The Optimization and Application of Cut-off Grades of Multiple Metal Open-pit Mines Based on Equivalent Grade

GU Qing-hua
Xi’an University of Architecture & Technology, Xi’an, Shaanxi, China

Bai Chun-ni
Xi’an University of Architecture & Technology, Xi’an, Shaanxi, China

LI Fa-ben
Xi’an University of Architecture & Technology, Xi’an, Shaanxi, China

JOHN Abrand
College of Engineering, Ohio State University, Columbus, USA

Abstract

The cut-off grade is one of the important parameters of mining production. Different material cut-off grade is directed to different destinations. Based on reasonable and optimal cut-off grade, long term mining plan and investment decision analysis can be made for mining managers. In view of multiple metal open-pit, the paper has proposed that multiple metal cut-off grade optimization model is set using the objective function to maximize net present value (NPV). The method of equivalent coefficient calculation is proposed to converting multi metals cut-off grade optimization to a single metal cut-off grade optimization. Then the optimization model is applied in a molybdenum and wolfram large open-pit mine in china. The parameters of concentrate price are calculated by PRT in the model and the curve of grade-recovery rate is analyzed and studied according to actual production data. Based on 3D metal deposit model, the distribution of grade-tonnage is analyzed. At final step, a optimal cut-off grade is proposed to guarantee to maximize NPV. The example shows that the model of multiple metal cut-off grade optimization can provide reasonable analysis for optimal multi-metals cut-off grade and develop optimal production strategies.

Keywords: EQUIVALENT GRADE, MULTIPLE METAL, CUT-OFF GRADE, NPV, OPTIMIZATION

1. Introduction

Cut-off grade is one of the most important parameters in metal ore mining because of its influence on the overall economic profit of mining production. Along with the development of the mining production, dynamic optimization of cut-off grade is necessary through the analysis of technical economy. It is better to manage the mining production according to the variation of market. The cut-off grade plays an important role in mining production.
Cut off grade is defined as by Taylor[1-2]; it is used to discriminate ore and rock. From the perspective of economics, it is a grade value which makes profit and cost equal in mining. When useful components grade in mining ore is higher than or equal to the cut off grade, it should be as ore; Otherwise, it should be divided into waste rock. The optimization theory of single metal cut off grade has been put forward earliest by Lane[3], which is considered to be a landmark in cut off grade optimization. Whittle and Wharton[4] had proposed opportunity costs, which includes two virtual costs, namely the delay cost and change cost. But this algorithm is not available for multi-metal mine. Annels[5] has proposed that the multi-metal valuation can be evaluated by net metal amount one ton ore(NSR). In this case, one metal grade can be converted into a different kind of metal grade. Osanloo[6] has proposed that equivalent grade factors is used to find the optimum cut-off grades of multiple metal deposits. M.W.A. Asad[7] has set up a net present value maximization model for optimum cut-off f grade of open pit mining operations considering the impact of stockpile. Guoqing Li[8] has proposed that profit method is adopted to calculate the equivalent copper grade of associated molybdenum in a copper and molybdenum mine. A underground metal cut-off grade optimization method is proposed from the perspective of ore spatial distribution by Xiaowei Gu[9]. A dynamic optimization model of cut-off grade in an open pit has been established through marginal analysis method combined with time cost of capital and analysis of opportunity cost by Yingliang Xie[10]. A few researchers usually use the approach of breakeven analysis to calculate cut-off grade and distinguish the ore block section. But this method is mostly not optimal. So net present value(NPV) is typically acknowledged to a more reasonable method of cut-off grade. The time cost of capital is fully considered in mining process. Maximum net present value is used to establish cut-off grade optimization model has also been different[11]. However, each multi-metal deposit has its own feature with different production process and constraints. The cut-off grade optimization model has also been different[12-13]. In this paper, the equivalent grade and net present value is adapted to establish cut-off grade optimization, especially considering dynamic recovery rate. Taking a large molybdenum and wolfram open pit as a example, the optimal equivalent cut-off grade of molybdenum and wolfram is figured out under the current market environment. This can provide production criterion for multi-metal open pit.

2. Multi-metal cut-off grade optimization model

Maximum net present value is used to establish multi-metal cut-off grade optimization model based on the whole life time operations of mining stage-treatment stage-refining stage. But in china the refining stage of mining group is usually independent. The concentrate is usually the final product for sale in most enterprises. So this paper mainly focus on the first two stage: mining and treatment. The cut-off grade optimization model is set up involving two phases:

(1)Mining stage: all kinds of ore are mined up to mining production capacity; (2) Treatment stage: ore is grinded and concentrated again up to processing capacity. In the above two stages, there are respectively production cost and production capacity constraints. So considering its own cost and sale income fully and multi-metal deposit contains two kinds of main useful elements, the overall profits are as follows:

\[
MaxNPV = \sum_{s=1}^{N} \frac{P_s}{(1+d)^n} \]

\[
P_s = s_1 Q_{1s} + s_2 Q_{2s} - m_1 Q_{m} - (C_{1s} + C_{2s}) Q_{c} - f - w_s Q_{w} \]

(2)

Where \(n=\)period(year) indicator, \(N=\)total life operation(years), \(P=\)profit($/year), \(d=\)discount rate(%), \(m=\)mining cost($/ton), \(w=\)stripping cost($/ton), \(C_1=\) milling or concentrating cost of metal 1($/ton), \(C_2=\) milling or concentrating cost of metal 2($/ton), \(f=\)fixed cost($/year), \(s_1=\)metal 1 concentrate selling price($/ton), \(s_2=\)metal 2 concentrate selling price($/ton), \(Q_{m}=\)quantity of ore mined(tons/year), \(Q_c=\)quantity of ore processed(tons/year), \(Q_w=\)quantity of rock mined(tons/year), \(Q_f=\)quantity of metal 1 concentrated(tons/year), \(Q_f=\)quantity of metal 2 concentrated(tons/year).

There are relationships between the quantities of metal 1 concentrated \(Q_1\) and metal 2 \(Q_2\) and the quantities of ore processed \(Q_c\) as follows:

\[
Q_1 G_1 = \bar{G}_1 y_1 Q_c \]

(3)

\[
Q_2 G_2 = \bar{G}_2 y_2 Q_c \]

(4)

Where \(\bar{G}_1\) is the average grade of ore processed containing metal 1, \(\bar{G}_2\) is the average grade of ore processed containing metal 2, \(y_1\) is the recovery rate of metal 1 from the ore, \(y_2\) is the recovery rate of metal 2 from the ore, \(G_1\) is the grade of metal 1 concentration, \(G_2\) is the grade of metal 2 concentration.
Substituting Eq.(3) and Eq.(4) into Eq.(2) yields

\[ P = \frac{s_1 y_1 G_1}{s_1 y_1 G_2} + \frac{s_2 y_2 G_1}{s_2 y_2 G_2} \]

\[ \frac{1}{G_1} \left( G_2 - (C_1 + C_2) \right) \]

\[ Q_{wf} - m Q_m - f - w Q_w \]

(5)

In Eq.(5), \( P \) is a grade function of metal 1 and metal 2. The grade of metal 2 is converting to grade of metal 1 by using equivalent factor. Equivalent factor is equal to

\[ F_{eq} = \frac{s_2 y_2 G_1}{s_1 y_1 G_2} \]

(6)

Substituting Eq.(6) into Eq.(5) yields

\[ P = \left[ \frac{s_1 y_1}{G_1} G_{eq} - \left( C_1 + C_2 \right) \right] Q_{eq} - m Q_m - f - w Q_w \]

(7)

The average equivalent grade of multi-metal ore can be calculated based on equivalent factor and average grade of each metal as follows:

\[ G_{eq} = \frac{G_1 + F_{eq} G_2}{s_1} \]

(8)

Substituting Eq.(8) into Eq.(7) yields

\[ P = \left[ \frac{s_1}{G_1} y_1 \left( G_{eq} - (C_1 + C_2) \right) \right] Q_{eq} - m Q_m - f - w Q_w \]

(9)

Maximizing the net present value for mining enterprises is necessary. Namely the sum of profits each year should be maximum. The earlier profit can be made, the less the net present value lost. Then more profits can be gained. Eq.(1) is the objective function for calculation of optimal cut-off grade of ore. The time \( N \) is related to the constraint production capacity. Two cases arise depending upon which of the two capacities is actually limiting factor.

Case 1: if the mining capacity \( M \) is the main limiting factor, then the time \( N \) is given by

\[ N = \frac{Q_m}{M} \]

(10)

Case 2: if the concentrating capacity \( C \) is the main limiting factor, then the time \( N \) is controlled by the concentrator.

\[ N = \frac{Q_c}{C} \]

(11)

The final objective function of maximizing net present value is as follows:

\[ \text{MaxNPV} = \sum_{s=1}^{N} \frac{\left[ \frac{F_{eq}}{G_{eq}} \right] y_{s-1} G_{eq} - \left( C_{s-1} + C_{s} \right) Q_{eq} - m Q_m - f_s - w Q_w}{(1+d)^s} \]

(12)

The equivalent factor is usually used to convert one metal grade to another metal grade in multi-metals deposits. Then the cut-off grade optimization model is converted into a single metal cut-off grade optimization. Based on Lane algorithm, the objective function is a unimodal function with above two constraint conditions. The maximum peak value can be found by iterative search method. However, due to the difference of the distribution to grade-ton for each deposit and the relationship of grade-recovery for each metal, it is very hard to calculate optimal grade. So the optimal grade is usually obtained by gradual approximation with different net present value scenarios.

3. Example

3.1 Molybdenum wofram deposit

A large molybdenum and wolfram deposit locates in Henan province, China. The shape of largest ore body actually is simple. It is a thick and bedded ore body. There also are small ore bodies in the bottom and top of the main ore body. Molybdenum and wolfram are mainly metal elements associated with few Iron and sulfur. With scope of the current mining license, the average grade of molybdenum is 0.10% and the associated wolfram is 0.07%. According to mining conditions, the mining approach of open pit is available. So shovels and trucks are adapted to mining ores. Milling and flotation process is used to obtain molybdenum concentrate and wolfram concentrate.

So far the mining production capacity and mineral treatment capacity is 9.9 million tons per year. It has been mined for many years. But wolfram did not recycle in the early production. So after wolfram recycled, the optimal grade of two metals needs to be calculated. This paper mainly focuses on how to optimize the grade of molybdenum and wolfram based on above cut-off grade optimization model.

Figure 1. 3D model of molybdenum and wolfram
Based on geological resource reserve report and supplementary drilling exploration in recent years, a total of 142 geological drilling data are normalized and geological database is set up. Using mining software Surpac as platform, the 3D entity model of molybdenum and wolfram are constructed according to geological database and geological profile morphology. Fig.1 shows this model. Molybdenum ore body is larger than wolfram ore body. Wolfram is mainly located in skarn and calcium silicate hornfels. In the middle of ore body, the shapes and distribution of molybdenum and wolfram ore body in the skarn are similar. Large difference appears in the edge of ore body. Wolfram ore body typically locates at bottom of molybdenum ore body. In the horizontal direction, the wolfram grade in the south is higher than in the north; the wolfram grade in the west is higher than in the east; in the vertical direction, the grade is higher in the bottom.

Based on above molybdenum and wolfram 3D model, the raw data of geological drilling is analysed by statistic method. Some grade values are too high to evaluate. All raw data is processed and grouped. All the sample data normally belong to normal distribution. A total of molybdenum sample 14950 groups is effective and a total of wolfram sample 8483 groups. With the variation functions, Kriging method is used to evaluate the grade of each block model, as shown in Fig.2.

![Figure 2. The grade distribution of molybdenum and wolfram block model](image)

### 3.3 The equivalent grade of multi-metals

**1. Production price**

With the price changing trend of molybdenum and wolfram concentrate in recent years and volatility of market price, the prices data of molybdenum and wolfram concentrate are done some statistical analysis. The maximum, most likely value and minimum of two kinds of concentrate can be obtained. Then the future prices of molybdenum and wolfram concentrates will be calculated through Program Evaluation and Review Technique (PERT) as follows\[14\]:

The price of molybdenum concentrate per unit is:

\[
P_1 = \frac{Max + 4 \times Mostlikely + Min}{6}
\]

\[
(460.5+4*275.9+212.9)/6=296.17\$/ton (13)
\]

The price of wolfram concentrate per unit is:

\[
P_2 = \frac{Max + 4 \times Mostlikely + Min}{6}
\]

\[
(413.4+4*270.2+150.5)/6=274.12\$/ton (14)
\]

**2. The recovery of molybdenum and wolfram**

The actual raw data of metal grade and recovery rate are collected respectively from molybdenum and wolfram processing plant, which belong to Luoyang Luanchuan Molybdenum Industry Group Inc. 1120 groups of molybdenum grade and recovery rate and 1636 groups of wolfram grade and recovery rate are acquired. Then all data is analysed and processed as samples. After doing some statistical analysis, it seems that the ratio between molybdenum recovery rate and wolfram recovery rate is mostly constant. Details is shown in [8,15], namely

\[
\xi = \frac{\gamma_2}{\gamma_1} = 0.9579 (15)
\]

Through the statistical analysis of molybdenum and wolfram grade-recovery, it is shown that when the metal grades become higher and higher, the corresponding recovery rates will increase gradually. However, due to some blocking conditions on the growth rate of recovery rate, such as production process, natural resources and environment, the blocking effects are more and more with the increase of the grade. So the Logistic model is adopted to set up the curve

© Metallurgical and Mining Industry, 2014, No. 6
relationship of molybdenum grade-recovery rate. The least square method is used to the fitting data of Logistic model. It makes points locate at two side of curve as many as possible. MATLAB is used to nonlinear data fitting for many times and finally parameters \( x_m \) and \( r \) are obtained with value 86.5095, 44.8110. Wolfram grade will convert into equivalent molybdenum grade. So the relationship curve model of molybdenum grade-recovery is obtained as follows:

\[
y = \frac{86.5095}{1 + \left( \frac{86.5095}{64} \right)^{e^{-44.8810(t-0.062)}}}
\]

The curve of molybdenum grade-recovery rate is shown in Matlab as follows:

Figure 3. The fitting curve of molybdenum grade-recovery rate

(3) Equivalent factor and equivalent grade

After milling and processing, the grade of molybdenum concentrate typically is 47% and the grade of wolfram concentrate is 30%. Substituting those parameters into Eq.(6) and Eq.(8), the equivalent factor for converting wolfram grade into molybdenum grade can be calculated as follows:

\[
F_{eq} = \frac{s_2 y_2 G_i}{s_1 y_1 G_2} = \frac{1699.3 \times 30 \times 0.47}{2036.5 \times 47 \times 0.30} = 0.79
\]

(17)

\[
g_{eq} = g_{wo3} + 0.79 \times g_{wo3}
\]

(18)

Using the above equivalent factor, all the wolfram grades are converted into molybdenum grades in the samples. The equivalent grades in all blocks are calculated and evaluated again. The block model of equivalent molybdenum grade is shown in Fig.4.

Figure 4. Distribution of equivalent molybdenum grade in block model

3.4 Parameters of equivalent cut-off grade optimization

The main input parameters in the model of equivalent cut-off grade optimization are shown in Table 1. The costs, prices, capacities, quantities and recoveries are demonstrated. After converting, the equivalent molybdenum grades are normally larger than before. The corresponding recovery rate can be calculated by Logistic model. This is second time to optimize open pit production. So the total investment is 229.1274 million dollars, including new construction investment is 99.6371 million dollars and the present value of fixed assets 43.2806 million dollars, the flow capital 86.2097 million dollars. The time for ream stripping is 1 year.

Table 1. Economic parameters for optimization example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molybdenum concentrate</td>
<td>Dollars per ton</td>
<td>13919.9</td>
</tr>
<tr>
<td>Wolfram concentrate</td>
<td>Dollars per ton</td>
<td>8223.6</td>
</tr>
<tr>
<td>Mine capacity</td>
<td>Tons per year</td>
<td>9900000</td>
</tr>
<tr>
<td>Dilution ratio</td>
<td>%</td>
<td>3</td>
</tr>
<tr>
<td>Ore lost ratio</td>
<td>%</td>
<td>3</td>
</tr>
<tr>
<td>Mining cost</td>
<td>Dollars per ton</td>
<td>8.96</td>
</tr>
<tr>
<td>Processing cost(Molybdenum)</td>
<td>Dollars per ton</td>
<td>8.39</td>
</tr>
<tr>
<td>Processing cost(wolfram)</td>
<td>Dollars per ton</td>
<td>2.49</td>
</tr>
<tr>
<td>Recovery(molybdenum)</td>
<td>%</td>
<td>84.19</td>
</tr>
<tr>
<td>Recovery(wolfram)</td>
<td>%</td>
<td>80.65</td>
</tr>
<tr>
<td>Discount rate</td>
<td>%</td>
<td>15</td>
</tr>
</tbody>
</table>
Mining production

<table>
<thead>
<tr>
<th>Fixed costs</th>
<th>Dollars per year</th>
<th>7601.49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stripping ratio</td>
<td>t/t</td>
<td>0.56</td>
</tr>
<tr>
<td>Waste ore cost</td>
<td>Dollars per ton</td>
<td>3.42</td>
</tr>
</tbody>
</table>

Converting wolfram grade into molybdenum grade, the grade-tonnage distribution

<table>
<thead>
<tr>
<th>Equivalent molybdenum grade(%)</th>
<th>Ore quantities(t)</th>
<th>Equivalent average grade (%)</th>
<th>Molybdenum grade(%)</th>
<th>Wolfram grade(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01-0.02</td>
<td>1606020</td>
<td>0.015</td>
<td>0.011</td>
<td>0.016</td>
</tr>
<tr>
<td>0.02-0.03</td>
<td>6710745</td>
<td>0.027</td>
<td>0.033</td>
<td>0.021</td>
</tr>
<tr>
<td>0.03-0.04</td>
<td>28793760</td>
<td>0.035</td>
<td>0.035</td>
<td>0.015</td>
</tr>
<tr>
<td>0.04-0.05</td>
<td>39301440</td>
<td>0.045</td>
<td>0.042</td>
<td>0.015</td>
</tr>
<tr>
<td>0.05-0.06</td>
<td>37556160</td>
<td>0.055</td>
<td>0.046</td>
<td>0.019</td>
</tr>
<tr>
<td>0.06-0.07</td>
<td>37921440</td>
<td>0.065</td>
<td>0.049</td>
<td>0.025</td>
</tr>
<tr>
<td>0.07-0.08</td>
<td>33568800</td>
<td>0.075</td>
<td>0.05</td>
<td>0.034</td>
</tr>
<tr>
<td>0.08-0.09</td>
<td>36442560</td>
<td>0.085</td>
<td>0.052</td>
<td>0.043</td>
</tr>
<tr>
<td>0.09-0.1</td>
<td>34824960</td>
<td>0.095</td>
<td>0.055</td>
<td>0.05</td>
</tr>
<tr>
<td>0.11-0.11</td>
<td>35478240</td>
<td>0.105</td>
<td>0.059</td>
<td>0.057</td>
</tr>
<tr>
<td>0.12-0.13</td>
<td>33014880</td>
<td>0.115</td>
<td>0.062</td>
<td>0.065</td>
</tr>
<tr>
<td>0.13-0.14</td>
<td>28624320</td>
<td>0.125</td>
<td>0.067</td>
<td>0.072</td>
</tr>
<tr>
<td>0.14-0.15</td>
<td>25619520</td>
<td>0.135</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>0.15-0.16</td>
<td>22289280</td>
<td>0.145</td>
<td>0.076</td>
<td>0.085</td>
</tr>
<tr>
<td>0.16-0.17</td>
<td>21408960</td>
<td>0.165</td>
<td>0.081</td>
<td>0.093</td>
</tr>
<tr>
<td>0.17-0.19</td>
<td>147447360</td>
<td>0.247</td>
<td>0.134</td>
<td>0.145</td>
</tr>
<tr>
<td>0.18-0.21</td>
<td>18980640</td>
<td>0.175</td>
<td>0.095</td>
<td>0.1</td>
</tr>
<tr>
<td>0.19-0.22</td>
<td>15805440</td>
<td>0.185</td>
<td>0.102</td>
<td>0.104</td>
</tr>
<tr>
<td>0.20-0.23</td>
<td>12018240</td>
<td>0.195</td>
<td>0.107</td>
<td>0.108</td>
</tr>
<tr>
<td>0.21-0.24</td>
<td>9891840</td>
<td>0.205</td>
<td>0.109</td>
<td>0.118</td>
</tr>
<tr>
<td>0.22-0.25</td>
<td>8620320</td>
<td>0.215</td>
<td>0.112</td>
<td>0.127</td>
</tr>
<tr>
<td>0.23-0.26</td>
<td>8580000</td>
<td>0.225</td>
<td>0.117</td>
<td>0.134</td>
</tr>
</tbody>
</table>

3.5 Optimization and analysis of equivalent cut-off grade

Mining capacity constraint is the limiting factor. So \( v_m \) is the net present value in the model of equivalent cut-off grade optimization. The cut-off grade is initialized with 0.01%. 0.01% is taken as step. Then 23 different scenarios are calculated with different cut-off grade. The ore quantities, average equivalent grade, metal, years of service, recovery rate and NPV are demonstrated in Table 3.

<table>
<thead>
<tr>
<th>Cut-off grade(%)</th>
<th>Ore quantities(tons)</th>
<th>Average grade(%)</th>
<th>Metal quantities(tons)</th>
<th>Total years</th>
<th>Recovery rate(%)</th>
<th>NPV(million dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>593077725</td>
<td>0.131</td>
<td>776610.2</td>
<td>60</td>
<td>68.72</td>
<td>-469.572</td>
</tr>
<tr>
<td>0.02</td>
<td>591471705</td>
<td>0.131</td>
<td>776369.3</td>
<td>60</td>
<td>68.72</td>
<td>-465.588</td>
</tr>
<tr>
<td>0.03</td>
<td>584760960</td>
<td>0.132</td>
<td>774557.4</td>
<td>59</td>
<td>68.75</td>
<td>-449.713</td>
</tr>
<tr>
<td>0.04</td>
<td>555967200</td>
<td>0.138</td>
<td>764479.6</td>
<td>56</td>
<td>68.78</td>
<td>-385.039</td>
</tr>
</tbody>
</table>
Based on the equivalent cut-off grade and NPV on the table 4, the function curve of cut-off grade and NPV can be fitting, as shown in Fig.4. It can be seen that when equivalent cut-off grade is 0.06%, NPV is close to zero. So the break even cut-off grade should be between 0.06% and 0.07%. As the cut-off grade increases, the value of NPV also grows gradually. The function curve of cut-off grade and NPV is a unimodal function. When the equivalent cut-off grade is 0.21%, NPV is close to maximum. So the optimal cut-off grade should be 0.21% and the average equivalent grade is 0.285%. In this case, the molybdenum average grade is 0.153% and wolfram average grade 0.169%. But the years of service is shorter. In china, many mining enterprises consider that the years of service should be as long as possible and the quantities of metal should be as many as possible. So the break even cut-off grade also can be considered. In this paper, the optimal scenario is shown with optimal parameters as follows:

**Table 4. The optimal scenario with optimal cut-off grade**

<table>
<thead>
<tr>
<th>Equivalent cut-off grade(%)</th>
<th>Ore quantities(tons)</th>
<th>Average grade(%)</th>
<th>Molybdenum grade(%)</th>
<th>Wolfram grade(%)</th>
<th>Metal quantities(t)</th>
<th>Total years</th>
<th>NPV(million dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.21</td>
<td>90751200</td>
<td>0.285</td>
<td>0.153</td>
<td>0.169</td>
<td>258640.9</td>
<td>9</td>
<td>1710.686</td>
</tr>
</tbody>
</table>

**Figure 4.** The function curve of cut-off grade and NPV
4. Conclusion
1. Cut-off grade optimization of multi-metal is proposed based on two stages: mining and processing in this paper. Considering grade-tonnage distribution concentrate market price and recovery rate, the model of multi-metals cut-off grade optimization is constructed using equivalent factor and NPV. Objective function is expressed to one variable function by equivalent factor. Multi-metal cut-off grade is converted into a single metal cut-off grade.
2. A verification example of molybdenum and wolfram is presenting for confirming the cut-off optimization model in this study. The results of optimal cut-off grade and break even cut-off grade are compared in the paper. It provides advice for production of multi-metals mine.
3. The cut-off grade is calculated with many parameters such as concentrate price, discount rate and costs. Those economic parameters are always variable as market changes. So the cut-off grade is optimal with those parameters and this period. Average value and statistical analysis are usually used to eliminate this influence in this study.

Acknowledgements
The work was supported by National Natural Science Foundation of China (Grant No. 51404182) and the key discipline of Shaanxi Province(E08001) and Yulin Municipal Science and Technology Project .The authors gratefully acknowledge the cooperation of the Luoyang Luanchuan Molybdenum Industry Group Inc. The authors also thank other participants for their support in OSU.

References
14. LI Ling, LU Cai-wu. The analysis and prediction of molybdenum concentrate market price in china[J]. Nonferrous