a result, low resistance to longitudinal relative movement of side frames. In the same time, they have high inter-axle stiffness at relative rotation of wheel pairs in plain
view.

Equations of truck side frame oscillations (l=1.2; k=1.2) look as follows

\[ m_b \ddot{y}_{blk} = -F_{blky} + \sum_{j=1}^{2} F_{byj} ; \]

\[ m_b \ddot{x}_{blk} = -F_{blkx} + \sum_{j=1}^{2} F_{bxijk} ; \]  \hspace{1cm} (3.67)

\[ J_b \ddot{\psi}_{blk} = -M_{blk} + \sum_{j=1}^{2} \sum_{i=1}^{2} M_{biij} + \left( F_{by2k} - F_{byl} \right) C . \]

Equations of wheel pair horizontal oscillations (k=1.2; j=1.2)

\[ m \ddot{y}_{jk} = F_{byjk} + \sum_{i=1}^{2} \left( S_{yijk} + H_{ijk} \right) ; \]

\[ J \ddot{\psi}_{jk} = -M_{bij} + \left( S_{x1jk} - S_{x2jk} \right) A - \left( F_{bx1jk} - F_{bx2jk} \right) B ; \] \hspace{1cm} (3.68)

\[ J_y \ddot{\phi}_{jk} = \sum_{i=1}^{2} S_{xijk} R_{ijk} . \]

Three-axle trucks of 18-102 type are produced for six-axle freight cars. These trucks have two joined type side semi-frames that provide even load transmission over all three-wheel pairs.

Each side semi-frame rests with its one end directly on the box body of the outermost wheel pair, and with the other end on the balance outermost end, which, in its turn, rests on the box body of the middle wheel pair.

Body horizontal oscillation equations of a six-axle freight car look as follows

\[ m_c \ddot{y}_c = \sum_{k=1}^{2} F_{ck} ; \] \hspace{1cm} (3.69)
\[ J_c \ddot{\psi}_c = (F_{c2} - F_{cl})D + \sum_{k=1}^{2} M_{ck}. \]

Equations of horizontal oscillations of truck body bolsters (k=1.2)

\[
\begin{align*}
m_{llj} \ddot{y}_{llk} &= -F_{ck} + \sum_{l=1}^{4} F_{blky}; \\
J_{lll} \ddot{\psi}_{llk} &= -M_{ck} + (F_{blkx} + F_{b3kx} - F_{b2kx} - F_{b4kx})L + \\
&+ (F_{b4k} + F_{b4k} - F_{blik} - F_{b5k})E + \sum_{l=1}^{4} M_{blk}.
\end{align*}
\]  (3.70)

Frames of trucks 18-102 and 18-100 have moving side frames that provide low parallelogram stiffness, but high inter-axle stiffness, when wheel pairs turn in plain view. Equations of truck side frame horizontal oscillations (l=1.2; k=1.2)

\[
\begin{align*}
m_{b} \ddot{y}_{lik} &= -F_{bliky} + \sum_{j=1}^{3} F_{byjk}; \\
m_{b} \ddot{x}_{lik} &= -F_{blikx} + \sum_{j=1}^{3} F_{bxjik};
\end{align*}
\]  (3.71)

\[
J_{b} \ddot{\psi}_{lik} = -M_{blk} + \sum_{j=1}^{3} \sum_{i=1}^{2} M_{bijk} + \left(F_{by3k} - F_{by1k}\right)C.
\]

Equations of wheel pair horizontal oscillations (k=1.2; j=1.3)

\[
\begin{align*}
m_{y} \ddot{y}_{jk} &= -F_{byjk} + \sum_{i=1}^{2} \left(S_{yijk} + H_{ijk}\right); \\
J_{y} \ddot{y}_{jk} &= -M_{bijk} + \left(S_{x1jk} - S_{x2jk}\right)A - \left(F_{bx1jk} - F_{bx2jk}\right)B; \quad (3.72)
\end{align*}
\]
\[ J_y \ddot{\theta}_{jk} = \sum_{i=1}^{2} S_{x_{ij}k} R_{ij}. \]

Eight-axle freight cars are equipped with four-axle trucks of 18-101 type. The trucks are assembled of two 18-100 trucks and longitudinal binder.

A body horizontal oscillation equation

\[ m_c \ddot{y}_c = \sum_{k=1}^{2} F_{ck} ; \quad (3.73) \]

\[ J_c \ddot{y}_c = (F_{c2} - F_{c1})D + \sum_{k=1}^{2} M_{ck} . \]

Horizontal oscillation equations of truck longitudinal beams

\[ m_{ll} \ddot{y}_{ll1} = -F_{c1} + \sum_{k=1}^{2} F_{llky} ; \quad (3.74) \]
\[ m_{ll} \ddot{y}_{ll2} = -F_{c2} + \sum_{k=3}^{4} F_{llky} ; \]
\[ J_{ll} \ddot{y}_{ll1} = -M_{c1} + (F_{ll2y} - F_{ll1y})E + \sum_{k=1}^{2} M_{llk} ; \]
\[ J_{ll} \ddot{y}_{ll2} = -M_{c2} + (F_{ll4y} - F_{ll3y})E + \sum_{k=3}^{4} M_{llk} . \]

Horizontal oscillation equations of truck lateral body bolsters (k=1.4)

\[ m_{ll} \ddot{y}_{llk} = -F_{llky} + \sum_{l=1}^{2} F_{blky} ; \quad (3.75) \]
\[ J_{ll} \ddot{y}_{llk} = -M_{ck} + (F_{blxx} - F_{b2xx})L + \sum_{l=1}^{2} M_{blk} . \]

Equations of truck side frame horizontal oscillations (l=1.2; k=1.4)
\[
m_b \ddot{y}_{blk} = -F_{byk} + \sum_{j=l}^{2} F_{byjk} ;
\]

\[
m_b \ddot{x}_{blk} = -F_{blkx} + \sum_{j=l}^{2} F_{bxjk} ;
\]

\[
J_b \dddot{\psi}_{blk} = -M_{blk} + \sum_{j=l}^{2} \sum_{i=1}^{2} M_{bijk} + \left(F_{by2k} - F_{bylk}\right)C .
\]

Equations of wheel pair horizontal oscillations (k=1.4; j=1.2)

\[
m_j \ddot{y}_{jk} = -F_{byjk} + \sum_{i=1}^{2} \left(S_{yjk} + H_{ijk}\right) ;
\]

\[
J_j \dddot{\psi}_{jk} = -M_{bijk} + \left(S_{x1jk} - S_{x2jk}\right)A - \left(F_{bx1jk} - F_{bx2jk}\right)B ; \quad (3.77)
\]

\[
J_j \dddot{\phi}_{jk} = \sum_{i=1}^{2} S_{xijk} R_{ijk} .
\]

Four-axle cars prevail in freight car fleets of Western Europe countries. R25C model in some modifications is the main truck type in European Unit countries.

Truck frames have closed and stiff structure, which is composed of two longitudinal beams, one body bolster and two end lateral beams.

Trucks, whose majority is similar to TsNII-Kh-O trucks, are used on railways of USA and Canada.

3.6. Simulation of the Contacting Parts of the Carriage

According to the principle of mathematical description of connections between separate rolling stock elements, they may be divided into two groups.
To the first group we relate the connections that have linear characteristics included into Lagrange equation as constant coefficients with derivative parts.

Longitudinal and lateral connections in box linkage assemblies that do not have lateral play are considered as linear connections. These connections are represented by stiffness with coefficient $\mathcal{K}_{bx}$, $\mathcal{K}_{by}$ and damping coefficients $-\beta_{bx}$, $-\beta_{by}$.

Parameters of the other connections, as a rule, are non-linear. According to the adopted coordinate system, we will consider only lateral and angular connections of the body and trucks in plain view.

The lateral connection of the body with trucks has to implementation variants, i.e. stiff or flexible one. The stiff connection is special to body resting on trucks through a flat cylindrical or spherical heel, or a stiff pivot.

Flexible connection is provided by various devices, and it possesses a wide range of features. There are following design variants:

- pendulum supports with a spring loaded counter mechanism;
- cradle suspension;
- pivot with spring loaded lateral movement;
- Flexy-coil suspension;
- pneumatic or rubber-and-metal shock absorbers.

The connection with pendulum supports is implemented in locos SS7100, VV1650, F, FP, VL60, TEP60. It is features by precompression of counter springs. Stiffness of one pendulum support in lateral direction is expressed by the formula

$$\mathcal{K}_y = \frac{F_0}{y} + \frac{\mathcal{K}_{pp}}{2} - \frac{P}{2\sqrt{1^2 - y^2}},$$

(3.78)

where $F_0$, $\mathcal{K}_{pp}$ – force of preliminary tightening and counter device spring stiffness;

$1$ – pendulum support height;

$P$ – body weight.

Pendulum mechanism roll stiffness in plain view in case of two supports installed at distance $C$ from each other may be expressed as

$$\mathcal{K}_\psi = 0.5\mathcal{K}_y C^2.$$

(3.79)
It shall be admitted that the classical pendulum suspension distinct by its simplicity has underwent signification modification, mainly, through weak damping of hunting oscillations, and it has been enhanced by installing additional supports with guides.

The structural combination of the pivot assembly with the spring loaded counter device in lateral direction is special to a large group of locos: VL10, VL80, TE10B, TE10M, TE116, TE109, TE120, TE132.

The spring loaded counter device can function in parallel with the rubber-and-metal supports that possess high visco-elastic resistance, or with guides applying dry friction resistance. The back moment can be also created by roller supports with profiled plates.

In ER1 electric trains and subway cars of G, D, E series the body through a central spherical support rests on an intermediate cradle beam, which, in its turn, rests on the cradle through shock absorber suspension second stage elements. Quasi-spring counter forces of the cradle mechanism have gravitational origin. The cradle suspension insufficiently damps hunting oscillations. Due to this, a number of enhanced structures of the cradle suspension were created. In particular, they are used in electric trains ER2, ER9, ER22.

Flexicoil spring suspension is considered as the simplest design solution of the supporting and counter device, where springs operate both in vertical and lateral directions. Such suspension is used in electric locos E151, E120, electric train TGV, diesel trains DR1, DR2, TG16, TEP70.

As it has been mentioned above, characteristics of wheel pair-truck frame connections in a box assembly linkage structure are close to linear ones, therefore during modelling it is possible to consider them as constant coefficients in Lagrange's partial differential equations. Let us consider the non-linear characteristics in connection with this: for a pedestal box in case of installing a wheel pair in the box with axial clearance (lateral play).

Though both manufacturers and operators try to decrease the longitudinal clearance in box assembly horn plates, in practice it always achieves values of 0.5...2.5 mm, which is, as it is well known, significantly influences on wheel pair hunting.

If one designates the total longitudinal clearance in a box assembly as $2\Delta xo$, characteristics of the longitudinal connection between a wheel pair and boxes will
look as follows

at \( \Delta x < - \Delta x_0 \), \( F_{bx} = \kappa_{bx}(\Delta x + \Delta x_0) \);

at \( -\Delta x_0 < \Delta x < \Delta x_0 \), \( F_{bx} = 0 \); \hspace{1cm} (3.80)

at \( \Delta x > \Delta x_0 \), \( F_{bx} = \kappa_{bx}(\Delta x - \Delta x_0) \),

where \( \kappa_{bx} \) – stiffness of the contact between the box body and horn plate guides;
\( \Delta x \) – longitudinal movement of the wheel pair neck middle toward box assembly horn plates.

If between the wheel pair and the box body there is axial clearance, the following designation shall be introduced:
\( \delta_o \) – one side free axial clearance (play) in a box assembly;
\( F_{tp} \) – friction force, which interferes with axial movement of the wheel pair axle neck against the box body;
\( F_n \) – precompression of axial thrust springs;
\( \kappa_{by} \) – box connection axial thrust stiffness.

Reactions in boxes may be described by the following relations

as: \( \Delta y < - \Delta y_o \); \( F_{by} = - (\Delta y + \delta_o) \kappa_{by} - F_n - F_{tp} \text{ sign}(\dot{\Delta}_y); \)

\( \Delta y_o < \Delta y < \Delta y_o \); \( F_{by} = - F_{tp} \text{ sign}(\dot{\Delta}_y); \) \hspace{1cm} (3.81)

\( \Delta y > \Delta y_o \); \( F_{by} = (\Delta y + \delta_o) \kappa_{by} + F_n - F_{tp} \text{ sign}(\dot{\Delta}_y). \)

4. SIMULATING THE EXTERNAL CONTROL ACTION

4.1. Generalized Contact and Axle Equipment Reactions

The force interaction of the wheel and track is hardly describable, and in the
same time, it is very important for studying dynamic issues of frictional interaction of the rolling stock and track, and guiding the rolling stocks with the track. In a general case, wheel-track contacting is done in two planes: on the rolling surface and flange. More simply, the contact is considered as a two-point contact. In this case the principle vector of the contact interaction of i-n wheel (i=1 – for the left wheel, i=2 – for the right wheel), j-n wheel pair, k-n truck has the following structure

$$\vec{F}_{ijk} = \vec{N}_{ijk} + \vec{N}_{ijkII} + \vec{S}_{ijk} + \vec{S}_{ijkII},$$  (4.1)

where $\vec{N}_{ijk}$, $\vec{N}_{ijkII}$, $\vec{S}_{ijk}$, $\vec{S}_{ijkII}$ – principle vectors of normal reactions and adhesion forces of the first (main) and second (flange) contacts

$$\vec{N}_{ijk} = \vec{P}_{ijkI} + \vec{H}_{ijkI}; \quad \vec{N}_{ijkII} = \vec{P}_{ijkII} + \vec{H}_{ijkII},$$

where $\vec{P}_{ijkI}$, $\vec{H}_{ijkI}$, $\vec{P}_{ijkII}$, $\vec{H}_{ijkII}$ – vertical and horizontal lateral constituents of normal contact reactions, while

$$\vec{H}_{ijkI} = \vec{P}_{ijkI} \cdot g_{ijkI}; \quad \vec{H}_{ijkII} = \vec{P}_{ijkII} \cdot g_{ijkII},$$  (4.2)

where $g_{ijkI}$, $g_{ijkII}$ = $\tan(\gamma_{ijkI})$, $\gamma_{ijkI}$, $\gamma_{ijkII}$ – conditional conicities of wheel profiles at corresponding contact points;

$\gamma_{ijkI}$, $\gamma_{ijkII}$ – inclination angles toward horizon in corresponding contact centers of wheel-rail profiles.

At static vertical loading on wheel $P_0$

$$P_0 = P_{ijkI} + P_{ijkII}. \quad (4.3)$$

Total gravitational constituent $H_{ijk}$ from two contacts is defined as a sum of two reactions $\vec{H}_{ijkI}$ i $\vec{H}_{ijkII}$, while

$$H_{ijk} = P_0[p_{ijkI}g_{ijkI} + (1 - p_{ijkI})g_{ijkII}]. \quad (4.4)$$
where \( k_{ijk} \) – the coefficient considering the vertical load distribution between contacts I and II with values within 0 ... 1. According to the linear behavior in case of the change from \( P_{ijkI} \) to \( P_{ijkII} \) with two-point contacting \( dy_1 \leq dy_{jk} \leq dy_{II} \), we obtain

\[
P_{ijkI} = P_0 k_{ijk}; \quad P_{ijkII} = P_0 (k_{ijk} - 1),
\]

(4.5)

where

\[
k_{ijk} = \frac{dy_{jk} - dy_1}{dy_{II} - dy_1};
\]

\( dy_{jk} \) – instantaneous lateral movement of the wheel profile against the rail;

\( dy_1, dy_{II} \) – lateral movements of the wheel profile against the rail at points of entering to/ exiting from the two-point contact.

Values \( dy_1 \) and \( dy_{II} \) that depend on certain profiles of the wheel/rail rolling surfaces.

Adhesion contact forces have a 3-D structure

\[
\begin{align*}
\bar{S}_{ijkI} &= \bar{S}_{xijk} + \bar{S}_{yijk} + \bar{S}_{zijk}; \\
\bar{S}_{ijkII} &= \bar{S}_{xijkII} + \bar{S}_{yijkII} + \bar{S}_{zijkII},
\end{align*}
\]

(4.6)

where \( \bar{S}_{xijk}, \bar{S}_{yijk}, \bar{S}_{zijk} \) – longitudinal, lateral, vertical constituents at corresponding contact points.

Reaction \( \bar{S}_{zijkI} \) may be ignored, since its value is associated with touching prevention, and therefore for I-st contact it is insignificant.

Values of the other constituents are determined by the following equations:

\[
\begin{align*}
S_{xijkI} &= N_{ijkl} \psi_0 K_x(\varepsilon_{xijkI}, \varepsilon_{yijkI}); \\
S_{xijkII} &= N_{ijkl} \psi_0 K_x(\varepsilon_{xijkII}, \varepsilon_{yijkII}); \\
S_{yijkI} &= N_{ijkl} \psi_0 K_y(\varepsilon_{xijkI}, \varepsilon_{yijkI}); \\
S_{yijkII} &= N_{ijkl} \psi_0 K_y(\varepsilon_{xijkII}, \varepsilon_{yijkII}); \\
S_{zijkII} &= N_{ijkl} \psi_0 K_z(\varepsilon_{zijkII}),
\end{align*}
\]

(4.7)

where: \( \varepsilon_{xijkI}, \varepsilon_{yijkI}, \varepsilon_{xijkII}, \varepsilon_{yijkII}, \varepsilon_{zijkII} \) – longitudinal, lateral, vertical constituents
of relative sliding in corresponding contacts;

\(N_{ijkI}, N_{ijkII}\) – normal reactions in I-st and II-nd contacts;

\(\psi_0\) – adhesion cutoff coefficient (physical);

\(K_x(\varepsilon_{xijk}, \varepsilon_{yijk}), K_y(\varepsilon_{xijk}, \varepsilon_{yijk}), K_z(\varepsilon_{zijk})\) – experimental adhesion features.

The adhesion features are introduced as external functions. The method and results of feature experimental studies are described in [104].

Relative sliding in contacts (longitudinal, lateral and vertical) are determined by the following formulas

\[
\varepsilon_{xijkI} = \frac{V_{sxijkI}^*}{\phi_{jk} R_{ijkI}}; \quad \varepsilon_{yijkI} = \frac{V_{syijkI}^*}{\phi_{jk} R_{ijkI}}; \quad \varepsilon_{xijkII} = \frac{V_{sxijkII}^*}{\phi_{jk} R_{ijkII}};
\]

\[
\varepsilon_{yijkII} = \frac{V_{syijkII}^*}{\phi_{jk} R_{ijkII}}; \quad \varepsilon_{zijkII} = \frac{V_{szijkII}^*}{\phi_{jk} R_{ijkII}},
\]

(4.8)

where \(\phi_{jk}\) – angular speed of wheel pair rotation.

When deriving equations for relative sliding, the following designations are adopted:

\(V_{sijkI}, V_{sijkII}\) – total speeds of sliding in contacts;

\(V_{sxijk}, V_{sxijkII}, V_{syijk}, V_{syijkII}, V_{szijkII}\) – longitudinal, lateral, vertical constituents of sliding speeds in corresponding contacts of fixed coordinate system XOY, linked to the track;

\(V_{sxijkI}^*, V_{sxijkII}^*, V_{sxijkI}, V_{syijkI}, V_{szijkI}\) – longitudinal, lateral, vertical constituents of sliding speeds in corresponding contacts of moving coordinate system \(X_jOY_j\) linked to wheels;

\(V_{jk}\) – absolute speed of wheel pair geometrical center movement.

The direction corresponding to a positive value of proper adhesion force is adopted as the positive direction of sliding speed.

\[
V_{sijkI} = V_{sxijkI}^* + V_{syijkI}^*\]
\[ V^*_{sx1jkI} = \dot{\phi}_{jk} R^*_{1jkI} - \dot{x}_{jk} - (\dot{y}_{jk} - \dot{y}_{p1jk})\psi_{jk} - A\psi_{jk}; \]

\[ V^*_{sx2jkI} = \dot{\phi}_{jk} R^*_{2jkI} - \dot{x}_{jk} - (\dot{y}_{jk} - \dot{y}_{p2jk})\psi_{jk} + A\psi_{jk}; \]

\[ V^*_{syljkI} = \dot{x}_{jk} \sin \psi_{jk} - \dot{y}_{jk} + \dot{y}_{p1jk}; \]

\[ V^*_{sy2jkI} = \dot{x}_{jk} \sin \psi_{jk} - \dot{y}_{jk} + \dot{y}_{p2jk}, \]

\[ \nabla^*_{sijkI} = \nabla^*_{sxijkI} + \nabla^*_{syljkI} + \nabla^*_{syzijkI}, \tag{4.9} \]

\[ V^*_{sx1jkII} = \dot{\phi}_{jk} R^*_{jkII} \cos \chi_{1jk} - \dot{x}_{jk} - (\dot{y}_{jk} - \dot{y}_{p1jk})\psi_{jk} - A\psi_{jk}; \]

\[ V^*_{sx2jkII} = \dot{\phi}_{jk} R^*_{2jkII} \cos \chi_{2jk} - \dot{x}_{jk} - (\dot{y}_{jk} - \dot{y}_{p2jk})\psi_{jk} + A\psi_{jk}; \]

\[ V^*_{syljkII} = \dot{\chi}_{jk} \psi_{jk} - \dot{y}_{jk} + \dot{y}_{p1jk}; \]

\[ V^*_{sy2jkII} = \dot{\chi}_{jk} \psi_{jk} - \dot{y}_{jk} + \dot{y}_{p2jk}; \]

\[ V^*_{syljkII} = \dot{\phi}_{R^*_{1jkII}} \sin \chi_{1jk}; \]

\[ V^*_{syljkII} = \dot{\phi}_{R^*_{2jkII}} \sin \chi_{2jk}, \]

where \( R_{1jkI}, R_{2jkI} \) – radii of rolling surfaces of 1-st and 2-nd wheels in I-st contacts;

\( R^*_{ijkII} \) – equivalent radii of II-nd contacts;

\( \chi_{ijkII} \) – touch precedence angles (fig.4.1.);

\( \psi_{jk} \) – wheel pair attack angle;

\( \dot{x}_{jk}, \dot{y}_{jk} \) – absolute speed projection of the wheel pair center on axis X-Y of fixed coordinate system XOY.
\[ \hat{y}_{ij} \] – speed of lateral movements of rail sections that contact with corresponding wheels.

![Diagram of contact centers and rail sections](image-url)

**Fig. 4.1** – Position of contact centers in case of two point contacting and wheel striking against the rail

The values \( R_{ijk}^* \) and \( \chi_{ijk} \) can be determined from the equations

\[
R_{ijk}^* = \sqrt{b_{ijk}^2 + (R_{ijk} - \Delta R_{\psi_{ijk}})^2};
\]

\[
tg\chi_{ijk} = \frac{b_{ijk}}{R_{ijk} - \Delta R_{\psi_{ijk}}},
\]

(4.10)

where \( R_{ijk} \) – radiiuses of wheels rolling surfaces in II contacts in the absence of an angle of attack;

\( b_{ijk}, \Delta R_{\psi_{ijk}} \) – precedence of touching and radiiuses increase due to set of wheels attack angles

\[
\Delta R_{\psi_{ijk}} = 1.235 \frac{\psi_{jk}^2}{g_{II}}; \quad b_{ijk} = \frac{\psi_{jk}}{0.15 \psi_{jk}^2 + 3.6 \cdot 10^{-4} g_{II}}.
\]

(4.11)

Normal loads on contacts can be determined according to a formulas
\[
\begin{align*}
N_{ijkl} &= \frac{P_{ijkl}}{\cos(\arctg(g_{il}))}; \quad N_{ijkl} = \frac{P_{ijkl}}{\cos(\arctg(g_{il}))}.
\end{align*}
\] (4.12)

Longitudinal and transversal journal-box reactions in general terms depend on gaps or deformations in the journal-box assemblies

\[
F_{bxijn} = F_{bx} (\Delta_{bxijn}); \quad F_{byijn} = F_{by} (\Delta_{byijn}).
\] (4.13)

Reactions in the journal-box assemblies can be described by the following interrelations:

Longitudinal reactions in the journal-boxes: \(F_{nxijn} = -F_{bxijn};\)

\[
\begin{align*}
\delta_{ox} \geq \Delta_{bxijn} \geq -\delta_{ox}; \quad F_{bxijn} &= 0; \\
\Delta_{bxijn} > \delta_{ox}; \quad F_{bxijn} &= (\Delta_{bxijn} - \delta_{ox})\mathcal{X}_{bx}; \\
\Delta_{bxijn} < -\delta_{ox}; \quad F_{bxijn} &= (\Delta_{bxijn} + \delta_{ox})\mathcal{X}_{bx};
\end{align*}
\] (4.14)

Transversal reactions in the journal-boxes: \(F_{myijn} = -F_{byijn};\)

\[
\begin{align*}
\delta_{oy} &\geq \Delta_{byijn} \geq -\delta_{oy}; \quad F_{byijn} = 0; \quad F_{myijn} = 0; \\
\Delta_{byijn} > \delta_{oy}; \quad F_{byijn} &= (\Delta_{byijn} - \delta_{oy})\mathcal{X}_{by} - F_{n}; \\
\Delta_{byijn} < -\delta_{oy}; \quad F_{byijn} &= (\Delta_{byijn} + \delta_{oy})\mathcal{X}_{by} - F_{n};
\end{align*}
\] (4.15)

\[
\Delta_{bxijn} = x_{bij} - x_{ijn}; \quad \Delta_{byijn} = y_{bij} - y_{ijn},
\]

where \(x_{bij}, y_{bij}, x_{ijn}, y_{jk}\) – longitudinal and transversal beating of the truck elements and wheels sets in the journal-box connections;

\(\delta_{ox}, \delta_{oy}\) – longitudinal and transversal one-sided gaps in the journal-box connection;

\(\mathcal{X}_{bx}, \mathcal{X}_{by}\) – longitudinal and transversal stiffness of the journal-box connection;

\(F_{bxijn}, F_{mxijn}, F_{byijn}, F_{myijn}\) – Reactions in the journal-box connection applied in the longitudinal direction to the journal-box housing, in the Reactions in the journal-boxes.
box connection direction – to the butt end of the wheel set axis and in both directions – to the truck frame;

Fₙ – Preliminary compression of the spring element blocked in the axial stop.

4.2. Analysis on the Control of Power Factors

The force factors operating in the 'carriage-track' system can be divided into guiding factors and resistance factors regarding the process of rail track guidance. The sign of the guiding factor is the positive direction of the moment that creates the considered factor in relation to a motion direction. The sign of the resistance factor is its negative value.

However, this division is conditional, because the forces or moments of the same origin may be of guides or resistance nature in different modes of motion or even in different wheel sets.

4.2.1. Structure of the Power Factors Affecting Wheel Pairs

The main vectors of transversal external forces that influence the wheel sets in the absolute coordinates system XOY

\[ F_{yjk} = \sum_{i=1}^{2}\left( S_{x_{ijkI}} + S_{x_{ijkII}} \right) \cdot \sin(\psi_{jk} - \psi_{mk}) + \sum_{i=1}^{2}\left( S_{y_{ijkI}} + S_{y_{ijkII}} \right) \cdot \cos(\psi_{jk} - \psi_{mk}) + \]

\[ + \sum_{i=1}^{2}(H_{ijkI} + H_{ijkII}) - F_{byjk}; \]

The main moments of the external forces that influence the wheel sets.

\[ M_{jk} = S_{x_{1jkI}}A_{I1} - S_{x_{2jkI}}A_{I1} + S_{x_{1jkII}}A_{II1} - S_{x_{2jkII}}A_{II2} + \left(F_{bx2jk} - F_{bxIjk}\right)B. \]

In equations (4.16)-(4.17):

\( \psi_{jk}, \psi_{jk} \) – angles of roll relative to the wheel sets and trucks in the absolute coordinates system, XOY, where \( \psi_{jk} = \psi_{rjk} + \psi'_{jk} \);
\( \psi_{jk} \) – angular position of the radial straight line that passes through the wheel set gravity center;

\( \psi'_{jk} \) – the angle of the radial straight line position that passes through the wheel set gravity center;

\( A_{ii}, A_{III} \) – the distance from wheel set center to relevant contact centers;

\( i \) – wheel number in the wheel set;

\( j \) – wheel set number;

\( k \) – truck number.

Equilibrium equation for \( N \)-axial trucks

\[
F_{nk} = \sum_{j=1}^{N} (F_{byjk} - F_{cyk}) + \frac{M_{ck}}{D_k} = 0; \tag{4.18}
\]

\[
M_{nk} = \sum_{j=1}^{N} (F_{bx1jk} - F_{bx2jk})B - \sum_{j=1}^{N} F_{byjk} C_{jk} - M_{ck} = 0.
\]

Equilibrium equation of \( L \)-truck carriage body

\[
F_c = \sum_{k=1}^{L} F_{ck} = 0; \quad M_c = \sum_{k=1}^{L} M_{ck} = 0. \tag{4.19}
\]

Equations (4.16)-(4.19) are stationary models of entering in the curves of the track sections.

The force factors considered, as it was noted, cannot be unambiguously referred to the determining factors, or the resistance factors. Specific values and directions of forces and moments depend on location of the wheel sets in the rail track.

### 4.2.2. Power Factors for Carriages Control when Interaction between Wheel Pairs and Tracks

Longitudinal adhesion forces components create a directing moment which influences the wheel set. The moment's positive value will increase in case of lateral displacement of the wheel set to the outer side of the curve. Partial longitudinal sliding at the wheels and rails contacts are caused by these displacements, depend on the
profiles of wheels rolling surfaces, wheel sets transversal displacement relative to the track axis $\delta_{jk}$ and the curve radius $-\rho$:

$$
\varepsilon_{1jkl} = 1 - \frac{R_0}{R_{1jkl}} \left( 1 + \frac{s}{\rho} \right); \quad \varepsilon_{2jkl} = 1 - \frac{R_0}{R_{2jkl}} \left( 1 - \frac{s}{\rho} \right); \quad \text{(4.20)}
$$

$$
\varepsilon_{1jkII} = 1 - \frac{R_0}{R_{1jkII}} \left( 1 + \frac{s}{\rho} \right); \quad \varepsilon_{2jkII} = 1 - \frac{R_0}{R_{2jkII}} \left( 1 - \frac{s}{\rho} \right),
$$

where $2s$ – the track width;

$R_0, R_{ijkl}, R_{ijkII}$ – consequently, the mean radius of the wheel rolling surface and radiuses in I and II contact points.

When conicity of the rolling surface is 1:20, maximum slip values that correspond to the motion in straight lines without touching a flange, reach not more than 0.1%. In the curves having radius less than 1,000 m longitudinal adhesion forces components become a resistance factor. Thus, longitudinal adhesion forces components as a force factor is a guiding one only in the straight lines and curves with radius more than 1,000 m. In the mid-size and and small curves, Longitudinal adhesion forces components are the forces of motion resistance.

Transversal adhesion forces components are determined by the transversal sliding (4.8) which depend on wheel sets attack angles on the rails $\psi_{jk}$.

Depending on the wheel sets installation, maximal, middle and minimal sliding values can be determined according to equations

$$
\varepsilon_{yjk}^{\text{max}} = \sin\left[ \frac{\pi}{2} - 2\frac{\rho\delta_{jk} + C_k^2}{C_k^2\rho + \delta_{jk}} \right];
$$

$$
\varepsilon_{yjk}^{\text{mid}} = \frac{C_k}{\rho + \delta_{jk}};
$$

$$
\varepsilon_{yjk}^{\text{min}} = \sin\left[ \frac{\pi}{2} - 2\frac{\rho\delta_{jk} - C_k^2}{C_k^2\rho - \delta_{jk}} \right].
$$

\text{(4.21)}
In fig. 4.2, dependences of maximal and minimal relative sliding of the outer wheel set $\epsilon_{yjk}$ from truck wheelbase $C_k$ and the curve radius are shown as profile maps $\rho$.

*Figure 4.2 – Dependences of maximal (a) and minimal (b) relative sliding of the outer wheel set climbing ($\epsilon_{yjk}$) on truck wheelbase ($C_k$) and the curve radius ($\rho$)*

Analysis of the given ratios indicates that Transversal adhesion forces components can play the role of guiding factors only under condition of the truck's installation with skewness and the first wheel set climbing the internal rail.

In the event of other installation options the factor under study resists entering.

The resistance moments level is characterized by high values, because the transversal sliding values are in the critical and overcritical boundaries.

Besides, the nature of the resistance moment dependence on the attack angle, that is, increase in the resistance moment when the attack angle is increased, intro-
duces certain instability to the process of entering.

This is particularly evident when using directive wheel sets mounted on the front of the truck.

Experimental data for 2TE126 diesel locomotive indicated that removal of its one-sided installation even on the straight track sections requires a greater moment in the rotary support.

Gravitational components of the normal reactions are equal to each other, though opposite directional under condition of equal effective conicities of wheels rolling surfaces of the same wheel set with respect to equations (4.3) and (4.4).

In case of transversal displacement of the wheel set prior to the flange touching the rail head, conicity difference is rather insignificant for the majority of profiles which virtually does not affect the entering process.

In case of the flange contacting the guiding role of the gravitation components can be rather large depending on normal loads redistribution between I and II contacts of the two-point wheel and rail contact.

For instance, the flange gravitational component in case of the wheel vertical loading of 115 kN can reach $\gamma = 60^\circ - 196$ kN for the flange cone, while for $\gamma = 70^\circ - 325$ kN.

In fig. 4.3 dependences of gravitational adhesion forces components on the truck type TE116 position in the rail track are shown.

Reverse moment in the body connection with the trucks is an internal moment, easily creating the motion resistance indirectly due to the above force factors.

it is customary to assume that in the track's curve segments the Reverse moment should be minimal, or even negative which would enable us to improve the guidance characteristics due to forcible trucks installation relative to the track axis.

In fig. 4.4. the profile maps of the main vector and main moment dependence $\vec{F}$ $\vec{M}$ of the external forces on the trucks installation.
Figure 4.3 – Dependences of the normal loads gravitational components in wheels contacts with rails for TE116 truck on its installation in the curve with 900 m radius.

In fig. 4.5 dependence on the trucks equilibrium position on the curve radius and limitations as to the derailing is shown in the form of the blocking circuit.

Calculations indicate that the trucks are installed with negative skewness for any curve radius. Kinematic entering is possible in the curves with radius exceeding 900 m. If the curves radiiuses are within the range of 900–470 m one-sided flange contact of the first wheel set occurs with the external rail. In the curves with radius less than 470 m, two-sided flange contact always occurs: of the wheel set – the external rail, the third wheel set – the internal rail. Longitudinal and transversal sliding of both, the leading and free wheel sets, cause the motion resistance and are surpassed by the guiding efforts.
Figure 4.4 – Dependence of the main vector (a) and the main moment (b) of lateral contact forces on the 2TE116 diesel locomotive truck installation in the curve with 900 m radius.

Figure 4.5 – Blocking circuit of 2TE116 diesel locomotive truck installation in the different radius curves: 1 – 4,000 m; 2 – 2,000 m; 3 – 900 m; 4 – 700 m; 5 – 500 m; 6 – 400 m; 7 – 250 m
5. MODELING THE KINEMATIC RESISTANCE TO THE MOVEMENT

This section provides basic provisions of research methods for rail carriages kinematic motion resistance, based on the four-square torque loops theory. The theory explains some phenomena associated with the kinematics and dynamics of carriages frictional interaction in the track, guidance of the wheel sets with rail track and motion resistance, of both the individual wheel set and a group of wheel sets interconnect with a common drive.

5.1. The Nature of the Resistance to the Movement Associated with the Control of the Wheel Pairs by the Railway Track

For decades, the motion resistance characteristics have not been analyzed at the stage of new rolling stock types design. Locomotives and cars rolling stock perfection often resulted in worsening in the motion resistance characteristics. Increase in the rolling wheels surfaces wear when operating new types of multi-engine locomotives is indirect confirmation of this data. The flanges intense cutting and lateral wear of the rails heads which are the result of the motion resistance influence deserve special analysis.

The nature of motion associated with carriages guidance with the track. Insufficient exploration maturity of the motion resistance associated with the carriages guidance with the track, kinematic motion resistance is one of the reasons for increased wear of wheels and rails. Four-square torque loops in the system of rail carriages guidance are the source of kinematic motion resistance. The kinematic motion resistance emerges due to parasitic sliding in the assemblies of four-square torque loops. With reference to the theory of four-square torque loops, the wheels contacts with rails are nothing but decoupling node points of the frictional type.

The carriage kinematic motion resistance is caused by kinematic non-conformity of geometrical parameters of wheels rolling surfaces and kinematic motion parameters that causes parasitic sliding.

Kinematic resistance has two components:
- differential (w_d)
- circulatory (w_u).

The first one manifests itself when each specific wheel contacts with the rail
through the contact spatial geometry.

The other one is a result of the wheels, or wheel sets group interaction with the rail track during a guided motion in the rail track due to parasitic losses circulation within the limits of one wheel set, or a group of wheel sets interconnect with the truck frame. The value of this resistance depends on the distribution of relative slidings speeds in the wheels contacts with rails, which are influenced by both, the carriage structural parameters, and its motion mode. Energy losses due to surpassing frictional effects, associated with these differential and circulatory sliding, determine the kinematic motion resistance.

As it is demonstrated in work [75], the flange contact of the wheel and rail is characterized by emergence of additional differential sliding which leads to the increase in the rolling resistance. The picture of differential sliding depends on the wheel attack angle to the rail. When the wheel set is installed vertical to the track axis, the both wheel contact points in case of the two-point contact, are located in the axial plane of the wheel set, and in case the attack angle emerges, the flange contact is displaced forward by a value of the touching prevention $b$ and upwards by the value $\Delta R_\phi$ [75].

The resistance nature under study is associated with parasitic sliding in four-square torque loops that are created in the wheel set guidance system with the rail track. The research is based on the energy hypothesis according to which the mechanical energy of surpassing parasitic sliding frictions is the motion resistance energy.

It is suggested that the motion resistance component should be named which is associated with wheel sets guidance with the rail track, kinematic motion resistance. According to the accepted classification, the kinematic motion resistance has signs of the basic and additional resistance. That is why, in case of motion along the track straight segments, it should be viewed as a part of the basic one, whereas in case of the motion along the curve segments, it should be viewed as an additional motion resistance.

In the system of rail carriages trucks guidance by the rail track several four-square torque loops can be distinguished.

A four-square torque loop is created with two nod points in the basic and flange contacts in case of two-point wheel contact with the rail. Differential sliding emerges in this loop which can be the reason for the additional kinematic resistance to motion through the increase in the rolling resistance.
Fig. 5.1 shows an example diagram of the possible normal reactions distribution \((N_1, N_2)\) and adhesion forces \((S_1, S_2)\) in the two-points flange contact of the wheel and rail.

\(F_t\) – an external longitudinal reaction of the truck to the wheel. \(F_t\) reaction is motion resistance force which should be surpassed to ensure the wheel rolling.

Distribution between the normal reactions \(N_1, N_2\) depends on many factors:

- motion speed
- the curve radius
- the external rail raising
- the wheel set location in the track
- the truck design
- profile of the wheel rolling surface, etc.

Based on fig. 5.1 one can put down the following system of the equilibrium equations:

\[
\begin{align*}
\sum M &= S_1R_1 - S_2R_2 = 0; \\
\sum F &= S_2 - S_1 + F_1 = 0.
\end{align*}
\]  
(5.1)

Based on equations (5.1) the value of kinematic resistance to motion \(W_k\) can be obtained.

\[
W_k = F_t = S_1 \left(1 - \frac{R_1}{R_2}\right)
\]

Based on (5.2) an important conclusion can be drawn: the motion resistance force \(W_k = F_t\) in any case, cannot be equal to zero, in case there is the wheel flange contact with the rail.

In different conditions, the difference between radiuses of the main and the flange contacts reaches 15-30 mm. That is why, kinematic motion resistance can reach up to 6% of the adhesion force \(S_1\).

Wheel sets together with the rail track also create four-square torque loops. Absence of the longitudinal sliding in the wheels contacts with rails is only possible in the ideal case. An ideal case is the case, when a single wheel set is rolling freely without the flange touching the rails. In real conditions of the truck wheel set motion,
sliding is regularly emerges in the wheel sets contacts with rails. This sliding is parasitic and create an additional kinematic motion resistance in both, straight and curved track segments.

*Figure 5.1* is a diagram of the normal reactions distribution \((N_1, N)\) and adhesion forces \((S_1, S_2)\) in the two-points flange contact of the wheel and rail.

\(K_1, K_2\) – consequently the basic and flange contact; \(R_1, R_2\) – rolling radiiuses in the basic and the flange contact.

Fig. 5.2 shows a simplified diagram of the adhesion forces \(S_{k1}, S_{k2}\) and the journal-box reactions \(F_{b1}, F_{b2}\), which affect the wheel set. A case is being modeled when the wheel set is installed with transversal displacement \(\Delta y\) relative to the track axis and is forcibly rolling in the direct line on the rails. In this case, truck hunting is restrained with journal-box reactions \(F_{b1}, F_{b2}\).

Using fig. 5.2 the following Equilibrium equations can be composed

\[
\begin{align*}
(S_{k1} + S_{k2})A - (F_{b1} + F_{b2})B &= 0; \\
S_{k1}R_1 - S_{k2}R_2 &= 0; \\
F_{b1} - F_{b2} - S_{k1} + S_{k2} &= 0.
\end{align*}
\]

(5.3)

The following value of kinematic motion resistance can be obtained from equations (5.3)

\[
W_k = S_{b1} - S_{b2} = S_{k1} \left(1 - \frac{R_1}{R_2}\right)
\]
Equations (5.2) and (5.4) reveal the nature of the kinematic motion resistance emergence. The kinematic motion resistance emerges when the wheel set is guided with the rail track due to parasitic sliding in four-square torque loops assemblies. Such nods are the wheels and rails contacts.

5.2. Mathematical Modeling for Carriages Control by Rails in the Curve

5.2.1. Calculation Scheme of Truck Guiding into the Curve of the Track

The car two-axle truck steady motion is under consideration in the circular curve at the absence of an external influence except for the rail track.

Fig. 5.3 shows the computational model of the car two-axle truck entering provided the installation is free in the track.

Fig. 5.3 shows that the following designations were used:

δ – full gap of the wheel set in the rail track;

δ_{jk} – the gaps in the flange contacts of the relevant wheels (j=1, 2) of the relevant wheel sets (k=1, 2);

α – an angle between the truck transversal symmetry axis and the radius of the curve that passes through the truck’s coupling bolt;
ψ\(_k\) – attack angles of the relevant wheel sets to the rails (k=1, 2);  
2C – truck wheelbase;  
ρ, ρ\(_1\), ρ\(_2\) – radiuses of the curve symmetry line, as well as internal and external rails;  
G – the point of the truck turning relative to vertical axis – the truck's coupling bolt;  
V – direction of the truck motion, vertical to the radial straight line that passes through the truck's coupling bolt;  
OXY – absolute coordinate system;  
O\(_k\)X\(_k\)Y\(_k\) – coordinate system associated with relevant wheel sets.

Figure 5.3 – The computational model of the car double-bogie truck entering provided the installation is free in the track.

Axes y\(_k\) directions coincide with radial straight lines passing through the centers of the relevant wheel sets;  
The first wheel set is a directive one and moves flange pressed to the external rail, while the other is in free position. The actual position of the other wheel set is determined during calculation.  
Fig. 5.4 provides a diagram of the contact forces for wheels of the wheel sets:  
a – vertical cross sections in the contact planes;  
b – the contact forces horizontal projections
Figure 5.4 – The contact forces diagram for wheel sets wheels

a – vertical cross sections in the contact planes; b – the contact forces horizontal projections
The following designations were used:

\[ K_{ijk} \] – designation for relevant contacts of the wheels with rails: \( i \) – the contact number as per type (main contacts \( i = 1 \), the flange contacts \( i = 2 \)); \( j \) – designation of contacts by the wheel number (left wheels \( j = 1 \), right wheels \( j = 2 \)); \( k \) – contacts designation by the wheel set number (the first wheel set \( k = 1 \), the second wheel set \( k = 2 \));

\[ N_{ijk} \] – normal loads on contacts;
\[ P_{ijk} \] – vertical components of the normal loads on contacts;
\[ H_{ijk} \] – horizontal transversal components of the normal loads on contacts in the coordinate systems \( OkYkZk \);
\[ S_{ijk}, S_{xijk}, S_{yijk} \] – the adhesion force and their longitudinal and transversal components in the relevant contacts in the coordinate systems \( O_kX_kY_k \);
\[ F_x \] – longitudinal external force that affects the truck from the body side and models tractive power necessary to surpass the additional motion in the curve. \( F_x \) force applied to the truck's coupling bolt and vertical to the radius of the curve that passes through the coupling bolt;
\[ F_y \] – the transverse force that influences the truck from the side of the body and models unsuppressed centripetal inertia force.

\( F_y \) force applied to the truck's coupling bolt and directed along the curve radius that passes through the coupling bolt.

In order to study the truck and track parameters influence on the motion resistance associated with the truck guidance with the rail track, the principle of quasi-static dynamics was used. The carriage is considered during the motion in the circular curve being affected by the contact track forces (including the motion resistance forces), the locomotive tractive power and unsuppressed inertia force which is due to the circular motion.

5.2.2. The System of Equations for Truck Equilibrium when Guided into the Curve

The main vectors of the external force influence on the wheel sets, namely the forces main vector \( (\vec{F}) \) and main vector of the horizontal forces moments \( (\vec{M}_c) \), are equal to zero.
\[ \vec{F} = \vec{F}_k + \vec{F}_x + \vec{F}_y; \quad \vec{M}_G = 0, \]  

(5.5)

Where \( \vec{F}_k \) – the main vector of horizontal combined forces in the contacts of wheels with rails.

Based on fig. 5.3, 5.4, a chain of relations can be put down.

\[ \delta_{11} + \delta_{21} = \delta_{12} + \delta_{22} = \delta; \]

\[ \alpha = \cos(\tau) - \sin\left(\frac{C}{\rho_2}\right) + \sin\left(\frac{\rho_2 \sqrt{1 - d^2}}{2C}\right) \]

\[ \psi_1 = \frac{\pi}{2} - \sin\left(\frac{\rho_2 - \delta_{22}}{2C}\sqrt{1 - d^2}\right) \]

\[ \psi_2 = \frac{\pi}{2} - \sin\left(\frac{\rho_2 \sqrt{1 - d^2}}{2C}\right) \]

In equations (5.6):

\[ d = \frac{2\rho_2 (\rho_2 - \delta_{22}) - 4C^2 + \delta_{22}^2}{2\rho_2 (\rho_2 - \delta_{22})} \]

The adhesion forces in contacts \( K_{ijk} \) were calculated after the procedure provided in [75].

The main vector of horizontal combined forces in the contacts of wheels with rails.

\[ \vec{F}_k = \sum_{i,j,k=1}^{2} (\vec{H}_{ijk} + \vec{S}_{xijk} + \vec{S}_{yijk}). \]

(5.8)

The main vector of horizontal forces moments relative to the vertical axis that passes through the truck's coupling bolt (point G):

\[ \vec{M}_G = \sum_{i,j,k=1}^{2} (\vec{H}_{ijk} l_{Hijk} + \vec{S}_{xijk} l_{xijk} + \vec{S}_{yijk} l_{yijk}). \]

(5.9)
where \( l_{Hijk}, l_{Sxijk}, l_{Syijk} \) – arms of relevant forces for moments calculation relative to G point.

Based on (5.5)–(5.9) one can compose the system of equilibrium equations.

\[
\begin{align*}
\sum_{i,j,k=1}^{2} \left( \vec{H}_{ijk} + \vec{S}_{xijk} + \vec{S}_{yijk} \right) &= 0; \\
\sum_{i,j,k=1}^{2} \left( l_{Hijk} \vec{H}_{ijk} + l_{Sxijk} \vec{S}_{xijk} + l_{Syijk} \vec{S}_{yijk} \right) &= 0.
\end{align*}
\]

5.2.3. Calculation Results of the Resistance to the Movement Associated with the Control of Wheel Pairs by Rail Tracks: A Study on the Carriages of 18-100 Type

The solution of equation system (5.5) resulted in obtaining the dependence of the additional motion resistance associated with a car guidance with the rail track in the curve.

The following was accepted as the input calculation parameters: the motion speed, the curve radius, the external rail raising level, the wheel set gap in the track, the car load.

The calculations output data were dependences of the motion resistance on the above parameters.

Fig. 5.5 shows calculation dependences of the specific motion resistance in the curve \( \omega_r \) on the motion speed \( V \), the curve radius \( \rho \) and raising of the external rail \( h \). Fig. 5.6 shows in the form of lines calculation dependences of the specific motion resistance in the curve \( \omega_r \) on the truck wheelbase \( 2C \) and the curve radius \( \rho \) for the motion speed fixed values \( V=30 \text{ m/s} \), relative raising of the external rail \( h=120 \text{ mm} \) and two options of the gap values for the wheel set in the rail track – \( \delta=20 \text{ mm} \) and \( 40 \text{ mm} \).

Dependences represented in fig. 5.6 demonstrate the carriages motion resistance dependence on the truck wheelbase and the wheel sets gap in the rail track. The results obtained can be used as justification for the selection of the trucks rational parameters for their designing. These results can also be used when selecting permissible parameters of the rails lateral displacement.
Figure 5.5 Calculation dependences of the specific motion resistance in the curve $\omega_r$ (N/kN) on the motion speed $V$ (m/s), the curve radius $\rho$ (m) and raising of the external rail $h$ (mm): a – $h=50$ mm; b – $h=80$ mm; c – $h=120$ mm; d – $h=150$ mm
Figure 5.6 – Calculation dependences of the specific motion resistance in the curve $\omega_r$ (N/kN) on truck wheelbase and the curve $\rho$ radius, m for the two options of the wheel set gap in the rail track: a – $\delta=20$ mm; b – $\delta=40$ mm (motion speed $V=30$ m/s; the external rail raising $h=120$ mm)

Mathematical modelling of the carriage guidance with the track in the curve based on quasi-static dynamics principles. The carriage steady motion is being modeled in the circular curve with permanent speed influenced by the track forces, the locomotive tractive power and unsuppressed inertia force. The contact forces system is given a special detailed consideration. The adhesion forces modeling is performed based on procedure set forth in [75]. The truck remains in a loose placement in the circle. Its actual position is determined by the wheel sets gaps in the track. The gaps values are defined during solution of equilibrium equations system. the kinematic motion resistance is defined as a force that should be applied to the truck's coupling bolt for the external forces counterbalancing.

Mathematical model and the motion resistance assessment procedure allow obtaining the calculation dependences of the motion resistance on the following parameters:

- motion speed;
- track parameters: raising the external rail, the track radius, deviation of the track width influencing the wheel sets gap in the track;
- the truck parameters: truck wheelbase, diameter and profile of wheels,
disturbance of the wheel sets position in the truck frame.

The calculation results confirmed the presence of particularly evident minimal dependence of the motion resistance on speed (fig. 5.5). Though, this minimum does not correspond to the equilibrial speed in the curve. The minimal speed is on average by 15–20 % less than the design equilibrial.

One more important result has been obtained: the trucks with wheelbase from 2.5 to 6.0 m (fig. 5.6) have the least motion resistance. Thus, the truck wheelbase parameters 18-100 are not the best ones with reference to the motion resistance.

The conclusion that an increase in the gap of wheel sets in the track from 20 to 40 mm increases the motion resistance in the curve by up to 25% is of particular interest (fig. 5.6).

The elucidation of the motion resistance nature associated with the rolling stock guidance by the rail track offers certain challenges for reducing the kinematic motion resistance due to the design parameters of the carriages and the track. The kinematic motion resistance can be an additional criterion for the optimal choice of the carriages mechanical part characteristics. The same refers to the permissible deviations of the rolling stock and track parameters.

As the analysis of publications shows, the vast majority of research on the to rolling stock motion resistance have been experimental. The purpose of experiments was to obtain formulas for traction calculations. By their very nature, they were passive, stating consistent patterns associated with motion resistance. The results presented are active since they confirm the possibility of influencing of the rail carriages motion resistance through the structural parameters of the truck arrangement and the track.

The obtained results show that the component of the motion resistance associated with the track guidance acquires a significant value only in curves with a radius less than 350 m. Because of this, the study results can produce the maximum effect only on the railways with the presence of small radius curves. Also, the studies will be useful for the development and modernization of the urban rail transport rolling stock.

The research is an attempt to confirm the exploitability of reducing the motion resistance based on the analysis of its being impacted by the carriages design parameters. Research can develop, at least, in three directions. The first one is associated with the choice of the carriages optimal design parameters based on traditional
schemes. The second one is the research of truck arrangement perspective designs, on the basis of so-called, wheel sets controlled motion. The third one is the study of the admissible deviations of rolling stock and track parameters effect on the motion resistance. This is, first of all, the distortions and wheel sets climbing in the trucks, the deviation of the wheel sets wheels diameters, etc.

The first two of the said directions are related to the need for changes, sometimes essential, in the design of wheel sets and trucks. This can be quite problematic with regard to the modernization economic efficiency. More promising is the third direction, which is limited to the technological requirements for the rolling stock maintenance.

5.3. Method to Calculate the Kinematic Resistance to the Movement

The transversal displacement of the wheel set relative to the track axis determines the radiiuses of the wheels rolling surfaces. The difference in wheel radiiuses, in its own turn, determines the instantaneous radius of rotation, at which the wheel set can be rolled without slipping in the contacts of the wheels with rails. Such wheel set motion will be called the kinematic movement along the so-called equilibrium trajectory.

However, due to the interaction between the wheel sets through the truck frame, the actual trajectory of the wheel sets rolling differs from the equilibrium trajectory. A rather tight connection between the wheels leads to the circulation of power in the contours of 'rail track – wheel set', the redistribution of the power flow between the wheels and, as a consequence, increase in the motion resistance. In the case of locomotives, this also leads to a deterioration of cohesive properties. In the case of the wheel sets group operation, the power circuit has several branching circuits. The power circuit power circulation is absorbed, mainly in the contacts of the wheels with rails, and partly in the trucks dissipative connections. The uneven distribution of the power flux between the wheels depends on several factors:

- the rigidity of the cohesion characteristics;
- rigidity of the axle, i.e. parameters of the wheels connection;
- geometric characteristics of the wheel set, including conicity and diameter of the rolling surfaces, the track width and the truck wheelbase;
parameters of longitudinal and transversal journal-box connections of the wheel set with the truck frame;
• radius of the track curve segment.

As it was indicated, the circulatory resistance is the result of the power flows circulation of four-square torque loops. Contacts of wheels with rails act as decoupling node points. The level of circulating power is limited by the limit values of the cohesive forces in the contacts, due to the full or pseudo-slipping. Therefore, rolling stock motion resistance under unsatisfactory conditions the wheel-rail adhesion is lower than under good ones.

5.3.1. Reduction of Forces and the Moments of the Movement Resistance

The calculation of the kinematic motion resistance is based on the normalization of the horizontal contact reactions, or their moments, to the gravity carriage center. The normalization method employs the provision on the equality of the sum of works performed by each of the considered contact forces on possible movements, and the work of the total resistance force.

\[ A_W = \sum_{i=1}^{N} A_{F_i} \]

where \( A_{F_i} \) – i work of the force that caused the possible relocation;
\( A_W \) – work of the normalized force of kinematic motion resistance;
\( N \) – the number of normalized forces.

The current value of the normalized relative motion resistance can be determined according to formula

\[ w_k = \frac{\sum_{i=1}^{N} F_i V_i \cos(\bar{F}_i, \bar{V}_i)}{V_C Q} \]  \hspace{1cm} (5.12)

where \( F_i \) – contact forces;
\( V_i \) – absolute speed of relevant \( F_i \) force application point;
\( V_C \) – absolute speed of the carriage gravity center motion;
\( Q \) – the carriage weight.

Taking into account the contact forces structure (4.1) and using expressions (4.7)-(4.9), we obtain expressions for the equation numerator (5.12), which is a sum of the contact forces powers.

\[
F_i V_i \cos(\vec{F}_i, \vec{V}_i) = \sum_{i=1}^{2} \left[ S_{xijkl} V_{xijkl}^* + S_{xijkl} V_{xijkl}^* + (S_{yijkl} + S_{yijkl} + H_{ijkl})(V_{yijkl}^* - \ddot{y}_{ijkl}) + S_{zijkl} V_{zijkl}^* \right].
\]

(5.13)

5.3.2. Impact of Carriage Truck Parameters on the Kinematic Movement Resistance

The study of serial locomotives and cars allowed to reveal the main factors influencing the kinematic motion resistance: truck wheelbase, the profiles of the rolling wheels surfaces, the stiffness in the wheel sets spring connections with the truck frame and the gaps in the journal-box assemblies.

In this section, the influence of these factors on the motion resistance in curved segments of the track is investigated on the example of a four-axle two-truck carriage.

Fig. 5.7 shows dependences of the angles of the directive wheel set striking against the rail on the truck wheelbase and the curve radius for the gap in the rail-track \( \delta \) = of 30 and 40 mm. As it is evident from the results obtained, when the gap in the rail track exceeds 30 mm and the truck wheelbase is less than 3 m, the striking angles reach significant values (up to 0.025 rad.), and their dependence on the curve radius is insignificant.

Fig. 5.8 shows profile maps of the motion kinematic resistance equal level depending on to these values: truck wheelbase, the curve radius and the gap in the rail track. The dashed lines show the minimum levels of kinematic motion resistance. The smallest values of the kinematic motion resistance correspond to the truck wheelbase 2.5 ... 8.0 m. Increasing the gap in the track from 20 to 80 mm leads to an increase in resistance in the curve by 25%.
Studies of locomotive and carriages with different wheel profiles on mathematical models have shown a significant influence of the rolling surfaces shape on kinematics and the dynamics of frictional interaction between carriages and the track.

Figure 5.7 – Dependences of the directive wheel set attack angles against the rail on the truck wheelbase and the curve radius: \( a - \delta = 30 \text{ mm}; b - \delta = 40 \text{ mm} \)

Figure 5.8 – Dependence of the kinematic motion resistance on the truck wheelbase (C), the curve radius (\( \rho \)) and the gap in the rail track (\( \delta \)): \( a - \delta = 30 \text{ mm}; b - \delta = 40 \text{ mm} \)

Fig. 5.9 shows a histogram of the kinematic motion resistance calculation results of 2 TE116 carriage with different wheel profiles for a speed of 30 m/s in a curve of
1000 m with the external rail elevation by 120 mm.

It is difficult to give an unambiguous answer to the question of what exactly has a decisive influence on the indicators of this interaction - the shape of the rolling wheel surface or the shape of the flange. However, we can say for sure that the higher values of the kinematic motion resistance are characteristic for profiles with a large angle of cone.

![Figure 5.9](image)

*Figure 5.9* - The value of the kinematic motion resistance of the TE116 (N/kN) carriage with different wheel profiles for a speed of 30 m/s in a curve of 1000 m with an elevation of the outer rail by 120 mm I - Hayman-Lotter; II - the stand. Germany until 1953; III - DBII (Germany); IV - type I (Germany); V - type II (Germany); VI, VII - type R2, R6 (England); VIII - stand. (Japan); IX, X - stand. and stud. of type Uni-Point (USA); XI - type Sc (France); XII - stand. car (Ukraine).

For example, profiles with flange cone of 70° are characterized by higher values of resistance - 1.30 ... 1.46 N/kN, while for profiles with flange cone 60° lower – 0.85 ... 1.08N/kN. Influence of the longitudinal coupling stiffness of wheel sets with the truck frame on the motion resistance indicators motion was investigated in the range of longitudinal stiffness of the axlebox links $\zeta_{bx} = 1.00 ... 20.0$ kN/mm.

Figure 5.10 shows the dependence of the kinematic motion resistance on the curve radius (200 ... 1000 m) and the velocities of the motion with the stiffness $\zeta_{bx} = 1, 5, 10$ and 20 kN/mm depending on the radius of the curves in the form of iso diagrams. The calculations results also indicate that for speeds higher than 15 m/s, increase in stiffness reaching more than 5 kN/mm virtually does not affect the parameters of the motion resistance, especially in small radius curves.

Within the limits of the change in the longitudinal stiffness of the axlebox links
from 5 to 1 kN/mm, the resistance increase is observed in 1.5 ... 1.7 times. The minimal values of motion resistance occur at combinations of velocities and the curve radiiuses which are close to the equilibrium motion mode:

$$\rho = \frac{163}{h} \cdot V^2, \text{ or when } h = 120 \text{ mm}: \quad \rho = 1.36 \cdot V^2.$$  \hspace{1cm} (5.14)

**Figure 5.10** - Dependence of the kinematic motion resistance of the TE116 carriage on the longitudinal stiffness of the axlebox links ($\mathcal{K}_{bx}$) and the curve radius ($\rho$):

- $a$ – $V = 5$ m/s;
- $b$ – $V = 50$ m/s

The sharp drop in the motion resistance in the region of the equilibrium curve is due to the guiding wheel sets striking angles close to the minimum absolute values. Curve 2 ($\rho = 0.675 \cdot V^2$) in Fig. 5.14 corresponds to a combination of velocity and radius of the curve for the maximum allowable comfort rate of the unsuppressed centripetal acceleration $a_0 = 0.75 \text{ m/s}^2$. However, it is difficult to indicate the optimal value of the longitudinal connection stiffness in the journal-box assemblies. The kinematic motion resistance in coordinates ($V$) is significantly influenced by the motion speed. The value of $\mathcal{K}_{bx} = 4.2 \ldots 7.5 \text{ kN/mm}$ could be chosen for equilibrium motion mode, as the optimum value. For the comfort zone, this value is $\mathcal{K}_{bx} = 3.6 \ldots 6.8 \text{ kN/mm}$. For the motion with speeds higher than the comfort speed – $\mathcal{K}_{bx} = 8.0 \ldots 12.0 \text{ kN/mm}$.  

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6. WAYS TO IMPROVE CONTROLABILITY AND REDUCE THE RESISTANCE TO THE MOVEMENT OF RAIL CARRIAGES

The analysis of the carriages motion resistance structure indicates its dependence on the factors, whose significant part is difficult to be changed currently.

First of all, it refers to the state of the rail track, which requires large capital costs for maintenance and repair. Also, it is not possible to significantly reduce the motion resistance due to parameters of the wheel with the rail contact. On the contrary, with the current tendency of increasing the wheels diameter with simultaneous increase in axial loads, an increase in the resistance of the rolling motion should be expected.

For the high-speed rolling stock the aerodynamic resistance becomes decisive, with great potential for its reduction.

This is confirmed, in particular, by the fact that up to 90% of publications dedicated to motion resistance over the past 40 years, were from the area of the rolling stock aerodynamics study. Unfortunately, for Ukrainian railways, especially for urban railroad transport this area is not yet relevant.

A large reserve for reducing the motion resistance by replacing the journal plain friction bearings with roller bearings in the rolling stock, has already been used and their further research in terms of motion resistance is not promising.

Investigation of the rail carriages motion resistance components allows us to make inference on certain prospects availability for reducing it, due to the reduction in the kinematic resistance caused by the group interaction of the wheel sets dynamic system with a rail track in the course of the carriages track guidance. However, with the framework of this area, comprehensive research is required on the frictional interaction of carriages and track and the search on their basis of new technical solutions for the construction truck arrangement that allow to control the kinematics and dynamics of frictional interaction of wheels with rails.

This, above all, relates to the investigation of horizontal forces in the contacts of wheels with rails, which arise when the carriages are guided by rail track.

The development of rolling stock truck arrangement designs of the railroad transport is associated with two main problems.

The first one, is the stability and level of horizontal impact on the track on the part of high-speed motion.

The second one, is the resistance to the urban rail transport entering small radius
curved tracks. Both problems are combined with a common problem of transversal controllability of carriages.

With the frame design of the carriage part, through the parallelism of all wheel sets axles and relatively small gaps in the rail track, the possibility of passing the small radius curves was very limited. The transition to the carriage design was the designers first step to implement the principle of carriages guided motion. The radius of curves for truck carriages, as a rule, is 6 ... 8 times smaller than for the frame ones.

6.1. Linked Carriages Design Development

Trucks articulation is employed on many serial rail carriages. At first, it was a forced technical solution, because the longitudinal efforts were transmitted through drawbars located on the trucks frames, for example, on electric locomotives VL8, VL22, VL23, and others. The disadvantage of rigid longitudinal intertruck connections is increasing the moment of inertia when the group of connected trucks turns in the plan which, consequently, increases the lateral impact on the track. However, along with the disadvantages, the articulated trucks have a number of advantages associated with improving the curved tracks passage. In this connection, intertruck connections, often spring ones, have started to be used in recent years, the intertruck connections transmitting horizontal transversal forces from one truck to another.

There are many design types of spring intertruck connections, but the majority of them have one- or two-spring devices with horizontally located springs.

For example, in a K series electric locomotives (*Siemens-Schuckertwerke and Krupp*) (Fig. 6.1), the spring unit is connected to a single truck, and its rod, which passes between the springs, and the body of the ball insert are connected to another truck. Depending on the direction of the trucks relative motion, the right or left springs are compressed, transmitting the reciprocal horizontal forces from one truck to the other.

Electric locomotives VL80 are equipped with one-spring device with the same operating principle. Spring intertruck connections are used in electric locomotives ChS2, Re 2/2, and others.
It is believed that the spring intertruck connection significantly reduces the guiding efforts in the small radius curves of both the front and rear trucks, although when driving in curves with a radius exceeding 600 m and in straight sections of the track it is ineffective, and sometimes harmful. Therefore, in a spring connection, a transversal gap is usually made, which excludes the trucks interaction when their small mutual deviations are present.

### 6.2. Carriages with Radial Installation of Wheel Pairs

The advantage of traditional wheel sets is their ability to self-adjust with reference to the track axis and ability to achieve certain motion speeds without flanges touching the rails. One of the conditions for wheel set pure rolling in the curve, i.e. its motion without wheels slipping the relative to the rails, is its radial installation in the track.

*Figure 6.2 - Schemes of carriages of the traditional design (*a*) and those with radial wheel sets (*b*) [146].*
According to the generally accepted view, trucks with wheel sets, installed radially in curves, have a number of advantages over carriages with conventional 'rigid' wheel sets. The latter loading the rails to a much greater extent.

Many theoretical and experimental studies have proved that the radial installation of wheel sets can significantly reduce the parasitic slipping in the contacts, the load on the flange contacts and, consequently, reduce the flanges wear and motion resistance.

Wheel pairs of traditional trucks do not have a guiding mechanism for the axis rotation in the plane. Contrary to popular belief, in the ideal case, the wheels flanges should not touch the rails. The flanges should be a preventive measure against the wheels derailing. The wheel sets, as a rule, have a fairly rigid connection with the trucks frames with respect to the rotation in the horizontal plane.

The idea of using wheels rotating in the plane and their installation in the truck along the axis radiuses of the rail track came to the railroad, of course, from the automotive industry. The possibility to control the motion trajectory within the gap in the track and move in any modes without touching the flanges and rails, turned out to be very attractive. This would make it possible to significantly reduce lateral slip and wear in the contacts and reduce the motion resistance.

Radial installation of the wheel sets in the truck frame reduces the contact forces in the flange contacts of the wheels with rails. As a consequence, there is a reduction in the wear of the wheels and rails, reduction in the motion resistance and improvement the carriages guidance in the curved segments of the track.

6.2.1. *HTCR-II Trucks Locomotives Developed by Electro-Motive Diesel*

Three-axle locomotive trucks HTCR-II Trucks developed by the Electro-Motive Diesel company (Figure 6.3) can be considered as one of the modern developments in the rolling stock with the trucks controlled guidance by rail track. The letter R in the truck index indicates a modification with the radial installation of the wheel sets in the curve.
Figure 6.3 - The three axle locomotive truck HTCR-II Trucks (Electro-Motive Diesel company) with the wheel sets radial installation.

6.2.2. Truck of 2TE25k Locomotive Being Studied

2TE25κ Locomotive pilot truck is another example of the truck with wheel sets radial installation – 2TE25κ Locomotive pilot truck developed by VNDKTI (The All-Russian Scientific-Research and Design-Technological Institute) and built at BMZ (Bryansk Machine-Building Plant) (fig. 6.4).

Figure 6.4 – 2TE25κ Locomotive truck design with controlled position of the wheel sets in curves.
The radial installation mechanism consists of: locking studs 1 and 7 of the outermost journal boxes are connected with transverse equalizers 2 and 6. Rods 3 and 5 are hinged to the equalizers ends, the rods are connected with vertical two-arm lever 4 with their other ends. The lever upper end extension is connected to the hydraulic shock absorber 8. The trucks with the wheel sets radial installation are characterized by a lesser motion resistance and a reduced flanges wear in the curved track segments.

Nevertheless, despite all these advantages, the trucks with wheel sets radial installation have not been used extensively due to the complexity of the mechanism controlling the wheel sets position, and consequently, their high cost.

6.2.3. Wheel Blocks of Talgo High-Speed Spanish Electric Trains

Wheel blocks of high-speed Spanish trains Talgo (Fig. 6.5) can be considered the most progressive implementation of the wheel sets radial installation principle. The application of body tilting systems along with the wheel sets radial installation in Talgo trains, allows increasing the train speed in the curved sections of the track.

This reduces the negative impact of centrifugal unsuppressed acceleration on the passenger travel comfort level.

In this case, it is particularly important that the body tilt occurs automatically under the action of gravity and centrifugal force and does not require the use of a complicated servo drive with electronic control, onboard gyroscopes, as it takes place in systems of rolling stock's body forcible tilt.

The truck arrangement of Talgo passenger cars is constructed without the use of the classic car truck and wheel set. This technical solution is based on the Talgo-designed wheel block. The rigid steel frame of the wheel block is made in the form of a yoke in which the wheel
assemblies are fixed. In this case, the wheels, not connected to each other with a rigid axle, can rotate with a different angular speed, which prevents the slipping of the wheels while moving in the curve, as it is usually the case with the traditional design of the wheel sets. Application of the system of 'the wheel block radial installation' in the curved sections of the track allows increasing the service life of the train truck arrangement elements while reducing the wear of the rail infrastructure.

The system of 'the wheel block radial installation' is installed on all wheel assemblies (except for final wagons) and consists of a system of longitudinal and transverse rods and equalizers that ensure an automatic radial installation of the wheel block when entering the curve. The use of air suspension in combination with the systems of the pendulum tilt of the body and the 'the wheel block radial installation' in the curves, allows ensuring a high degree of the car riding smoothness and, consequently, significant increase in the passenger comfort travel.

6.2.4. Technical Solutions for Designs of Wheels with Radial Installation of Wheel Pairs

There are a large number of technical solutions for the trucks wheel sets position control. Some of them are made in real constructions, though, most of them remain on paper. They can be divided into three groups according to the actuating mechanisms operating principle used for turning the wheel sets. The first group includes trucks, in which wheel sets are installed in a position close to the radial due to the guiding forces that arise in the contacts of the wheels with rails.

The second group includes structures based on the control action of inertial forces. The third group designs use the principle of forcible turning of wheel sets in order to improve entering the curved sections of the track.

Trucks in which the radial installation of wheel sets is achieved due to the guiding forces in the contacts of the wheels with rails.

The electric trains Re4/4-46 trucks can serve as an example of wheel sets trucks, that are self-installing due to the guiding forces.

A similar scheme is described in the patent [152] 'Two-axle truck' (fig. 6.6). The dual self-installing truck comprises frame 1, the bracket 2 and tractive motor 4. Each motor 4 is connected to the frame 1 by four spring rods 3, which are mounted
with a slope toward the center of the truck gravity center. Moreover, rods 3 form a hinged truncated pyramid, symmetric to this gravity center.

During the truck motion, rods move under the action of transverse guiding forces acting on wheel sets. In this case, the gravity center of the wheel-engine unit is moved in a transversal direction, which eliminates the wheel sets skewness. When moving in a curve, the wheel set is installed to a position close to the radial, as a result of the action of the contact forces and inertia forces of the carriage elements.

The transversal displacements of the wheel set – \( y \), the angular rotation in the plane – \( \psi \) and the longitudinal displacement – \( x \) are connected by the following relationships:

\[
\begin{align*}
y_1 &= l(\sin \alpha + \beta_1) - A + B; \\
y_2 &= A - B - l \cdot \sin(\alpha - \beta_2); \\
x_1 &= l[\cos \alpha - \cos(\alpha + \beta_1)]; \\
x_2 &= l[\cos(\alpha - \beta_2) - \cos \alpha];
\end{align*}
\]  

\[ (6.1) \]
\[
\psi = \frac{x_1 + x_2}{2B + y_1 - y_2}
\]

A self-adjusting wheel set, shown in Fig. 6.7, as an example, can be considered the most simple variant of the wheel sets spring coupling with a truck frame. Wheel set 2 is so connected with the frame 3 with its spring grouping 1. A cradle suspension with the axles of suspension 4, which intersect above the axle of the wheel set is created.

![Figure 6.7 - Self-installing spring-based wheel set.](image)

The Liechti truck structure belongs to the same group (fig. 6.8). The wheel sets 1 are inclining along with the controlling guides 2. The guides can rotate relative to the truck frame 7 around coupling bolts 3. Both the guides 7 are interconnected in the middle point 4 by an angle lever 5 with the bracket 6 of the body 8.

![Figure 6.8 - The diagram of Liechti truck.](image)

The two-axle truck with radially adjustable wheel sets with cross-linkage (European patent application EP 2157007 A1) [147]. A two-axle truck for a railroad vehicle with radially adjustable wheel sets with cross-linkage which consists of two cross-
wires that are diagonally connected by the both wheel sets of the truck, contains L-shaped dimensional adapters 3 (fig. 6.10), which are connected by two crossed rods 4 and 7 which are firmly connected to the axle box 1 through the carrier of axle boxes 2, by which the dimensional adapter brackets 3 do not lie in the same plane, and therefore the dimensional adapter 3 has at least one other bend, through which its points of connection with axle box carrier 2 and the rod 4 and/or 7 will get into the desired position.

The Controlled Railroad Truck (patent US 8276522 B2) [148].

A truck for a railroad vehicle includes a frame, two or more wheel sets, and steering linkage that connect wheel groups to support motion control (fig. 6.10).

The truck has sensors for monitoring the angle of roll and speed. The output signal of the sensor is processed to determine the curvature of the track and to determine the trains motion speed and the speed of the body motion.

The processor starts the alignment bars to adjust the body position relative to the frame in response to the track curvature and the frame current position to increase the stability of the wheel sets.

Fig. 6.11 shows an embodiment of the construction of a two-layer connection of a truck with the body 10 that has frame 11 and wheel sets 12 and 13. The front wheel set 12 and rear wheel set 13 include axles 15, conical wheels 16 and wheel journal boxes 18. The front wheel set 12 is connected with with a pivoting frame 21, through the journal boxes 18.
The swivel frame 21 is connected through a pivot element 24 with an average frame 25. The middle frame 25 is substantially rectangular and is also connected to the second pivoting frame 26 through the rotating element 27. The pivoting frame 26 is connected to the journal boxes 18 of the other wheel set 13. The intermediate frame 25 is connected to the body of the vehicle by coupling 28.

Thus, the wheel sets 12, 13 are connected in such manner that the orientation of the front wheel set 12 affects the orientation of the rear wheel set 13.

When the truck 10 moves along the curved track, wheel groups 12, 13 move so that the wheel sets 12, 13 wheels 16 touch the flanges in the same positions. When the front wheel set 12 moves together with the frame 21, this causes the intermediate frame 25 rotation and in its turn results in the frame 26 rotation and setting the second wheel set 13 in a position that is symmetrical to the position of the first the wheel set.
12 relative to the transversal axis of the frame 11. The intermediate frame 25 deviates transversely due to the connection with the body 28, when the anterior wheel set 12 follows the track curved segment.

In addition, the truck 10 has rods 30 that connect the truck frame 11 with the body. Rods 30 are arranged longitudinally on the opposite sides of the frame 11. Rods 30 may include a pneumatic, hydraulic or electric actuator that responds to the output signals of the sensors indicating the truck position in the curve.

When the truck 10 enters the track curved segment, the processor processes the output signal of the sensor according to the program and triggers the rods 30 operation in such a way that the supports for the equalizer 30 coincide to reduce the wheels slipping 16. When the truck 10 moves along the curve, the internal rod 30 is shortened, while the outer one is lengthened. The design allows the wheels guidance 16th by the track at minimum levels of wheels and rails wear and minimal values of the motion resistance.

The truck rods 30 response to the track curvature assessment. The use of rods 30 in this mode is a semi-active method for controlling the wheel sets motion.

Fig. 6.12 illustrates option 2 of the three-axle truck design according to US patent No. 8276522 B2) [148].

Alternatively, rods 30 can be activated from the train position input and a database of known rail curvature, centrifugal signal, and variation in trucks deviations 10. Data on the track curvature may come from a GPS receiver and processed with a rail curvature database. The rods 30 response to the design truck deviation 10 in accordance with the track curvature. This operating mode is a complete active control method and can be used to control the truck motion 10 in the curve. The truck 50 has a frame 51 and wheel sets 52, 53, 54. The truck 50 is connected with the body via the rods 70. The wheel sets 52, 53, 54 have axles 56, conical wheels 57 and journal box- es assemblies 58. The front wheel 51 has a movable frame 61 connected by a tipping mechanism 62 with the intermediate frame 63. The intermediate frame 63 joins the other pivoting frame 65. The movable frame 61, the intermediate frame 63, and the other movable frame 65 are interconnected by connecting links 62.
Figure 6.12 - A truck with the wheel sets radial installation in the curve as per US patent code 8276522 B2) [148] - option 2.

The railroad truck with the wheel sets radial control (US patent 5375533 A).

In US patent No. 5375533 A [149] proposes a technical solution for a three-axle truck with the possibility of radial installation of wheel sets is suggested. The extreme wheel sets are installed in a radial position by means of longitudinal guide rails positioned with a slope to the longitudinal direction of the vehicle. The intermediate wheel set is displaced in the transversal direction due to the centrifugal force in the curve. The inclined arrangement of the longitudinal guiding rod provides a longitudinal displacement of the journal-box assembly which is in opposite directions to the left and to the right. However, the intermediate wheel pair is not part of the control in this process. It is specifically transversely directed, regardless of the outermost wheel sets. Moreover, such a design operates in the sense of radial orientation of wheel sets.
only in those cases where there is an excess of centrifugal force is noted. At low motion speeds, the noticeable effect will be absent.

Fig. 6.13 depicts two options of the truck arrangement diagram of the three-axle truck according to US patent No. 5,375,533 A [149]. When the truck enters the curve, the intermediate wheel set 3 is displaced in transversal direction. Levers 4 of the intermediate rotate around the supports 6 fixed on the truck arrangement frame. In this case, the outermost wheel sets are set at an angle of 1 to the transversal axis of the truck, and their geometric axes are oriented approximately in the direction to the curve center. When traveling at relatively high speeds and, thus, with relatively high transversal acceleration, the centrifugal force causes the truck arrangement frame accept the shear force oriented towards the outside of the curve.

Figure 6.13 - Scheme of a three-axle truck with radial installation of the wheel sets according to US patent 5375533 A [149].

A truck of a railroad vehicle (patent EP 0161729 A1) [150]. In the truck patented as EP 0161729 A1, in order to control the wheel sets hunting fluctuations in the track straight segments, the wheel sets are interconnected through two attachments,
each of which is coupled with the cross-coupling mechanism. The railroad car truck comprises the frame 2 and two wheel sets 3.

Wheel sets rotate in bearings 4. Wheels 5 of the wheel sets 3 move along rails 6. Each bar 4 has a rotary bracket 7 which is connected to the central part 9 of the vertical lever 10 by means of a spherical axis 8.

The lever 10 upper end is hingedly suspended to the neck 13 of the frame 2 by means of a spherical rod 12. The necks 13 are supported by a spring pack 14 on the journal box assemblies 4. The lower end 15 of each lever 10 is connected by means of a spherical axle 16 with transversal rods 17.

The rods 17 are crossed with each other and connect the levers 10 arranged diagonally opposite each other.

*Figure 6.14 - The truck of the railroad vehicle with the controlled position of the axles according to EP 0161729 A1 [150].*

Thus, they form, the so-called cross anchor. The spherical axles 16 can transpose each other in the direction of the arrows 18. Thus, a steady turn of the wheel sets 3 in a horizontal plane is obtained. Crossings 17 are arranged at a low level, at least, at a much lower level than the axles level 19 of the wheel sets 3.
A railroad truck containing a frame and two controlled axles (EP 0387744 A2) [151].

The truck in fig. 6.15 consists of a frame 10 and two controlled axles 20, each of which is mounted on the frame 30. Each axle is mounted in the journal boxes pair of axle boxes 25 connected to the frame via the primary spring suspensions 40 and the vertical connecting rod 50.

![Diagram of a railroad truck with controlled axles](image)

Figure 6.15 - A railroad truck with a controlled axles position according to patent EP 0387744 A2 [151].

The both frames are connected by two diagonal rods 71, 72. The transversal rods 32 are connected with a pair of links 55 by the central levers 60. The levers 60 are hingedly installed on the frame 11. When the levers 60 are rotated, a relative rotation of the wheel sets in the horizontal plane is achieved for their radial installation in the track.
CONCLUSIONS

The presented study results aimed at reducing the motion resistance of the rolling stock which is associated with the guidance of the railroad carriages by the track, enabled making the following conclusions.

1. The existence of a component of the motion resistance of railroad carriages due to the wheel sets guidance by the rail track is substantiated. The authors called this resistance the kinematic motion resistance.

   The nature of the kinematic motion resistance occurrence is revealed. The kinematic motion resistance is caused by the group interaction of the multi-axle wheel system of carriages with a rail track. The nature of this resistance occurrence lies in the discrepancy between the geometric and kinematic parameters of contacting of each of the wheels and the kinematic parameters of the entire carriage motion. The kinematic motion resistance is associated with circulatory processes four-square torque loops, formed by the carriage part and track elements.

2. The method of four-square torque loops analysis in mechanical transmission of power in relation to rail rolling stock is suggested. The kinematic diagrams of the wheel sets drive and guidance include four-square torque loops formed by wheel sets and track subsystem. The essential uneven distribution of power flows along the circuits branches and the circulation of flows within the contours are their characteristic features.

   Parasitic power flows are the cause of an increase in mechanical losses and a decrease in the efficiency of the multi-axle wheel propulsion. The level of mechanical losses depends on the characteristics of the kinematic pairs. The circulation coefficient of was introduced to estimate the level of losses in closed loops. The main circulating flows that determine the level of the kinematic motion resistance of rail carriages are the flows in the axial and inter-axial four-square torque loops. The circulation energy is absorbed in the decoupling nodes, especially in the contacts of the wheels with rails. In this case, the contacts of the wheels with rails act as friction dampers with a high degree of scattering.

   The coefficients of circulation in the specified loops are of greatest importance when the carriages are moving in the curves of medium and small radiuses reaching values of 0.85 ... 1.0.

3. The procedure for mathematical modeling of two-point contact of a wheel
with a rail as a statically undetermined supporting system has been developed, taking into account the contact elasticity of the material and the spatial geometry of the rolling surfaces profiles. The laws of the distribution of vertical and normal loads, longitudinal and transversal forces and sliding speeds in contacts, as functions of the mutual positioning of the wheel and the rail in the spatial coordinate system, are obtained. The procedure relaying of the main and flange contacts is presented as a linear dependence of the vertical load on the relative transversal displacement of the wheel and the rail within the two-point contact zone.

A comparative analysis of the results of the integration of the carriage motion equations with various options of the contact interaction description showed the expediency of taking into account two-point contacting when modeling the movement of carriages in the mode of the flange touching the wheels and rails.

4. Based on the analysis of four-square torque loops and geometrical characteristics of wheels and rails contacting, a procedure for simulating the kinematic motion resistance has been developed. Mathematical models for most of the design schemes of the serial rolling stock have been developed: steam engines, electric locomotives, electric and diesel locomotives, passenger and freight cars.

5. The description of the frictional interaction of wheels with rails has been refined base on taking into account the two-point contacting features. This allowed obtaining a satisfactory level of reliability of mathematical modeling and evaluation of the kinematic motion resistance to carriages, as well as assessing its being impacted by design factors. As the main criterion for the simulation validity, the kinematic motion resistance, which can be estimated by experimental characteristics, as an additional motion resistance in the curves, is accepted. The trajectory parameters of the carriage motion according to the study results of horizontal dynamics in known works, are accepted as additional criteria.

6. As a result of the research based on mathematical models, the characteristics of the kinematic motion resistance for the serial rolling stock were obtained.

The following results were obtained:

- the kinematic motion resistance of the rolling stock most units in the long-distance curves, depending on the speed and parameters of the track, is 20 ... 50%, and in the curves with radius of 30 ... 80 m typical for urban rail transport, up to 80% of the total motion resistance;
- the formulas for calculating the motion resistance in the curves used in
traction calculations do not reflect the dependence of the specific motion resistance on structural features and modes of the rolling stock motion, giving an error of 1.5 ... 2.5 times;

- in case of the traditional truck arrangement, the carriages with two-axle trucks have a specific kinematic motion resistance which is by 15 ... 30% lower than the carriages with three-wheel trucks;

- profiles of the wheels rolling surfaces with a larger angle of conicality are characterized by higher values of the motion resistance. In particular, under otherwise equal conditions, at an angle of the flange wheel cone of 60°, the motion resistance of the four-axle carriage is 1.3 ... 1.6 times lower than at the flange angle of 70°.

7. Analysis of the guiding force factors of the carriages entering the track curves, indicated that the controllability of the carriages by the rail track has a rigid characteristic associated with the transversal resilience of rail lines. The longitudinal forces of adhesion play the role of guides only in curves with a radius of more than 1000 ... 1250 m. In the curves of medium and small radius, they, on the contrary, cause motion resistance. Transversal adhesion forces components can act as determining factors only in case of the wheel sets installation with skewness when first wheel set is climbing the internal rail. In other installations, this factor causes the moment of resistance to the entering. In this case, the dependence of the resistance moment on the angle angle is unstable, which manifests itself in the wheel set being prone to jamming.

8. Taking into account the high level of transversal sliding and transversal coupling rigidity characteristics, these forces are the main resistance factor in the entire range of the curves radiiuses. The only really acting guiding factor is the flange gravitational forces.

9. When carriages move in the curves an approximately a linear relationship between the guiding forces and the kinematic motion resistance is observed. It has been established that a carriage with less motion resistance creates a less horizontal effect on the rails. Thus, the reduction in the influence on the track can be achieved through the use control of entrance quality – the kinematic motion resistance as an integral criterion.

10. The truck arrangement design principle for rolling stock with low motion resistance, based on reducing the circulation coefficient in the axle and inter-axle
four-square torque loops has been proposed. Certain limits of the power circulation reduction can be achieved by reducing the loop stiffness. However, in pseudo-stationary modes of the loop operation, for example, when moving in curves, the limitation of loop flows can only be achieved by introducing the drive and wheel set guidance of constructive decoupling node points into the kinematic chains.

These methods allow reducing the kinematic motion resistance by 15 ... 60%, which corresponds to a reduction in the total motion resistance by 8 ... 20%, as well as fuel economy and fuel consumption for trains by 5 ... 12%.
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