Analysis of complete preorders provided by aggregation methods: application for the public landfill site selection

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Abstract
The purpose of this article is to analyze the complete preorders resulting from the aggregation of a set of preferences by the methods of Borda and Condorcet, and with the SMART model. The aggregation methods of the candidates' evaluations, according to Borda and Condorcet, do not require weights for the considered criteria. On the other hand, the SMART model, which is a multi-criteria aggregation procedure, requires it. In multi-criteria decision making, the criteria weights are directly assigned by the decision maker or indirectly determined starting from methods of calculation. In this work, four neutral methods are used to elicit the criteria weights. They are: 1) standard deviation method, 2) entropy method, 3) statistical method and 4) total variation method. The criteria weights obtained by those four neutral methods, which have the advantage of neutrality and objectivity, are separately used in the SMART model. And that, to analyze the impact of the obtained criteria weights on the selection of the best solution. The choice of the best alternative and the ranking of the remaining alternatives (considered) are studied for each complete preorder provided by the Borda and Condorcet methods as well as by the SMART model (with four variants of the criteria weights). In this article, an illustrative application is performed. It concerns a public landfill site selection problem.
Keys words: SOCIAL CHOICE, MULTI-CRITERIA DECISION MAKING, COMPLETE PREORDERS, PUBLIC LANDFILL.

1. Introduction

In current research, the problems of selection or ranking of alternatives are generally solved using the theory of the social choice or using the multi-criteria decision making. For the social choice theory, the best known and most frequently methods are the Borda method (Borda, 1781) and the Condorcet method (Condorcet, 1785). For the multi-criteria decision making, the aggregation methods are numerous. They are divided into two categories: 1) aggregation methods based on a single synthesis criterion such as the SMART method (Edwards, 1971), the MAUT method (Keeny and Raiffa, 1976), the AHP method (Saaty, 1980), the TOPSIS method (Hwang and Yoon, 1981), the weighted sum method (Timmerman, 1986), the weighted product method (Pomerol and Barba-Romeo, 1993), etc.; 2) methods of alternatives outranking such as ELECTRE methods (Roy, 1996) and PROMETHEE methods (Brans and Mareschal, 2002).

In this article, we study the complete preorders provided by the Borda and Condorcet methods, and by the SMART model (with four variants of criteria weights elicited by the neutral methods) when solving a public landfill site selection problem.

The problem to be solved is stated as follows: "select a site for public landfill among seven alternatives $\{a_1,\ a_2,\ a_3, a_4, a_5, a_6 \text{ and } a_7\}$ associated with four criteria $\{C_1, C_2, C_3 \text{ and } C_4\}$. The criteria are: 1) criterion of distance (distance between municipality and the potential site to serve as public landfill), 2) criterion of transport of household waste per tonne per kilometer, 3) criterion of residents opinion (in connection with the establishment of a public landfill) and 4) criterion of emission of N_2O flux by the soil". After aggregation of the alternatives' evaluations (potential sites for being useful as public landfills), the six obtained complete preorders denote the alternative $\{a_4\}$ as being the solution to the posed problem. The detailed analysis of the complete preorders is presented in section 3.

2. Social choice theory and multi-criteria decision making approach

2.1 Classical Borda Method (Borda, 1781):

The classical Borda method consists in adding the obtained ranks $r_j(a_{ij})$ by a candidate a_i (i = 1, …, m) relatively given to each of the criteria j (j = 1, …, n (n ≥ 3)). For a criterion j, Borda gives 1 point to a candidate who arrived first, 2 points to a candidate who came in second, 3 points to the third candidate, etc.

If there is equal candidates, Borda awards each of the equal candidates the average points that they would have obtained if they had not been equal.

The social choice, or the aggregated preorder, is obtained by making the sum of all the obtained points for all the criteria j by each of the candidates.

For any candidate a_i, the Borda score $b(a_i)$ is expressed by: $b(a_i) = \sum_j \left(r_j(a_{ij})\right)$. The ranking of the candidates a_i (i = 1, …, m) is done in ascending order of $b(a_i)$. We rank first, the candidate who has the least points, second, the one who has a number of points immediately higher, and so on until the last candidate with the most points.

According to (Bouyssou et *al*., 2005), the Borda method has an important advantage compared to the Condorcet method.

This method permits not only to designate one or more winners, but provides a ranking of all candidates from the best to the worst. In addition, it respects four axioms of Arrow's theorem (1963): universality, unanimity, transitivity and non-dictatorship; therefore, it does not satisfy the axiom of independence.

2.2 Method of Condorcet (Condorcet, 1785):

The Condorcet aggregation method is a procedure which, for any pair of candidates $\{a_i, a_k\}$ (i = 1, …, m et k ≠ i), consists in posing $a_i > a^k$ if and only if the number of criteria j for which a_i dominates a_k is strictly greater than the number of criteria j for which the inverse is true.

In case of equality of the number of criteria "for" and the number of criteria "against", the candidates are indifferent ($a_i \approx a_k$).

The winner of Condorcet is the candidate who, in the social relation resulting from the Condorcet method, dominates all the other candidates.

According to (Bouyssou et *al*., 2005), the Condorcet method leads sometimes to the not-transitive preferences, not allowing to rank the candidates nor same to choose a subset of good candidates. Moreover, it respects the axioms of universality, unanimity, independence and non-dictatorship of Arrow's theorem (1963); therefore, it cannot satisfy the axiom of transitivity.

2.3 SMART Model / Simple Multi-Attribute Technique Ranking (Edwards, 1971):

The SMART model is a multi-criteria aggregation procedure. Since, its conception by Edwards (1971), it is applied to solve decision problems of choice in the presence of multiple criteria.

Relevant and recent publications on the SMART model include: (Mardani et *al*., 2015), (Praveen, 2014), (Fentahun, 2013), (Schramm and Morais, 2012), (Kabli, 2009), etc. The SMART model is presented as follows:

1) Have the original decision matrix A, A: =(a_{ij}) where a_{ij}: alternative i (i = 1, …, m) associated with the criterion j (j = 1, …, n (n ≥ 3)).

2) Normalize evaluations a_{ij} according to the procedure No. 3 of Roberts (1979):

$$e_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}, \quad 0 \le e_{ij} \le 1$$

Where e_{ij}: normalized evaluation of alternative i associated with criterion j.

3) Have the weight vector of the criteria $W^T = (W_1, W_2, W_3, \ldots, W_n)$ satisfying $\sum_{j=1}^{n} W_j = 1$. According to (Tzeng et *al*., 1998), criteria weights are directly given by a decision maker or indirectly determined starting from the calculation methods.

4) For each considered criterion j, calculate the value of the plausible measure, $u_j(e_{ij})$:

$$u_j(e_{ij}) = \frac{e_{ij}^{max} - e_{ij}}{e_{ij}^{max} - e_{ij}^{min}}, 0 \le u_j(e_{ij}) \le 1$$

5) For each alternative a_i, determine the overall score, U(a_i):

$$U(a_i) = \sum W_j . u_j(e_{ij}); \; i = 1, \ldots, m \text{ et } j = 1, \ldots, n(n \ge 3).$$

6) Rank alternatives a_i in ascending order of U(a_i).

7) Select the best solution.

2.4 Methods for calculating criteria weights

In this article, the weights of the criteria are determined using the neutral methods below:

2.4.1 Standard deviation method (Fleiss, 1981):

The standard deviation method is used to determine the weights of the criteria, it is presented as follows:

1) Have the original decision matrix A, A: = (a_{ij}) where a_{ij}: alternative i (i = 1, …, m) associated with the criterion j (j = 1, …, n (n ≥ 3)).

2) Normalize evaluations a_{ij} according to the procedure No. 3 of Roberts (1979):

$$e_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}, 0 \le e_{ij} \le 1$$

Where e_{ij}: normalized evaluation of alternative i associated with criterion j.

3) For each considered criterion j, calculate the average arithmetic value, $\overline{e_{ij}}j$:

$$\overline{e_{ij}}j = \sum_j e_{ij} / m$$

Where m: number of alternatives considered in the decision matrix A.

4) For each considered criterion j, calculate the standard deviation, S_j:

$$S_j = \sqrt{\frac{\sum\left(e_{ij} - \overline{e_{ij}}j\right)^2}{m}}$$

5) For all the considered criteria j (j = 1, …, n (n ≥ 3)), calculate the sum of the standard deviations, $\sum_{j=1}^{n} S_j$:

$$\sum_{j=1}^{n} S_j = S_1 + \ldots + S_n$$

6) Determine the weight of the criterion j (W_j) according to the following formula:

$$W_j = \frac{S_j}{\sum_{j=1}^{n} S_j}, \; j = 1, \ldots, n(n \ge 3).$$

2.4.2 Entropy method (Zeleny, 1982):

The entropy method is part of the methods of the criteria weights elicitation, it is presented as follows:

1) have the original decision matrix A, A: = (a_{ij}) where a_{ij}: alternative i (i = 1, …, m) associated with the criterion j (j = 1, …, n (n ≥ 3)).

2) Normalize evaluations a_{ij} according to the procedure No. 3 of Roberts (1979):

$$e_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}, 0 \le e_{ij} \le 1$$

Where e_{ij}: normalized evaluation of alternative i associated with criterion j.

3) For each considered criterion j, calculate the entropy, E_j:

$$E_j = -\frac{1}{\log m} . \sum\left(e_{ij} . \log e_{ij}\right)$$

Where m: number of alternatives considered in the decision matrix A.

4) For each considered criterion j, calculate the measure of dispersion, D_j:

$$D_j = 1 - E_j$$

5) For all the considered criteria j (j = 1, …, n (n ≥ 3)), calculate the sum of the measures of dispersion, $\sum_{j=1}^{n} D_j$:

$$\sum_{j=1}^{n} D_j = D_1 + \ldots + D_n$$

6) Determine the weight of the criterion j (W_j) according to the following formula:

$$W_j = \frac{D_j}{\sum_{j=1}^{n} D_j}, \; j = 1, \ldots, n(n \ge 3).$$

2.4.3 Statistical method (Diakoulaki et *al.,* 1992):

The statistical method, which is a method for calculating the weights of the criteria, is based on the standard deviation and the correlation coefficient. It is presented as follows:

1) Have the original decision matrix A, A: = (a_{ij}) where a_{ij}: alternative i (i = 1, …, m) associated with the criterion j (j = 1, …, n (n ≥ 3)).

2) Normalize evaluations a_{ij} according to the procedure No. 3 of Roberts (1979)

$$e_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}, 0 \le e_{ij} \le 1$$

Where e_{ij}: normalized evaluation of alternative i associated with criterion j.

3) For each considered criterion j, calculate the average arithmetic value, $\overline{e_{ij}}j$:

$$\overline{e_{ij}}j = \sum_j e_{ij} / m$$

Where m: number of alternatives considered in the decision matrix A.

4) For each considered criterion j, calculate the standard deviation, S_j:

$$S_j = \sqrt{\frac{\sum\left(e_{ij} - \overline{e_{ij}}j\right)^2}{m}}$$

5) Determine the weight of the criterion j (W_j) according to the following formula:

$$W_j = S_j . \sum\left(1 - r_{jk}\right), j = 1, \ldots, n\left(n \ge 3\right) \text{ et } k \ne j.$$

Where r_{jk}: coefficient of correlation between criteria j and k.

2.4.4 Total variation method (Bouhedja and Pousin, 2011):

The total variation method is used to determine the weights of the criteria. It is based on the linear distance and presented as follows:

1) Have the original decision matrix A, A: = (a_{ij}) where a_{ij}: alternative i (i = 1, …, m) associated with the criterion j (j = 1, …, n (n ≥ 3)).

2) Normalize evaluations a_{ij} according to the procedure No. 3 of Roberts (1979):

$$e_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}, 0 \le e_{ij} \le 1$$

Where e_{ij}: normalized evaluation of alternative i associated with criterion j.

3) For each considered criterion j, calculate the sum of the linear distances between the pairs $\left(e_{ij}, e_{kj}\right), \sum_j d_j$:

$$\sum_j d_j = \left|e_{1j} - e_{2J}\right| + \ldots + \left|e_{(m-1)j} - e_{mj}\right|$$

i = 1, …, m–1; k = i + 1; j = 1, …, n (n ≥ 3).

4) For all the considered criteria j (j = 1, …, n (n ≥ 3)), calculate the sum of the sums of the linear distances between all pairs $\left(e_{ij}, e_{kj}\right), \sum_{j=1}^{n}\sum_j d_j$

$$\sum_{j=1}^{n}\sum_j d_j = \sum_1 d_1 + \ldots + \sum_n d_n$$

5) Determine the weight of the criterion j (W_j) according to the following formula:

$$W_j = \frac{\sum_j d_j}{\sum_{j=1}^{n}\sum_j d_j}, j = 1, \ldots, n\left(n \ge 3\right).$$

3. Illustrative application

To analyze the complete preorders provided by the Borda and Condorcet methods and the SMART model with criteria weights elicited using the neutral methods, an illustrative application is performed. It concerns a public landfill site selection problem whose data are as follows: "The Technical Service (TS) of a municipality seeks to localize a non-arable site for being useful as public landfill. After the commodo and incommodo study, the TS identified seven potential sites which could be lands for storing municipal wastes. The information collected by the TS, in order to solve the public landfill site selection problem, is presented in the decision matrix A (see Table 1).

Let us note that the illustrative application discussed here is not a case study, but rather a numerical example (adapted) for the analysis of the complete preorders provided by the aggregation methods envisaged in this article.

The criteria selected for the illustrative application form a family of criteria relating to the environment domain. This family of criteria satisfies five conditions imposed by (Keeny and Raiffa, 1976) which are: exhaustivity, non-redundancy, operability, decomposability and minimality.

– The criterion of distance: it represents the path between the municipality and the potential site that will serve as public landfill (criterion to be minimized).

– The criterion of transport: it represents the cost of hauling of the household wastes per tonne per kilometer. As it is known, if the distance decreases, the transport cost decreases and vice versa (criterion to be minimized).

– The criterion of residents' opinion: it represents the opinion expressed by the local residents regarding the implementation of a public landfill in the vicinity of their homes (criterion to be maximized).

Table 1. Original decision matrix A Public landfill site selection

Potential sites a_i	Criteria C_j			
	C_1 criterion of distance, Km	C_2 Criterion of transport, 10^3 DA/t/Km	C_3 Criterion of residents' opinion,	C_4 Criterion of emission of the N_2O flux by the soil, Kg N. ha^{-1}. $year^{-1}$
a_1	10	1.5	Favorable	2.8
a_2	7	1.9	Unfavorable	3.5
a_3	13	1.1	Favorable	1.8
a_4	9	1.4	Favorable	1.6
a_5	12	1.7	Unfavorable	2.3
a_6	8	1.6	Favorable	3.2
a_7	11	1.2	Unfavorable	3.4

DA: Algerian Dinar (Currency of Algeria).

Note:

In case of categorical refusal expressed by the residents concerning a given site, the decision-maker, for the sake of maintaining the public order, does not insert it in the decision matrix A, and consequently this site in dispute will not be taken in consideration.

– The criterion of emission of the N_2O flux by the soil: it represents the quantity of Nitrogen per hectare per year dissipated by the non-arable soil in the atmosphere. Specialists in the field of the environment report that Nitrogen Protoxide (N_2O) is a powerful greenhouse gas (Petitjean, 2013; Viard et *al.*, 2013; etc.). This criterion is therefore to be minimized.

3.1 Final results

3.1.1 Normalisation of potential sites

The normalized evaluations $e_{ij}\left(0 \le e_{ij} \le 1\right)$, according to the procedure No. 3 of Roberts, are presented in Table 2.

The residents' opinion criterion, which is a qualitative criterion, must be converted into a quantitative criterion. For this, we selected a 5-point scale. We assigned the value 1 for "very unfavorable", 2 for "unfavorable", 3 for "favorable", 4 for "very favorable" and 5 for "totally favorable".

The values 2 for "unfavorable opinion" and 3 for "favorable opinion" are inserted into their respective places at the original decision matrix A and normalized.

The obtained results are presented in the column of the criterion of residents' opinion (see Table 2).

3.1.2 Aggregation results according to the classical Borda method

3.1.2.1 Matrix of aggregation results

The score obtained by each potential site a_i and the ranking of all the considered potential sites are shown in Table 3.

3.1.2.2 Complete preorder according to the classical Borda method (see Table 4).

3.1.3 Aggregation results according to the Condorcet method

3.1.3.1 Couples and pairwise relations

For each pair of potential sites considered (a_i, a_k), the relations of preference or non-preference is presented in Table 5.

3.1.3.2 Complete preorder according to the Condorcet method (see Table 6).

3.1.4 Results of plausible measurements according to the SMART model

The values of the plausible measures $u_j\left(e_{ij}\right)$, $0 \le u_j\left(e_{ij}\right) \le 1$, determined according to the SMART model, are presented in Table 7.

Table 2. Normalized decision matrix A

Potential sites a_i	Criteria C_j			
	C_1 Criterion of distance, Min	C_2 Criterion of transport, Min	C_3 Criterion of residents' opinion, Max	C_4 Criterion of emission of the N_2O flux by the soil, Min
a_1	0.14 285	0.14 423	0.16 666	0.15 053
a_2	0.10 000	0.18 269	0.11 111	0.18 817
a_3	0.18 571	0.10 576	0.16 666	0.09 677
a_4	0.12 857	0.13 461	0.16 666	0.08 602
a_5	0.17 142	0.16 346	0.11 111	0.12 365
a_6	0.11 428	0.15 384	0.16 666	0.17 204
a_7	0.15 714	0.11 538	0.11 111	0.18 270

Min means criterion to be minimized; Max means criterion to be maximized

Table 3. Matrix of aggregation results according to Borda

Potential sites a_i	Criteria C_j, score and ranking of potential sites a_i					
	C_1 Criterion of distance, Min	C_2 Criterion of transport, Min	C_3 Criterion of residents' opinion, Max	C_4 Criterion of emission of the N_2O flux by the soil, Min	Score obtained by each potential site a_i	Ranking of potential sites a_i in ascending order of $b(a_i)$
	r_1	r_2	r_3	r_4	$b(a_i)$	
a_1	4	4	2.5	4	- 9.5	3
a_2	7	1	6	1	- 3.0	6
a_3	1	7	2.5	6	- 11.5	2
a_4	5	5	2.5	7	- 14.5	1
a_5	2	2	6	5	- 3.0	6
a_6	6	3	2.5	3	- 9.5	3
a_7	3	6	6	2	- 5.0	5

Table 4. Complete preorder according to the classical Borda method

Aggregation method	Complete preorder
classical Borda	$a_4 > a_3 > a_1 \approx a_6 > a_7 > a_2 \approx a_5$

Table 5. Couples and binary relations between potential sites

Couple (a_i, a_k)	Relation a_i/a_k	Couple (a_i, a_k)	Relation a_i/a_k	Couple (a_i, a_k)	Relation a_i/a_k	Couple (a_i, a_k)	Relation a_i/a_k
(a_1, a_2)	$a_1 > a_2$	(a_2, a_7)	$a_2 < a_7$	(a_4, a_6)	$a_4 > a_6$	(a_6, a_4)	$a_6 < a_4$
(a_1, a_3)	$a_1 < a_3$	(a_3, a_1)	$a_3 > a_1$	(a_4, a_7)	$a_4 > a_7$	(a_6, a_5)	$a_6 > a_5$
(a_1, a_4)	$a_1 < a_4$	(a_3, a_2)	$a_3 > a_2$	(a_5, a_1)	$a_5 < a_1$	(a_6, a_7)	$a_6 > a_7$
(a_1, a_5)	$a_1 > a_5$	(a_3, a_4)	$a_3 < a_4$	(a_5, a_2)	$a_5 > a_2$	(a_7, a_1)	$a_7 < a_1$
(a_1, a_6)	$a_1 > a_6$	(a_3, a_5)	$a_3 > a_5$	(a_5, a_3)	$a_5 < a_3$	(a_7, a_2)	$a_7 > a_2$
(a_1, a_7)	$a_1 > a_7$	(a_3, a_6)	$a_3 > a_6$	(a_5, a_4)	$a_5 < a_4$	(a_7, a_3)	$a_7 < a_3$
(a_2, a_1)	$a_2 < a_1$	(a_3, a_7)	$a_3 > a_7$	(a_5, a_6)	$a_5 < a_6$	(a_7, a_4)	$a_7 < a_4$
(a_2, a_3)	$a_2 < a_3$	(a_4, a_1)	$a_4 > a_1$	(a_5, a_7)	$a_5 < a_7$	(a_7, a_5)	$a_7 > a_5$
(a_2, a_4)	$a_2 < a_4$	(a_4, a_2)	$a_4 > a_2$	(a_6, a_1)	$a_6 < a_1$	(a_7, a_6)	$a_7 < a_6$
(a_2, a_5)	$a_2 < a_5$	(a_4, a_3)	$a_4 > a_3$	(a_6, a_2)	$a_6 > a_2$	*	*
(a_2, a_6)	$a_2 < a_6$	(a_4, a_5)	$a_4 > a_5$	(a_6, a_3)	$a_6 < a_3$	*	*

> means "preferred to" and < means "not preferred to".

Table 6. Complete preorder according to the Condorcet method

Aggregation method	Complete preorder
Condorcet	$a_4 > a_3 > a_1 > a_6 > a_7 > a_2 > a_5$

Table 7. Results of plausible measurements

Potential sites a_i	Criteria C_j			
	C_1 Criterion of distance, Min	C_2 Criterion of transport, Min	C_3 Criterion of residents' opinion, Max	C_4 Criterion of emission of the N_2O flux by the soil, Min
a_1	0.5000	0.4999	0	0.3684
a_2	1	0	1	0
a_3	0	1	0	0.8947
a_4	0.6666	0.6249	0	1
a_5	0.1667	0.2499	1	0.6316
a_6	0.8333	0.3750	0	0.1579
a_7	0.3333	0.8749	1	0.0526

Table 8. Weights of criteria elicited by the neutral methods

Methods of elicitation of the criteria weights	Weights of criteria W_j			
	W_1 Criterion weight of distance	W_2 Criterion weight of transport	W_3 Criterion weight of residents' opinion	W_4 Criterion weight of emission of the N_2O flux by the soil
Standard deviation method	0.28 262	0.19 680	0.21 745	0.30 313.
Entropy method	0.21978	0.16 653	0.20 611	0.40 758
Statistical method	0.29 406	0.17 192	0.26 264	0.27 138
Total variation method	0.30 878	0.20 786	0.26 104	0.22 232
Difference ΔW_j: $W_j^{max} - W_j^{min}$	0.08 900	0.04 133	0.05 653	0.18 526

3.1.5 Results of the criteria weights

The weights of the criteria, elicited using neutral methods, are shown in Table 8.

3.1.6 Aggregation results according to the SMART model

The score obtained by each potential site considered a_i - for each method of calculation of the criteria weights considered - is presented in Table 9.

Table 9. Aggregation results according to the SMART model

Potential sites a_i	Elicitation methods of the criteria weights			
	Standard deviation method	Entropy method	Statistical method	Total variation method
	Score	Score	Score	Score
a_1	- 0.35 136	- 0.34 328	- 0.33 294	- 0.34 019
a_2	- 0.06 517	- 0.01 367	- 0.03 142	- 0.04 774
a_3	- 0.46 801	- 0.53 119	- 0.41 472	- 0.40 676
a_4	- 0.61 450	- 0.65 814	- 0.57 483	- 0.55 804
a_5	- 0.07 029	- 0.12 955	- 0.00 073	- 0.01 722
a_6	- 0.35 716	- 0.30 993	- 0.35 236	- 0.37 034
a_7	- 0.06 486	- 0.03 426	- 0.00 005	- 0.03 541

3.1.6.1 Ranking of potential sites according to the SMART Model

The ranking of the considered potential sites being useful like public landfills is shown in Table 10.

Table 10. Ranking of potential sites according to the SMART Model

Potential sites a_i	Elicitation methods of the criteria weights			
	Standard deviation method	Entropy method	Statistical method	Total variation method
	Rank	Rank	Rank	Rank
a_1	4	4	3	4
a_2	6	5	7	5
a_3	2	2	2	2
a_4	1	1	1	1
a_5	5	6	5	7
a_6	3	3	4	3
a_7	7	7	6	6

3.1.6.2 Complete preorders provided by the SMART model (see Table 11).

Table 11. Complete preorders according to the SMART model

SMART model	Complete preorders
With the standard deviation method	$a_4 > a_3 > a_6 > a_1 > a_5 > a_2 > a_7$
With the entropy method	$a_4 > a_3 > a_6 > a_1 > a_2 > a_5 > a_7$
With the statistical method	$a_4 > a_3 > a_1 > a_6 > a_5 > a_7 > a_2$
With the total variation method	$a_4 > a_3 > a_6 > a_1 > a_2 > a_7 > a_5$

3.2 Discussion of results

The analysis of the six complete preorders, provided by the classical Borda and Condorcet methods, and by the SMART model (with four variants of the criteria weights elicited by the neutral methods above-mentioned), clearly shows that the potential site $\{a_4\}$ is the solution to the posed problem and is at the head of each complete preorder obtained (see Tables 4, 6 and 11).

The Technical Service of the municipality, as decision-maker, can therefore select the potential site $\{a_4\}$ for solving the problem of choice of a site to store the municipality household wastes.

Moreover, the worst alternative is not clearly identified; sometimes, it is designated by the alternative $\{a_5\}$ in the case of the classical Borda and Condorcet methods and the SMART model with the criteria weights elicited by the total variation method (see Tables 4, 6 and 11). Sometimes, it is presented by the alternative $\{a_2\}$ in the case of the SMART model with the criteria weights elicited by the statistical method (see Table 11). Or again, it is given by the alternative $\{a_7\}$ in the case of the SMART model with the criteria weights elicited respectively by the standard deviation method and the entropy method (see Table 11).

Furthermore, the instability of potential sites' ranks, encountered at the level of the six complete preorders (obtained), mainly concerns the alternatives $\{a_2, a_5$ and $a_7\}$. See Tables 4, 6 and 11. From our point of view, the instability of the alternatives' ranks, at the level of the obtained complete preorders, could be explained by the methodological or philosophical conception of each aggregation method envisaged in this article.

Concerning the equal alternatives, only the complete preorder provided by the classical Borda method gives two pairs. Those are the alternatives $\{a_1 \approx a_6\}$ and $\{a_2 \approx a_5\}$. See Table 4.

4. Conclusions

Following this research, the conclusions that we have deduced, are as follows:

1. The methods of Borda and Condorcet, although they have been developed since over two centuries to solve the problems of elections, remain operational for choosing and ranking alternatives (see Tables 4 and 6).

2. According to the complete preorder obtained by the Condorcet method (see Table 6), we can say that this last method can satisfy the axiom of transitivity.

3. The principle of repeatability, relating to the satisfaction of the transitivity axiom by the Condorcet method, remains an open line of research. Because the question is: is there transitivity or not?

4. Neutral methods, determining the weights of criteria, can be used to avoid partiality and subjectivity in the ordinal ranking of criteria by a decision maker. Which ordinal ranking of the criteria is required by the methods that use the decision maker to elicit the criteria weights. Neural methods can contain vague or erroneous perception of the decision maker.

In this article, the results of criteria weights, obtained by neutral methods, are presented in Table 8. In accordance with the Table 8, the differences between the maximum and minimum weights are: 0.08900 for the criterion of distance; 0.04133 for the criterion of transport; 0.05653 for the criterion of residents' opinion and 0.18526 for the criterion of emission of the N_2O flux by the soil.

5. By analyzing those differences, it turns out that they have no impact nor incidence on the result concerning the best alternative, in fact $\{a_4\}$. See Table 10.

6. The instability of the alternatives' ranks, encountered at the level of the six complete preorders, exists. For the illustrative application, it concerns alternatives $\{a_2, a_5$ and $a_7\}$. See Tables 4, 6 and 11.

7. Do the social choice theory and the multi-criteria decision making methodology formally do they have a relationship and do they are recommended for solving decision problems in the presence of multiple criteria? The answer is yes, it comes inter alia from (Vansnick, 1986), (Pomerol and Barba-Romeo, 1993), (Bouyssou et *al*., 2005), etc.

8. For helping the decision maker to select best alternative among a set of the alternatives considered, and in order to avoid any difficulty of choice resulting from the instability of the alternatives' ranks obtained by various aggregation methods, we recommend two variants hereafter:

1) The use of the Borda method or the Condorcet method (not both methods at the same time) or

2) The use of a multi-criteria aggregation method chosen by referring to the practical recommendations of (Guitouni and Martel, 1998) or (Mardani *et al*., 2015).

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