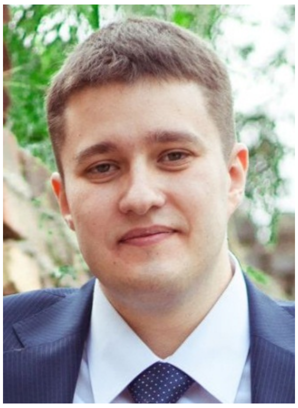
Research of adaptive algorithms of laguerre model parametrical identification at approximation of ore breaking process dynamics



Mykhailenko Oleksii

Assistant Lecturer, Department of Power Supply and Energy Management,
State Institution of Higher Education «Kryvyi Rih National University»,
Kryvyi Rih, Ukraine
epem.mykhailenko@gmail.com
ORCID ID: orcid.org/0000-0003-2898-6652

Abstract
In the article, the questions of dynamic characteristics modeling of breaking process using the Laguerre orthonormal function device are considered. The model parameters were estimated by using of algorithms, which are controlled independently, based on the least square method (algorithm of the least mean squares, normalized algorithm of the least mean squares and recursive algorithm of the least squares). The modification of algorithms considering the structural features of Laguerre model is carried out. The comparative analysis of quality of the model output approximation to the studied object characteristics is conducted.
Key words: LAGUERRE FUNCTIONS, ADAPTIVE ALGORITHMS, PARAMETRICAL IDENTIFICATION, MODELING, ORE BREAKING PROCESS

Relevance of work

It is necessary to have a mathematical model of the object describing its characteristics with high accuracy in order to create the process units control systems of technological processing production units of concentrating production. Considering the nonlinear properties of technological processes of ore concentration and preparation [1], it is reasonable to carry out the modeling of their characteristics by using of the structures based on Volterra series [2], nonlinear input-output models (NARX, NARMAX, NOE), fuzzy neural network models [3, 4] or block-oriented models (Hammerstein, Wiener or Hammerstein-Wiener model) [5]. From the control point of view, the preference is given to the use of the latest models, which have the form of in-series connection of nonlinear blocks, which are described by static functions, in a certain order, and the linear block, which reflects dynamics of controlled object. Such configuration of model results in simplicity of its practical implementation.

In this work, the question of parameters identification of block-oriented structures linear part, which is used for the description of breaking process dynamic characteristics, is considered. At that, the model, which is based on the orthonormal Laguerre functions, is used. This selection is associated with some features of this model. The system of orthonormal Laguerre functions is linear in parameters; it allows using of a method of the least squares when identifying. Due to orthogonality property, the sufficient quality of approximation to the object characteristics is reached at a small order of model. Also, at parametrical identification of the model, there is no need for priori information on time lags and time constants.

The modification of adaptive algorithms of parametrical identification considering the structure features of breaking process model on the basis of the orthonormal Laguerre functions and the quality analysis of approximation of dynamic properties of technological object are the objective of the work.

Materials and researches results

Dynamics of controlled object can be described by the model based on the orthonormal Laguerre function [6], which in state space digital form is represented as follows:

$$L[k+1] = F L[k] k + G u[k], \qquad y[k] = C^T L[k], \tag{1}$$

where p – order of Laguerre model; F -lower triangular matrix of the size ($p \times p$); G - column vector of the size ($p \times 1$); $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_p \end{bmatrix}^T$ – model parameters vector; $L[k] = \begin{bmatrix} l_1[k] & l_2[k] & \cdots & l_p[k] \end{bmatrix}^T$ – the state vector consisting of Laguerre functions:

$$F = \begin{bmatrix} \psi & 0 & 0 & \cdots & 0 \\ \vartheta & \psi & 0 & \ddots & 0 \\ -\psi\vartheta & \vartheta & \psi & \ddots & \vdots\, 0 \\ \vdots & \vdots & & \ddots & 0 \\ (-\psi)^{p-2}\vartheta & (-\psi)^{p-3}\vartheta & \cdots & \vartheta & \psi \end{bmatrix}; \; G = \sqrt{\vartheta}\begin{bmatrix} 1 & -\psi & \psi^2 & \cdots & (-\psi)^{p-1} \end{bmatrix}^T, \tag{2}$$

where $\vartheta = \left(1-\psi^2\right)$, ψ – scale coefficient.

The problem of parametrical identification of a Laguerre network (1) of order p is reduced to determination of vector components of coefficients C and scale coefficient ψ.

In order to estimate the accuracy of approximation of breaking process characteristics, we use the mean-square error of the form:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(y[k+i] - \hat{y}_i[k+i]\right)^2, \tag{3}$$

where $y[k+i]$ – vector component of the object Y initial value; $\hat{y}[k+i]$ – vector component of the initial value of model of process $\hat{Y} = \left[\hat{y}[k] \quad \hat{y}[k+1] \quad \dots \quad \hat{y}[k+N]\right]^T$; N – test set volume.

For computing experiments carrying out, the analytical model of breaking process [7] was linearized via “cone rotation rate - coefficient of variation of fineness properties”. And let us use the obtained mathematical description for test set formation. The quantization interval is selected equal to Δt = 0.1 s, which allows validated modeling of technological process lag effect. As a result, it was created the test set consisting of N = 2000 couples of values. At the input of linearized model, the test stochastic sequence $U = \left[u[k] \quad u[k+1] \quad \dots \quad u[k+N]\right]^T$ is fed.

It is distributed under the uniform law on an interval $\{u[k+i] \in \mathrm{R} \mid 0 < u[k+i] < 1\}$, where k – counting number in set.

The conducted researches of Laguerre model static identification showed that drift of breaking process parameters causes loss of model relevance until the end of identification process. Thus, it is reasonable to use the adaptive algorithms of estimation of model operating parameters in real time.

Considering the structure features of Laguerre model, a number of adaptive algorithms were modified, and researches of model identification quality when using of them were conducted. The identification quality was compared by three indicators: process modeling accuracy, convergence rate and stability. At first, the algorithm of the least mean squares, which does not demand the direct calculation of correlation functions and matrix inversions operations performing [8, 9], was considered. This algorithm is based on calculation of gradient value of a mean-square error of the form (3).

In order to determine the vector components of parameters *C* of Laguerre model of an order *p*, the algorithm of the least mean squares was reduced to the following form.

Algorithm 1. Algorithm of the least mean squares (LMS).
Input: *p* - Laguerre filter order,
μ - convergence rate,
C[0] = *Z*, where *Z* – null vector of the size (p × 1).
For *k* = 0.1, …,*N*

$$e[k] = y[k] - C^T[k]L[k],$$
$$C[k+1] = C[k] + \mu e[k]L[k].$$

The computing experiment graphic results, which show the change nature of mean-square error of approximation of the linearized model transitional characteristics of breaking process by Laguerre system of the 5th order with time constant ψ = 0.97 at various values of parameter μ, are given in Figure 1. The diagrams show that at $\mu = 10^{-9}$ and $\mu = 5\cdot10^{-8}$, algorithm of the least mean squares coincides after more than 1000 readings; therefore we will not consider these data in the analysis. Indicators of algorithm identification and convergence quality for three values μ are shown in Table 1. In the column mean-square error, the average value in "quasi-constant" mode is given, i.e. from the counting number, where the algorithm coincides, and until the end of the set.

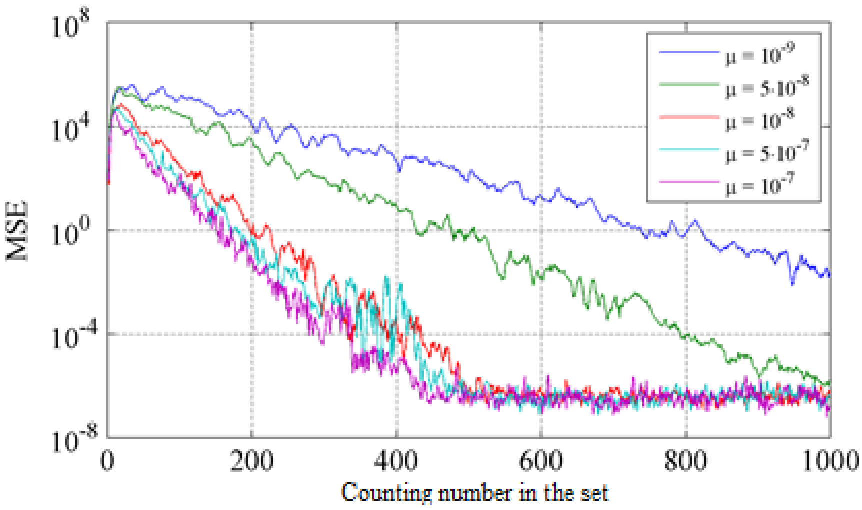


Figure 1. Comparison of identification efficiency of the Laguerre model by algorithm of the least mean squares with various values of parameter μ

The experimental data show that with increase of μ in the range $\{\mu \in \mathrm{R} \mid 10^{-8} \le \mu \le 10^{-7}\}$, the counting number, which is necessary for reaching of mean square error minimum, decreases; consequently, the parameters estimation time is reduced. Thus, the model accuracy is improved.

Table 1. A mean-square error and convergence time when identifying of Laguerre model by the algorithm of the least mean squares

μ	k	t, s	MSE
10^{-7}	486	0.0079	$4.323\cdot10^{-6}$
$5\cdot10^{-7}$	499	0.0086	$5.017\cdot10^{-6}$
10^{-8}	512	0.0092	$6.084\cdot10^{-6}$

Let us note that in comparison with the use of values $\{\mu \in R \mid 10^{-8} \le \mu \le 10^{-7}\}$, the convergence rate increases almost twice. The best indicators of time and accuracy of identification are reached at $\mu = 10^{-7}$. Additional computing experiments showed that at $\mu > 10^{-7}$, the algorithm loses stability (the average value of an error is $1.852 \cdot 10^{28}$). Therefore, the algorithm of the least mean squares demands preliminary control before parametrical identification in real working conditions.

One more drawback of the analyzed adaptive algorithm consists in its sensitivity to the range of input sequence *U*. Thus, it is necessary to specify μ variation range for ensuring of algorithm stability. In order to eliminate this problem, the input signal standardization is carried out [9-11].

The algorithm of normalized least mean squares, where parametrical identification of Laguerre model can be carried out, is of the following form.

Algorithm 2. Algorithm of normalized least mean squares (NLMS)

Input: *p* - Laguerre filter order,

ξ – convergence rate,

$C[0] = Z$, where Z – null vector of the size (p × 1).

For $k = 0.1, \ldots, N$

$$\mu[k] = \frac{\xi}{u^T[k]u[k]},$$

$$e[k] = y[k] - C^T[k]L[k],$$

$$C[k+1] = C[k] + \frac{\mu[k]e[k]L[k]}{L^T[k]L[k]}.$$

The experimental graphic and numerical data characterizing the Laguerre model identification process by adaptive NLMS-algorithm for five values of parameter ξ are given in Figure 2 and in Table 2.

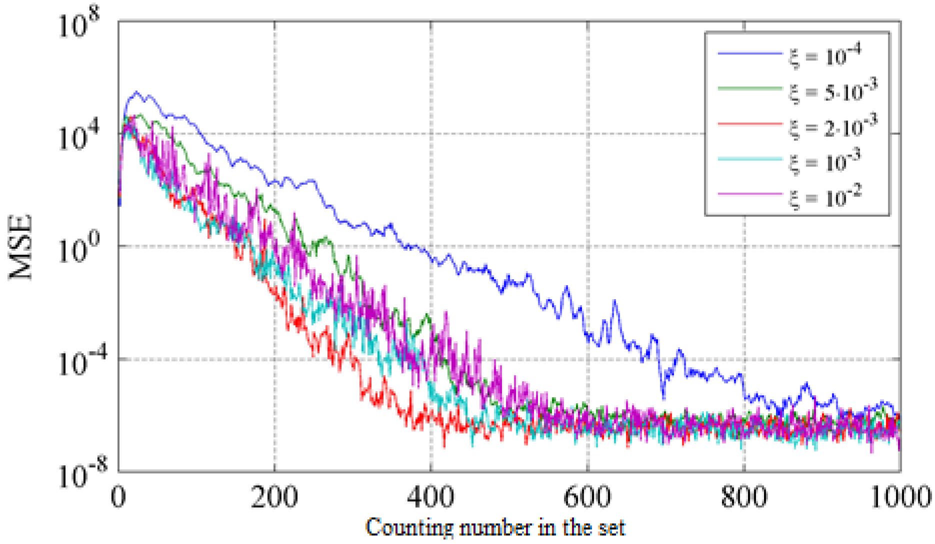


Figure 2. Comparison of identification efficiency of the Laguerre model by algorithm of normalized least mean squares with various values of parameter ξ

The worst indicators are reached at $\xi = 10^{-4}$. Within the values $\{\xi \in R \mid 5\cdot10^{-3} \le \xi \le 2\cdot10^{-3}\}$, the convergence rate increases from 512 to 318 readings, i.e. by 38.9% and by 54.6% in comparison with $\xi = 10^{-4}$. Thus, the identification time is reduced (from 12.2 ms to 9.3 ms), and also the accuracy of orthonormal basis functions system (OBF system) increases from $11.2\cdot10^{-6}$ to $3.474\cdot10^{-6}$. Further increase of $\xi > 2\cdot10^{-3}$ causes the convergence accuracy and rate reduction.

Table 2. A mean-square error and convergence time when identifying of Laguerre model by the algorithm of normalized least mean squares

ξ	k	t, s	MSE
10^{-2}	571	0.0134	$3.996 \cdot 10^{-6}$
10^{-3}	469	0.0108	$5.263 \cdot 10^{-6}$
$2 \cdot 10^{-3}$	381	0.0093	$3.474 \cdot 10^{-6}$
$5 \cdot 10^{-3}$	512	0.0122	$11.2 \cdot 10^{-6}$
10^{-4}	839	0.0202	$12.36 \cdot 10^{-6}$

Thus, compared to algorithm of the least mean squares, the considered algorithm keeps the stability at big values of ξ; however, the quality indicators of identification are reduced. Therefore, the algorithm of normalized least mean squares also demands preliminary control before use.

Let us consider the possibility of the use of another type of adaptive algorithms, namely recursive algorithm of the least squares [12], at parametrical identification of Laguerre model. The recursive algorithm of parametrical identification of Laguerre model can be presented as follows:

Algorithm 2.3. Recursive algorithm of the least squares (RLS)

Input: *p* - Laguerre filter order,

λ – "forgetting" coefficient,

$P = \delta I$,

where δ – parameter for estimation of input signal power;

I – unit matrix.

For $k = 0.1,\ldots,N$

$$e[k] = y[k] - L^T[k]C[k],$$

$$P[k] = \frac{1}{\lambda}\left(P[k-1] - \frac{P[k-1]L[k]L^T[k]P^T[k-1]}{\lambda + L^T[k]P^T[k-1]L[k]}\right),$$

$$C[k+1] = C[k] + e[k]P[k]L[k].$$

If necessary, calculate:

$$\hat{y}[k] = C^T[k]L[k],$$

$$e[k] = y[k] - \hat{y}[k].$$

The process of model identification by recursive algorithm of the least squares for four values of λ is represented in Fig. 3. Indicators of estimation quality of algorithm parameters and convergence are shown in Table 3. From diagrams, it can be seen that the algorithm loses stability at λ = 1.5. It is connected with negative determination of the matrix P. In the range of values $\{\lambda \in R \mid 1,05 \le \lambda \le 1,2\}$, the algorithm is steady, the mean-square error is reduced by 77% on average in comparison with λ = 1.5. The identification time is from 2.7 ms (at λ = 1.05) to 3 ms (at λ = 1.2).

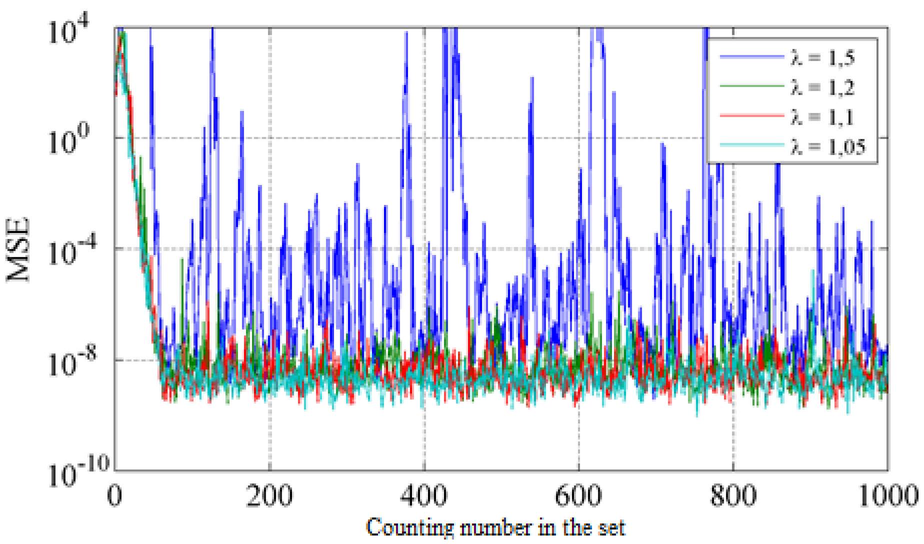


Figure 3. Comparison of identification efficiency of the Laguerre model by the recursive algorithm with various values of parameter λ

It should be noted that there is no significant change of obtained model accuracy and convergence rate in the field of algorithm stability. Thus, the recursive algorithm control process consists in determination of value λ variation limits, i.e. finding of optimum value of algorithm parameter is optional, as in the case of algorithms LMS and NLMS.

Table 3. A mean-square error and convergence time when identifying of Laguerre model by the recursive algorithm of the least mean squares

λ	k	t, s	MSE
1.05	68	0.002729	$1.722 \cdot 10^{-8}$
1.1	70	0.002834	$4.054 \cdot 10^{-8}$
1.2	75	0.003013	$6.874 \cdot 10^{-8}$
1.5	81	0.003193	$18.33 \cdot 10^{-8}$

For evident comparison of estimation quality of Laguerre model parameters by three adaptive algorithms, the diagram of a mean-square error change (Figure 4) for each case is constructed. The algorithms parameters, which provided the best quality and convergence rate at the previous experiments, were use. For algorithms of ordinary and normalized least mean squares, it is $\mu = 10^{-7}$ and $\xi = 2 \cdot 10^{-3}$ respectively, and for recursive algorithm, λ = 1.05. In general, the worst indicators of convergence rate were shown by LMS algorithm, where the identification is higher by 15.1% than in NLMS algorithm, and by 86% than in RLS algorithm. Thus, the high convergence rate (the minimum is reached during 68 readings) provides the lowest time of parametrical identification in spite of the fact that the computing load is increased at recursive algorithm due to increase in the number of actions with matrixes.

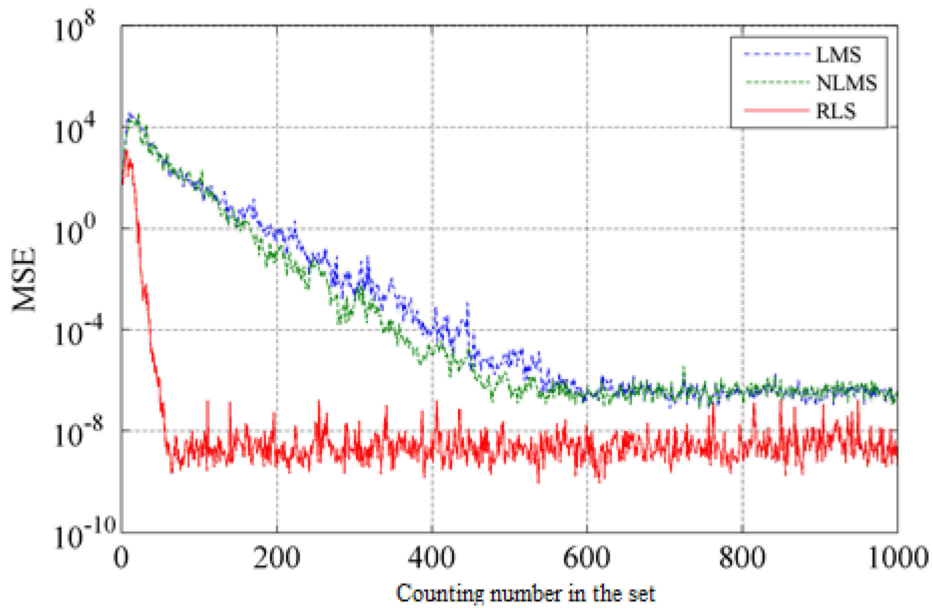


Figure 4. General comparison of identification efficiency of the Laguerre network using adaptive algorithms

At that, the accuracy of LMS and NLMS algorithms differs slightly, unlike the RLS algorithm, which allows reaching of higher accuracy (on average by 99.6% or practically twice) of approximation of breaking process transitional characteristics.

Having used nonlinearized analytical model of breaking process as an object, the quality estimation of approximation of its statics and dynamics by Laguerre linear OBF system, is conducted. The parameters identification was carried out by the adaptive algorithms considered above at their optimum control. The quantization interval and test set volume were not changed. The general quality of parametrical identification was estimated by means of variation coefficient of the mean-square error. In order to simplify the results interpretation, the option, where the width of an unloading slit is constant θ = 10 mm, and the cone rotation speed is changed in the range $\{\omega \in R \mid 6 \leq \omega \leq 14\}$ r/s according to the defined law.

The quality indicators of identification for the selected structure of Laguerre linear model and various adaptive algorithms, which are obtained at approximation of initial coordinates with a control class -9.1 + 6.7 mm and variation coefficient of the fineness properties, are shown in Table 4.

Table 4. The quality indicators of identification of Laguerre linear models with adaptive algorithms of parameters estimation

Model	Parametric identification algorithm	The coefficient of variation of mean-square error CV(RMSE), %	
		Control class output, %	The coefficient of variation of fineness properties
Laguerre OBF system	LMS	75.44	9.15
	NLMS	58.46	8.14
	RLS	52.78	6.37

Transition processes in the opened systems are presented in Fig. 5, 6.

As is seen from diagrams of Fig. 5, the Laguerre linear model describes change of variation coefficient of fineness properties to high precision. At that, the general indicator of modeling efficiency depending on algorithm of identification is within 6.37-9.15% (Table 4).

The recursive algorithm showed the best indicator (CV (RMSECV) = 6.37%), and the algorithm of the least mean squares showed the worst one (CV (RMSECV) = 9.15%). Let us note that the maximum accuracy of process statics modeling is provided by Laguerre model with recursive algorithm of parameters estimation (Fig. 5 d). The error in the steady mode on the temporary interval is $e_{st\ CV}$ = 0.0066. As a comparison, for algorithms of the least mean squares (LMS and NLMS), it is equal to $e_{ст\ CV}$ = 0.0148 and $e_{ст\ CV}$ = 0.016 respectively. Dynamics is described by OBF system adequately in three cases. The transition time of models process corresponds to the object lag effect.

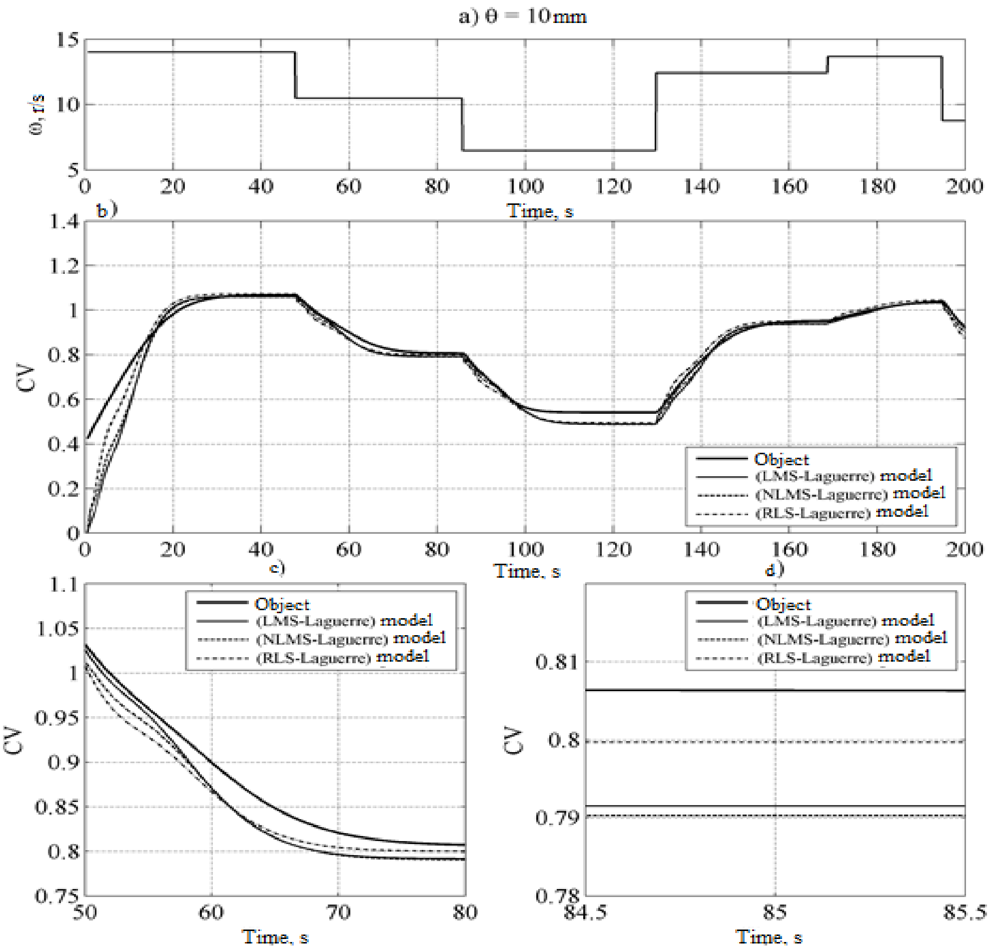


Figure 5. Transitional characteristics of the object and Laguerre linear models with adaptive algorithms of parametrical identification

On the other hand, the approximation of a partial output of the control class (Fig. 6) shows that the Laguerre model describes the breaking process dynamics to high precision (the time of model and object transition process coincides). However, the OBF system cannot reflect adequately the nonlinear static coefficient of strengthening (Fig. 6 d).

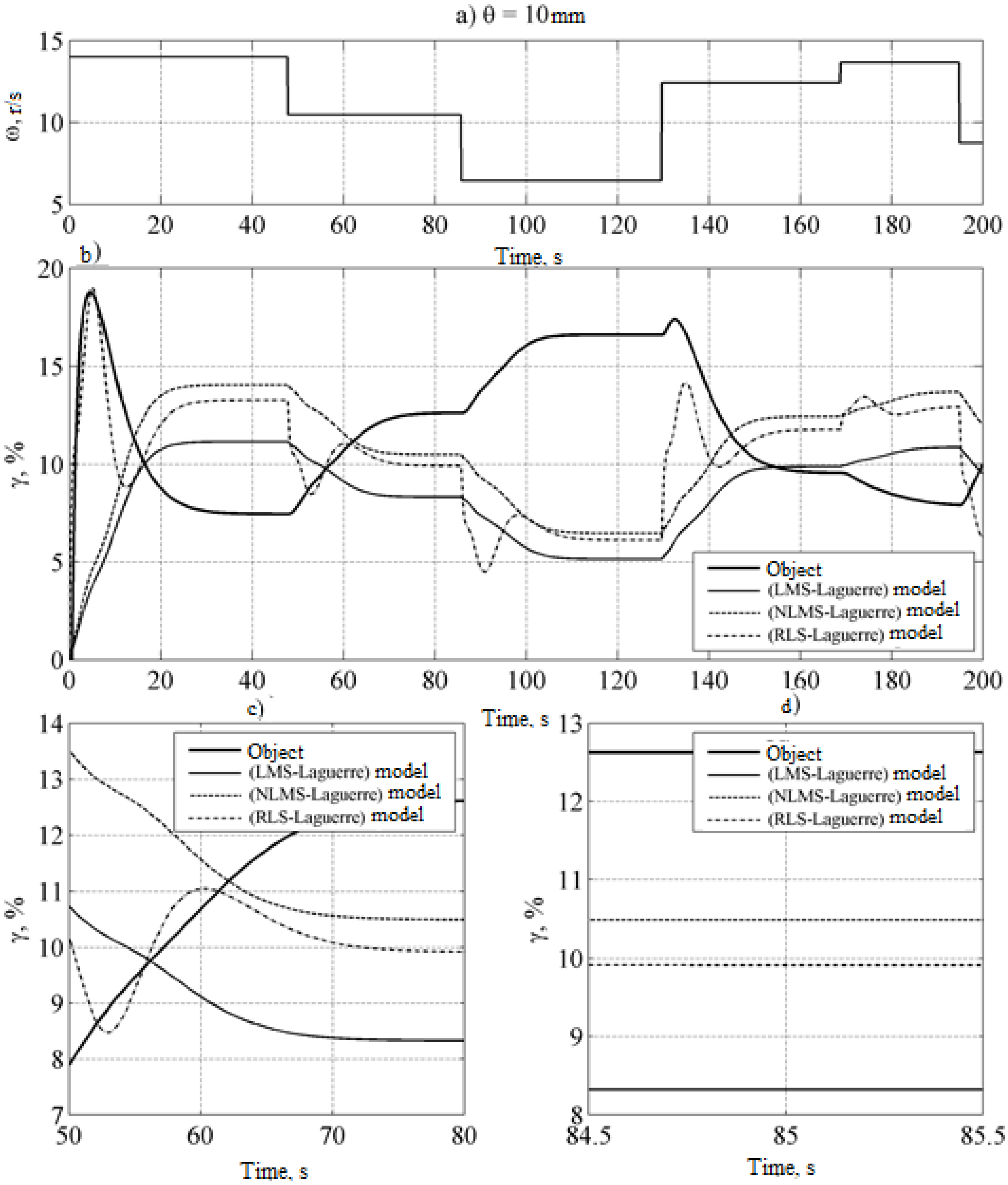


Figure 6. Transitional characteristics of the object and Laguerre linear models with adaptive algorithms of parametrical identification

The error in the steady mode on the temporary interval $\{t \in R \mid 84,5 \le t \le 85,5\}$ is $e_{st\ \gamma}$ = 4.3032 %, $e_{st\ \gamma}$ = 2.1366 % and $e_{st\ \gamma}$ = 2.7152 % for LMS, NLMS and RLS algorithms respectively. At that, the general indicator of modeling quality is minimum CV (RMSEγ) = 52.78% when using of recursive algorithm and maximum CV (RMSEγ) = 75.44% for algorithm of the least mean squares.

Conclusion

The modification of parametrical identification adaptive algorithms of system of the Laguerre orthonormal functions, which are used for approximation of ore breaking process dynamics concidering features of its implementation, is carried out. The conducted comparative analysis of identification algorithms efficiency showed that the recursive algorithm of the least squares shows the best rate of convergence and accuracy. Also, the experimental researches results showed that the Laguerre model can be used when forecasting of the object behavior or developing the regulator in the tasks, which do not demand the high precision, due to “inappreciable” nonlinearity of breaking process accotding to variation coefficient of the fineness properties. However, the low indicators of modeling accuracy with a partial output of a class -9.1 + 6.7 mm allow stating that in this case, the involvement of linear OBF system will negatively affect the quality of the automated regulators. Therefore, it is necessary to consider the possibility of its use in the other complex models structure.

Further researches will be connected with the analysis of breaking process modeling quality by the model based on the block-oriented structures including Laguerre dynamic models.

References

1. Johansson A. (2009) Modeling And Simulation Of Cone Crushers. *In Proceedings of IFACMMM. 2009 Workshop on Automation in Mining, Mineral and Metal Industry*. Viña del Mar, Chile, p.p. 13-18.
2. Singh Th.S.D. (2011) A Comparative Study of Adaptation Algorithms for Nonlinear System Identification based on Second Order Volterra and Bilinear Polynomial Filters. *Measurement*. Vol. 44(10), p.p. 1915-1923. ISSN 0263-2241.
3. Abonyi J. (2000) Identification and Control of Nonlinear Systems Using Fuzzy Hammerstein Models. *Industrial and Engineering Chemistry Research*. Vol. 39, No.11, p.p. 4302-4314.
4. Shchokin V. (2014) Automatization of agglomerative production on the base of application of Neuro-Fuzzy controlling systems of the bottom level [Electronic source]. *Metallurgical and Mining Industry*. No 6, p.p. 32–39. Available at: http://www.metaljournal.com.ua/assets/MMI_2014_6/7-Shchokin.pdf
5. Wills A. (2013) Identification of Hammerstein-Wiener models. *Automatica*. (49), 1, p.p. 70-81.
6. Wang L. Model Predictive Control System Design and Implementation Using MATLAB. London, Springer-Verilag, 2009. 375 p., ISBN 978-1-84882-330-3.
7. Mikhaylenko O. Yu. (2013) Improvement of mathematical model of cone crusher considering crushing chamber division into zones. *Visnik Krivoriz'kogo natsional'nogo universitetu.* Kryviy Rih, SHEI “Kryviy Rih National University”, No 35, p.p. 163-169.
8. Haykin S., Widrow B. Least-Mean-Square Adaptive Filters. Wiley & Sons, 2003. 491 p.
9. Diniz P.S.R. Adaptive Filtering: Algorithms and Practical Implementation [Third Edition]. New York, Springer Science, 2008. 627 p., ISBN 978-0-387-31274-3.
10. Haykin S.O. Adaptive Filter Theory [Third Edition]. New Jersey, Prentice-Hall, 1996. 989 p., ISBN 0-13-322760-X.
11. Douglas S.C. (1994) A Family of Normalized LMS Algorithms. *IEEE Signal Processing Letters*. Vol. SPL-1, No 3, p.p. 49-51.
12. José Antonio Apolinário Jr. QRD-RLS Adaptive Filtering. New York, Springer Science, 2009. 350 p., ISBN 978-0-387-09733-6.